

**CONVERGENCE ANALYSIS OF
PROPORTIONAL-DERIVATIVE -TYPE ILC FOR
LINEAR CONTINUOUS CONSTANT TIME DELAY
SWITCHED SYSTEMS WITH OBSERVATION NOISE
AND STATE UNCERTAINTIES**

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ABSTRACT. This article is concerned with the linear continuous time delay switching system with state uncertainties and observation noise. The goal of this study is to investigate how an internal switching mechanism and the efficacy of a conventional proportional-derivative ILC method is impacted by ambient noise for linear continuous-time switching systems. The findings demonstrate that learning gains and the dynamics of the subsystems, rather than the time-driven switching rule, are primarily responsible for the convergence and robustness of the control method. An appropriate selection of learning gains can ensure the control algorithm's convergence and resilience given any arbitrary time-varying switching rule.

Key Words: Iterative learning control, switched system, dynamical system, time delay, bounded state disturbance, bounded observation noise.

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1. INTRODUCTION

Theoretical research and practical applications of switched systems, which contain a given finite number of subsystems and switching signals, have recently received a great deal of interest. When a link in a network fails or is created, the connection topology frequently changes. Because the reference trajectory is established over a finite period, an ILC system repeatedly performs the same finite duration operation. The duration is referred to as the pass length, and each repetition is referred to as a pass. The system is brought back to its starting point when each pass is finished, so that the next pass may begin. The systems might diverge as a result of the states being reset, which could result in positioning problems. Through repeated completion of the same tasks, an ILC law that combines the knowledge from past passes with that from the current pass can eventually bring the output to the reference trajectory. Many ILC laws, including PID-type, P-type, and D-type ILC, PD-type, have been suggested for various kinds of systems. For instance, a hard disc drive's track following duty, a wafer manufacturing process's temperature management task, etc. When we refer to an extensive system, it means one that is made up of several subsystems that are connected by the system's extensive state vector, but each of which is managed based on its own input and output data. Examples of typical large scale systems include petrochemical operations, electricity systems, networked control systems, etc. According to Chen et al. [1], a system for learning at the beginning that operates between two successive iterations, establishes the starting position at a certain location, and asymptotically converges is suggested.

In 1993, according to Hwang et al. [2] the Derivative type ILC is built for reliable continuous-time systems, which are linear, and by this, we mean that the systems are fed a comparison of tracking error. One of the key issues that occurs with switched systems is stability, which has drawn the most attention. To analyze the stability of switched systems, a variety of techniques have been developed, and numerous helpful stability criteria have been defined in some articles. In order to ensure system stability and improve system performance, the dwell-time approach has been successful in determining the proper switching signals for switched systems that are subject to controlled switching signals

Ruan et al. [4] offer a PID type control update method that follows non-repeated goal trajectories. The technique is demonstrated to be limited in the L_p norm sense. It is well known that many engineering

systems inevitably have time delays. It is possible for the system to become unstable if the time delays are not properly managed. A type of time delay system known as a neutral system depends not only on state delay but also on state derivative delay. In [3], the requirements for switching delay systems' delay-dependent exponential stability are provided [5]-[10]. Due to its hybrid nature, a switched system typically does not inherit subsystem characteristics [12].

In some cases, alternating between these reliable sub-systems may even cause the switched system to become unstable. For instance, the stability which is global exponential, trait of all subsystems cannot ensure the switched system has the same stability attribute. Therefore, switched systems are not immediately applicable to typical design and analysis techniques for systems without switching. Evidently, switched systems are rife with uncertainty, which complicates the research of switched systems even further. It is anticipated that adaptive control, which is an effective method for researching ambiguous non-switched systems, will also be useful for research of switched systems with uncertainty [13]-[16]. In reality, this presumption is frequently unfounded [9].

For a class of LCTSSs, which may be recognized by random time-driven switching signals and observation noise interference, the learning performance of a classic PD-type ILC scheme was examined by Xaun Yang et al. in 2018 [18]. A necessary condition of convergence and robustness is derived by incorporating using some lemma, and the impact of switching and noise is examined.

The rest of this essay is structured as follows. Preliminary, concept property and lemma related information are given in Section 2. The tracking effectiveness of a class of linear continuous time delay switched systems with observation noise and state uncertainties using PD- type ILC is examined in section 3. The paper is wrapped up in the final part.

2. Basic and Mathematical Formulation

Take into consideration a class of linear continuous time delay switching systems with state uncertainties:

$$(2.1) \quad \begin{cases} \dot{x}_k(t) &= A_{\sigma(t)}x_k(t) + D_{\sigma(t)}x_k(t - \tau) + B_{\sigma(t)}u_k(t) + \xi_k(t), \\ y_k(t) &= C_{\sigma(t)}x_k(t) + w_{\sigma,k}(t), \quad t \in \Omega = [0, T], \end{cases}$$

here

- (1) $k \in \mathbb{N}$ represent the number of iterations, $\Omega = [0, T]$ denotes the time interval and $t \in \Omega$ denotes variable for time, τ denotes delay in time;
- (2) $x_k(t)$ be element \mathbb{R}^n , which stands for state vector, $u_k(t)$ is an element in \mathbb{R}^m , which stands for input vector and $y_k(t)$ is element in \mathbb{R}^l is output vector.
- (3) $w_{\sigma,k}(t) \in \mathbb{R}^l$ and $\xi_k(t)$ denotes the bounded state disturbance bounded observation noise with $\|w_{\sigma(i),k}\|_p \leq w_{i,0}$ and $\|\xi_k(t)\|_p \leq b_\xi$;
- (4) $A_\sigma(t)$ be the matrix in $\mathbb{R}^{n \times n}$, $B_{\sigma(t)}$ be the matrix in $\mathbb{R}^{n \times m}$ and $C_{\sigma(t)}$ are also matrix in $\mathbb{R}^{l \times n}$, this all type of matrix are known as system matrices;
- (5) $\sigma : [0, T] \rightarrow Q$, $Q = \{1, 2, \dots, q\}$ over a period of time, $[0, T]$ denotes a piecewise constant function which is known as the switching rule.

Without harming generality, it is thought to be characterized as

$$(2.2) \quad \sigma(t) = i = \begin{cases} 1, & t \text{ belong to } [0, t_1), \\ 2, & t \text{ belong to } [t_1, t_2), \\ \vdots & \\ q, & t \text{ belong to } [t_{q-1}, T]. \end{cases}$$

Therefore, the matrices group $(A_{\sigma(t)}, B_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)})$, for $\sigma(t)$ belong to $Q = \{1, 2, \dots, q\}$ are a component of the ensuing the following set

$$\{(A_1, B_1, C_1, D_1), (A_2, B_2, C_2, D_2), \dots, (A_q, B_q, C_q, D_q)\}$$

Satisfied (2.2), the system (2.1) is perhaps reformed as

$$(2.3) \quad \begin{cases} \dot{x}_k(t) &= A_i x_k(t) + B_i u_k(t) + D_i x_k(t - \tau) + \xi_k(t), \\ y_k(t) &= C_i x_k(t) + w_{i,k}(t), \quad t \text{ belong to } \Omega = [0, T], i \in Q. \end{cases}$$

Keep in mind that the dynamic system (2.3) can function repeatedly across the range $[0, T]$ of time, which is finite, even if the precise dynamics may not be known.

Consider the scheme, which is known as *PD*-type ILC as follows:

$$(2.4) \quad u_{k+1}(t) = u_k(t) + \Gamma_{p,i} e_k(t) + \Gamma_{d,i} \dot{e}_k(t), \quad i \in Q = \{1, 2, \dots, q\}, k = 1, 2, 3, \dots$$

is imposed the k^{th} term of error, which is denoted by $e_k(t)$ and define as $e_k(t) = y_d(t) - y_k(t)$, for any t belong to finite time interval $[0, T]$ is known as the tracking error, and $\Gamma_{d,i} \in \mathbb{R}^{m \times l}$ and $\Gamma_{p,i} \in \mathbb{R}^{m \times l}$ are

known as derivative gains and proportional gains, respectively. The purpose is that the output of the system (2.3) asymptotically converges to the given targeted or reference trajectory, which is indicated by $y_d(t)$, in time period $t \in [0, T]$ as exactly as feasible or when the iteration number tends to infinity follows into the vicinity of $y_d(t)$, that is,

$$\lim_{k \rightarrow \infty} \|e_{k+1}(\cdot)\|_p = 0 \text{ or } \lim_{k \rightarrow \infty} \sup \|e_{k+1}(\cdot)\|_p \leq \eta.$$

To object of this problem, to find the sequence $\{u_k(t) : k \in \mathbb{Z}_+\}$ such that $\{y_k(t)\}$ tends to $y_d(t)$ for (2.1) with PD-type ILC scheme(2.2).

Definition 2.1. [18] Consider the vector valued function $g : I \subseteq \mathbb{R}^+ \rightarrow \mathbb{R}^n$ defined by

$$g(t) = [g_1(t), g_2(t), \dots, g_n(t)]^T,$$

its Lebesgue -p norm is defined as

$$\|g(\cdot)\|_p = \left[\int_I \left(\max_{1 \leq j \leq n} \{|g_j(t)|\} \right)^p dy \right]^{\frac{1}{p}}, 1 \leq p \leq \infty.$$

Definition 2.2. [19]

For a given vector valued function $f(t) \in \mathbb{R}^n, g(t) \in \mathbb{R}^n$, the convolution integral is described as

$$(f * g)(t) = \int_I f(t - s)g(s)ds.$$

From definition (2.1) and (2.2), The convolution integrals generalized Young inequality (GYI) is stated as

$$(2.5) \quad \|f(\cdot)\|_q \|g(\cdot)\|_p \geq \|(f * g)(\cdot)\|_r,$$

for all $1 \leq p, q, r < \infty$ satisfying

$$1/r = 1/p + 1/q.$$

In particular, if p and r are equal, then inequality (2.5) , we get

$$(2.6) \quad \|f\|_1 \|g\|_p \geq \|f * g\|_p,$$

when $p = r$.

The following are the system’s (2.3) basic presumptions:

Assumption 1: Every operation begins at the same starting place. In this paper, it’s thought to be so $y_d(0) = y_k(0)$, for all $k = 1, 2, \dots$.

Assumption 2: The given targeted or reference or desired output $y_d(t)$ is invariant in the process of iteration over a time interval $[0, T]$.

Assumption 3: The switched sequence $\sigma(t)$ still maintains iteration invariance and at the first iteration, it is randomly chosen.

Assumption 4: For every $t \in [0, T]$, there is $\Delta x_k(-t) = 0$.

Assumption 5: Over every time sub-interval $[t_{i-1}, t_i], i \in Q$, the observation noise is arbitrarily constrained, it means, $w_{i,k}(t) \leq w_{i,0}$ where, each time subinterval's value of $w_{i,0}$ is to little enough non-negative constant.

Assumption 6: The state uncertainty (disturbance) is bounded, that is, $\|\xi_k\|_p \leq b_\xi$.

Assumption 7: Regarding the specified intended result $y_d(t)$, the only thing present is a desired control input $u_d(t)$ and a desired $x_d(t)$ s. t.

$$\begin{cases} \dot{x}_d(t) &= A_i x_d(t) + D_i x_d(t - \tau) + B_i u_d(t) + \xi_d(t), \\ y_d(t) &= C_i x_d(t), \quad t \in \Omega = [0, T], i \in Q. \end{cases}$$

Here τ denotes the time delay so that the dwell times of every subsystem exceed the delay times. That is,

$$\tau < t_i - t_{i-1}, \quad \forall i \in Q.$$

3. Main Results

Lemma 3.1. [19] *Assume that $\{b_k\}$ is a positive sequence of a real sequence defined as follows:*

$$b_k \leq c_1 b_{k-1} + c_2 b_{k-2} + \cdots + c_n b_{k-n} + \varepsilon_k, \quad k = n+1, n+2, \dots,$$

with the starting value b_l for every $l = 1, 2, \dots, n$, where $\{\varepsilon_k\}$ is another specified real sequence. If the coefficient satisfy $c_j \geq 0$ and

$$c = \sum_{j=1}^n c_j < 1,$$

then the $\limsup_{k \rightarrow \infty} \varepsilon_k \leq \varepsilon$ implies that

$$\limsup_{k \rightarrow \infty} b_k \leq \frac{\varepsilon}{1 - c}.$$

In particular, $\lim_{k \rightarrow \infty} b_k = 0$, provided that $\varepsilon = 0$.

Theorem 3.2. Consider the scheme (2.4) that is imposed on the system (2.3), which is defined by the switching rule (2.4) and is affected by uncertainties and noise. Assume that the system (2.3) satisfies assumptions from A1 to A7. If the A_i, B_i, C_i and D_i are system dynamics together with the learning gains $\Gamma_{d,i}$ and $\Gamma_{p,i}$ satisfy

$$(3.1) \quad \|C_i \exp(A_i \cdot (\cdot))(A_i B_i \Gamma_{d,i} + B_i \Gamma_{p,i})\|_1 + \|I - C_i B_i \Gamma_{d,i}\|_\infty = \rho_i < 1,$$

for every sub-system, then the system output $y_k(t)$ can approach the neighborhood of the targeted trajectory $y_d(t)$ in the whole time interval, as the iteration num tends to infinity.

Proof. Firstly, consider the input control signal $u_k(t)$ in the k^{th} trial over time sub interval $[t_{i-1}, t_i](i \in Q)$, the state response trajectory of the system (2.3) is formally represented as

$$\begin{aligned} x_{k+1}(t) &= \exp(A_i \cdot (t - t_{i-1}))x_{k+1}(t_{i-1}) \\ &+ \int_{t_{i-1}}^t \exp(A_i \cdot (t - s))D_i x_{k+1}(s - \tau)(s)ds \\ &+ \int_{t_{i-1}}^t \exp(A_i \cdot (t - s))B_i u_{k+1}(s)ds \\ &+ \int_{t_{i-1}}^t \exp(A_i \cdot (t - s))\xi_{k+1}(s)ds. \end{aligned}$$

Using the recursive relationship of tracking errors, the tracking error $e_{k+1}(t)$ is therefore described as follows:

$$\begin{aligned}
e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\
&= y_d(t) - y_k(t) - [y_{k+1}(t) - y_k(t)] \\
&= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&\quad - C_i \int_{t_i}^t \exp(A_i \cdot (t - s)) D_i [x_{k+1}(s - \tau) - x_k(s - \tau)] ds \\
&\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) B_i [u_{k+1}(s) - u_k(s)] ds \\
(3.2) \quad &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) [\xi_{k+1}(s) - \xi_k(s)] ds - (w_{i,k+1}(t) - w_{i,k}).
\end{aligned}$$

Now, we consider the PD type ILC as an updating law (2.4), which is a substitute in the above equation (3.2), we can easily calculate as follows:

$$\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&\quad - C_i \int_{t_i}^t \exp(A_i \cdot (t - s)) D_i [x_{k+1}(s - \tau) - x_k(s - \tau)] ds \\
&\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) B_i [\Gamma_{p,i} e_k(s) + \Gamma_{d,i} \dot{e}_k(s)] ds \\
&\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) [\xi_{k+1}(s) - \xi_k(s)] ds - (w_{i,k+1}(t) - w_{i,k}) \\
&= e_k(t) - C_i \exp(A_i \cdot (t - t_{i-1}))(x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
&\quad - C_i \int_{t_i}^t \exp(A_i \cdot (t - s)) D_i [x_{k+1}(s - \tau) - x_k(s - \tau)] ds \\
&\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) B_i \Gamma_{p,i} e_k(s) ds \\
&\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
(3.3) \quad &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t - s)) B_i \Gamma_{d,i} \dot{e}_k(s) ds - (w_{i,k+1}(t) - w_{i,k}).
\end{aligned}$$

By using the partial integration approach, it is possible to rearrange the last term in equation (3.3) to become as follows:

$$\begin{aligned}
 C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t-s)) B_i \Gamma_{d,i} \dot{e}_k(s) ds \\
 &= C_i \exp(A_i \cdot (t-s)) B_i \Gamma_{d,i} e_k(s) \Big|_{s=t_{i-1}}^{s=t} \\
 &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t-s)) (A_i B_i \Gamma_{d,i} \\
 &\quad + B_i \Gamma_{p,i}) e_k(s) ds.
 \end{aligned}
 \tag{3.4}$$

Substituting (3.4) into (3.3) yields

$$\begin{aligned}
 e_{k+1}(t) &= e_k(t) - C_i \exp(A_i \cdot (t-t_{i-1})) (x_{k+1}(t_{i-1}) - x_k(t_{i-1})) \\
 &\quad - C_i \int_{t_i}^t \exp(A_i \cdot (t-s)) D_i [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\
 &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t-s)) B_i \Gamma_{p,i} e_k(s) ds \\
 &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
 &\quad - C_i \exp(A_i \cdot (t-s)) B_i \Gamma_{d,i} e_k(s) \Big|_{s=t_{i-1}}^{s=t} \\
 &\quad - C_i \int_{t_{i-1}}^t \exp(A_i \cdot (t-s)) (A_i B_i \Gamma_{d,i} + B_i \Gamma_{p,i}) e_k(s) ds \\
 &\quad - (w_{i,k+1}(t) - w_{i,k}).
 \end{aligned}
 \tag{3.5}$$

Step 1: If t belongs to the first sub-interval $t \in \Omega_1$. The first subsystem is turned on in this situation. Taking $t_0 = 0$, the tracking error's

recursive connection (3.4) becomes

$$\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_i \exp(A_1 \cdot (t)) (x_{k+1}(0) - x_k(0)) \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) B_1 \Gamma_{p,i} e_k(s) ds \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
&\quad - C_1 \exp(A_1 \cdot (t-s)) B_1 \Gamma_{d,1} e_k(s) \Big|_{s=t_{i-1}}^{s=t} \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) (A_1 B_1 \Gamma_{d,1} \\
(3.6) \quad &\quad + B_1 \Gamma_{p,1}) e_k(s) ds. - (w_{1,k+1}(t) - w_{1,k}).
\end{aligned}$$

Using first assumption A1, which is $(x_{k+1}(0) - x_k(0)) = 0$. Thus, we have

$$\begin{aligned}
e_{k+1}(t) &= (I - C_1 B_1 \Gamma_{d,1}) e_k(t) \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) (A_1 B_1 \Gamma_{d,1} + B_1 \Gamma_{p,1}) e_k(s) ds \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 [x_{k+1}(s-\tau) - x_k(s-\tau)] ds \\
&\quad - C_1 \int_0^t \exp(A_1 \cdot (t-s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
(3.7) \quad &\quad - (w_{1,k+1}(t) - w_{1,k}).
\end{aligned}$$

Since $\Delta w_{1,k}(t) = w_{1,k+1}(t) - w_{1,k}$, then we get as follows:

$$\begin{aligned}
e_{k+1}(t) &= (I - C_1 B_1 \Gamma_{d,i}) e_k(t) - C_1 \int_0^t \exp(A_1(t-s)) (A_1 \cdot B_1 \Gamma_{d,1} \\
&\quad + B_1 \Gamma_{p,1}) e_k(s) ds - C_1 \int_0^t \exp(A_1 \cdot (t-s)) D_1 \Delta x_k(s-\tau) ds \\
(3.8) \quad &\quad - C_1 \int_{t_0}^t \exp(A_1 \cdot (t-s)) \Delta \xi_k(s) ds - \Delta w_{1,k}(t).
\end{aligned}$$

where $\Delta x_k(s-\tau) = x_{k+1}(s-\tau) - x_k(s-\tau)$, $\Delta \xi_k(s) = \xi_{k+1}(s) - \xi_k(s)$ and $\Delta w_{1,k} = w_{1,k+1}(t) - w_{1,k}(t)$. Firstly, applying the Lebesgue-p norm

on two sides of the equation (3.8) and using the definitions (2.1) and (2.2), we get

$$\begin{aligned}
 \|e_{k+1}(\cdot)\|_p &\leq (\|I - C_1 B_1 \Gamma_{d,1}\|_\infty + \|C_1 \exp(A_1(t-s))(A_1 \cdot B_1 \Gamma_{d,i} \\
 &\quad + B_1 \Gamma_{p,1})\|_1) \|e_k(\cdot)\|_p + \|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \|\Delta x_k(s-\tau)\|_p \\
 &\quad + \|\exp(A_1(t-s))\|_p \|\Delta \xi_k(t)\|_p + \|\Delta w_{1,k}(t)\|_p \\
 &\leq \|I - C_1 B_1 \Gamma_{d,1}\|_\infty \|e_k(\cdot)\|_p + \|C_1 \exp(A_1 \cdot (t-s)) \\
 &\quad (A_1 B_1 \Gamma_{d,1} + B_1 \Gamma_{p,1})\|_1 \|e_k(\cdot)\|_p \\
 (3.9) \quad &+ \|C_1 \exp(A_1(t-s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t-s))\|_p b_\xi + 2b_{w_{1,0}}
 \end{aligned}$$

Where $\|\Delta x_k(s-\tau)\|_p \leq \gamma_0$, and we can observe that

$$\begin{aligned}
 \|\Delta \xi_k(t)\|_p &\leq \|\xi_{k+1}(t)\|_p + \|\xi_k(t)\|_p \leq 2b_\xi, \\
 \|\Delta w_{1,k}(t)\|_p &\leq \|w_{1,k+1}(t)\|_p + \|w_{1,k}(t)\|_p \leq 2w_{1,0},
 \end{aligned}$$

So, $\|\Delta \xi_k(t)\|_p$ and $\|\Delta w_{1,k}(t)\|_p$ are bounded by $2b_\xi$ and $2w_{1,0}$ respectively. Taking the supremum of the equation (3.9) with the assumption $\rho_1 < 1$ and applying the Lemma 3.1, we conclude that

$$\begin{aligned}
 \lim_{k \rightarrow \infty} \|e_{k+1}(\cdot)\|_p &\leq \frac{\|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \gamma_0}{1 - \rho_1} \\
 (3.10) \quad &+ \frac{\|C_1 \exp(A_1 \cdot (t-s))\|_p b_\xi + 2b_{w_{1,0}}}{1 - \rho_1} = \frac{\delta_1}{1 - \rho_1},
 \end{aligned}$$

over $[0, t_1)$, where $\delta_1 = \|C_1 \exp(A_1 \cdot (t-s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t-s))\|_p b_\xi + 2b_{w_{1,0}}$. In other words, the 1st sub-system's output can follow the targeted trajectory towards a neighborhood on $\Omega_1 = [0, t_1)$.

Step 2: In the second step, t belongs to the second sub-system, $[t_1, t_2)$. The second subsystem is switched on in this situation. The tracking

error (3.6) is expressed as follows:

$$\begin{aligned}
e_{k+1}(t) &= e_k(t) - C_2 \exp(A_2 \cdot (t - t_1))(x_{k+1}(t_1) - x_k(t_1)) \\
&\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) D_2 [x_{k+1}(s - \tau) - x_k(s - \tau)] ds \\
&\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) [\xi_{k+1}(s) - \xi_k(s)] ds \\
&\quad - C_2 \exp(A_2 \cdot (t - s)) B_2 \Gamma_{d,2} e_k(s) \Big|_{s=t_1}^{s=t} \\
&\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) (A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2}) e_k(s) ds \\
&\quad - (w_{2,k+1}(t) - w_{2,k}) \\
&= (I - C_2 B_2 \Gamma_{d,i}) e_k(t) \\
&\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) (A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2}) e_k(s) ds \\
&\quad - C_2 \exp(A_2 \cdot (t - t_1)) \Delta x_k(t_1) + C_2 \exp(A_2 \cdot (t - t_1)) B_2 \Gamma_{d,2} e_k(t_1) \\
&\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) D_2 \Delta x_k(s - \tau) ds \\
(3.11) \quad &\quad - C_2 \int_{t_1}^t \exp(A_2 \cdot (t - s)) \Delta \xi_k(s) ds + \Delta w_{2,k}(t).
\end{aligned}$$

Where $\Delta x(t_1)$ is equal to $x_{k+1}(t_1) - x_k(t_1)$, $\Delta x_k(s - \tau) = x_{k+1}(s - \tau) - x_k(s - \tau)$, $\Delta \xi_k(s) = \xi_{k+1}(s) - \xi_k(s)$ and $\Delta w_{2,k} = w_{2,k+1}(t) - w_{2,k}(t)$.

Using the generalized Young inequality of the convolution integral and the taking Lebesgue -p norm on both sides of the equation (3.11),

and applying the definition (2.1) and (2.2), we can formulate as

$$\begin{aligned}
& \|e_{k+1}(\cdot)\|_p \\
& \leq (\|I - C_2 B_2 \Gamma_{d,2}\|_\infty + \|C_2 \exp(A_2 \cdot (\cdot))(A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2})\|_1) \|e_k(\cdot)\|_p \\
& + \|C_2 \exp(A_2 \cdot (t - t_1))\|_p \|\Delta x_k(t_1)\|_p + \|C_2 \exp(A_2 \cdot (\cdot)) D_2\|_p \|\Delta x_k(s - \tau)\|_p \\
& + \|C_2 \exp(A_2 \cdot (\cdot))\|_p \|\Delta \xi_k(t)\|_p + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \|e_k(t_1)\|_p \\
& \quad + \|\Delta w_{2,k}(t)\|_p \\
& \leq (\|I - C_2 B_2 \Gamma_{d,2}\|_\infty + \|C_2 \exp(A_2 \cdot (\cdot))(A_2 B_2 \Gamma_{d,2} + B_2 \Gamma_{p,2})\|_1) \|e_k(\cdot)\|_p \\
& + \|C_2 \exp(A_2 \cdot (t - t_1))\|_p \|\Delta x_k(t_1)\|_p + \|C_2 \exp(A_2 \cdot (\cdot)) D_2\|_p \gamma_1 \\
& + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \|e_k(t_1)\|_p + \|C_2 \exp(A_2 \cdot (\cdot))\|_p b_\xi + 2b_{w_{2,0}} \\
& = \rho_2 \|e_k(\cdot)\|_p + \|C_2 \exp(A_2 \cdot (t - t_1))\|_p \|\Delta x_k(t_1)\|_p \\
& \quad + \|C_2 \exp(A_2 \cdot (t - s)) D_2\|_p \gamma_1 \\
(3.12) \quad & + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \|e_k(t_1)\|_p + \|C_2 \exp(A_2 \cdot (t - s))\|_p b_\xi + 2b_{w_{2,0}}.
\end{aligned}$$

where $\|\Delta x_k(s - \tau)\|_p \leq \gamma_1$, $\|\Delta \xi_k\|_p \leq 2b_\xi$, $\|\Delta w_{2,k}\|_p \leq 2b_{w_{2,0}}$. It is seen that the proving procedure starts with the first sub-interval Ω_1 that

$$\begin{aligned}
& \lim_{k \rightarrow \infty} \|e_{k+1}(\cdot)\|_p \\
& \leq \frac{\|C_1 \exp(A_1 \cdot (t - s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t - s))\|_p b_\xi + 2b_{w_{1,0}}}{1 - \rho_1} \\
& = \frac{\delta_1}{1 - \rho_1},
\end{aligned}$$

Where $\delta_1 = \|C_1 \exp(A_1 \cdot (t - s)) D_1\|_p \gamma_0 + \|C_1 \exp(A_1 \cdot (t - s))\|_p b_\xi + 2b_{w_{1,0}}$ satisfies on first subinterval Ω_1 , which implies both $\lim_{k \rightarrow \infty} \sup \|\Delta x_k(t_1)\|_p < \infty$ and $\lim_{k \rightarrow \infty} \sup \|e_k(t_1)\|_p < \infty$ are satisfied. Now indicating $\lim_{k \rightarrow \infty} \sup \|\Delta x_k(t_1)\|_p = \alpha_1$ and $\lim_{k \rightarrow \infty} \sup \|e_k(t_1)\|_p = \beta_1$, the inequality (3.12) can be written as follows:

$$\begin{aligned}
(3.13) \quad & \lim_{k \rightarrow \infty} \sup \|e_{k+1}(\cdot)\|_p \leq \rho_2 \|e_k(\cdot)\|_p + \|C_2 \exp(A_2 \cdot (\cdot))\|_p \alpha_1 \\
& \quad + \|C_2 \exp(A_2 \cdot (\cdot)) D_2\|_p \gamma_1 \\
& \quad + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \beta_1 \\
& \quad + \|C_2 \exp(A_2 \cdot (\cdot))\|_p b_\xi + 2b_{w_{2,0}}.
\end{aligned}$$

Again, applying Lemma 3.1, it follows that

$$\begin{aligned}
\limsup_{k \rightarrow \infty} \|e_{k+1}(\cdot)\|_p &\leq \frac{1}{\rho_2} \|C_2 \exp(A_2 \cdot (\cdot))\|_p \alpha_1 + \|C_2 \exp(A_2 \cdot (\cdot)) D_2\|_p \gamma_1 \\
&\quad + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \beta_1 + \|C_2 \exp(A_2 \cdot (\cdot))\|_p b_\xi \\
&\quad + 2b_{w_{2,0}} \\
(3.14) \qquad &= \frac{\delta_2}{1 - \rho_2}.
\end{aligned}$$

where $\delta_2 = \|C_2 \exp(A_2 \cdot (\cdot))\|_p \alpha_1 + \|C_2 \exp(A_2 \cdot (\cdot)) D_2\|_p \gamma_1 + \|C_2 \exp(A_2 \cdot (\cdot)) B_2 \Gamma_{d,2}\|_p \beta_1 + \|C_2 \exp(A_2 \cdot (\cdot))\|_p b_\xi + 2b_{w_{2,0}}$. Comparably repeating the aforementioned proof procedure for $t \in \Omega_i$, $(i = 1, 2, \dots, q)$ and indicating $\lim_{k \rightarrow \infty} \sup \|\Delta x_k(t_{i-1})\|_p = \alpha_{i-1}$ and $\lim_{k \rightarrow \infty} \sup \|e_k(t_{i-1})\|_p = \beta_{i-1}$, In light of the inequalities, we may say

$$\begin{aligned}
\limsup_{k \rightarrow \infty} \|e_{k+1}(\cdot)\|_p &\leq \frac{\|C_i \exp(A_i \cdot (\cdot))\|_p \alpha_{i-1} + \|C_i \exp(A_i \cdot (\cdot)) D_i\|_p \gamma_{i-1}}{1 - \rho_i} \\
&\quad + \frac{\|C_i \exp(A_i \cdot (\cdot)) B_i \Gamma_{d,i}\|_p \beta_{i-1} + \|C_i \exp(A_i \cdot (\cdot))\|_p b_\xi + 2b_{w_{i,0}}}{1 - \rho_i} \\
(3.15) \qquad &= \frac{\delta_i}{1 - \rho_i},
\end{aligned}$$

satisfied on the time sub-interval Ω_i , $(i = 1, 2, \dots, q)$, where $\delta_i = \|C_i \exp(A_i \cdot (\cdot))\|_p \alpha_{i-1} + \|C_i \exp(A_i \cdot (\cdot)) D_i\|_p \gamma_{i-1} + \|C_i \exp(A_i \cdot (\cdot)) B_i \Gamma_{d,i}\|_p \beta_{i-1} + \|C_i \exp(A_i \cdot (\cdot))\|_p b_\xi + 2b_{w_{i,0}}$. In other words, throughout successive time intervals from Ω_1 to Ω_q , the output can converge into a neighborhood of the targeted or reference or desired output trajectory $y_d(t)$, and it also does for the whole time period Ω . This proof is complete. \square

Remark 3.3. If $x_k(t - \tau) = 0$ and $\xi_k(0) = 0, \forall k \in \mathbb{N}$, then result become same as in [18].

4. CONCLUSION

The impact of traditional PD-type ILC on the LCTDSS with state uncertainties and observation noise has been examined in this study. The findings demonstrate that the control method is convergent, despite the fact that switching may take place at any instant when noise is

present, and resilience may be ensured in the presence of bounded noise. We examine the impact of environmental noise and state time delay on tracking performance. There is also the option to analyze different ILC types for systems with many inputs and outputs that have a nonlinear continuous time delay.

5. CONFLICTS OF INTEREST

There are no conflicts to declare.

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