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ON BIPOLAR COMPLEX FUZZY H_v-SUBGROUPS

SANEM YAVUZ, SERKAN ONAR AND BAYRAM ALI ERSOY

Abstract. The target of this article is to peruse bipolar complex fuzzy subhypergroups (H_v-subgroups) by employing bipolar complex fuzzy sets and hyperstructures besides of this some related properties are debated.
Furthermore, the notion of bipolar complex anti-fuzzy subhypergroups (H_v-subgroups) is presented. Their characteristics and their connections with bipolar complex fuzzy and anti-fuzzy subhypergroups (H_v-subgroups) are proposed, with several examples demonstrating these concepts.

Key Words: Bipolar complex fuzzy set, bipolar complex fuzzy subhypergroup, bipolar complex anti-fuzzy subhypergroup.

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1. Introduction

Fuzzy mathematics constitutes a specialized branch of mathematics connected with the view of fuzzy sets and fuzzy logic, a concept introduced by Zadeh [16]. This theory extends the classical set framework. In traditional set theory, elements are categorized in binary terms, signifying that an element is either a part of the set or is not. However, in notion of fuzzy set, elements are assigned degrees of membership, enabling a gradual assessment of their belongingness to a set. This gradation is achieved through a membership function that operates within the real interval $[0, 1]$.

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∗Address correspondence to S. Yavuz; E-mail: ssanemy@gmail.com.

Following Zadeh's introduction of fuzzy sets, it has ignited a wide array of applications in mathematics and related disciplines. This has provided researchers with compelling reasons to investigate various facets and expand the domain of abstract algebra within the broader context of fuzzy environments. The exploration of fuzzy algebraic structures was initiated with Rosenfeld's introduction of fuzzy subgroups [9]. The examination of fuzzy hyperstructures has become an intriguing research topic within the realm of fuzzy sets. A substantial body of research exists that explores the connections between fuzzy sets and hyperstructures.

As for vision of bipolar fuzzy set, Zhang [17, 18] introduced the concept in 1998, extending the principles of fuzzy sets and intuitionistic fuzzy sets. Bipolar fuzzy set is an extension of fuzzy sets, enabling membership degrees within the range $[0, 1] \times [-1, 0]$. In this framework, positive knowledge denotes what is feasible, while negative knowledge indicates what is unattainable, prohibited, or unquestionably false. On the other hand, in the domain of complex sets, Ramot [10, 11] introduced complex fuzzy sets as a broadening of the view of fuzzy sets. Subsequently, bipolar complex fuzzy set theory [2, 7] was proposed, and its properties were thoroughly investigated.

When it comes to the implementation of mathematics in other scientific domains, it plays a crucial role and has been a key focus of research by experts in hyperstructure theory worldwide in recent decades. Algebraic hyperstructures, which extend classical algebraic hyperstructures, were proposed by Marty [8]. Within usual algebraic hyperstructures, combining two elements produces a distinct element, while with algebraic hyperstructures, it yields a set. Subsequently, numerous works and a wealth of literature have been dedicated to this field. Currently, hyperstructures have wide-ranging applications [3] in different fields of mathematics and computer science, and researchers are studying their potential in numerous countries globally.

When considering H_v-structures, a generalized form of algebraic hyperstructures, Vougiouklis [12, 13, 14] introduced significant contributions. In classical hyperstructure axioms, equality is replaced by non-empty intersection. Recently, various researchers have proposed H_v-structures; for instance, Davvaz et.al. explored both hyperstructures and H_v-structures [1, 4, 5]. Additionally, Davvaz introduced the vision of fuzzy subhypergroups (or H_v-subgroup) within the framework of hypergroups (or H_v-group). A concise overview of fuzzy algebraic hyperstructures can be found in [6].

Some theorems and results in the 3rd section of this study were presented [15] at the *15th Algebraic Hyperstructures and Its Applications Conference-AHA2023*.

The primary objective of this article is to explore the algebraic attributes of subhypergroups using bipolar complex fuzzy set theory. The key contributions of this research can be summarized as follows:

1) Following an Introduction, Section 2 presents various definitions and outcomes related to hyperstructures and bipolar fuzzy hyperstructures.

2) With Section 3, the sight of bipolar complex fuzzy subhypergroups is introduced, and their algebraic properties are thoroughly examined.

3) In Section 4, it proposes the notion of bipolar complex anti-fuzzy subhypergroups, explaining their properties. Additionally, the relationship between bipolar complex fuzzy subhypergroups and bipolar complex anti-fuzzy subhypergroups are elucidated.

2. Preliminaries

In this part, we will establish foundational knowledge requisite for understanding bipolar complex fuzzy H_v-subgroups. To begin, we will provide descriptions and theorems concerning hyperstructures and bipolar fuzzy subhyperstructures, which are crucial for the subsequent analysis.

Definition 2.1. [5] Consider a non-empty set as S. A binary hyperoperation $* : S \times S \to P^*(S)$ is defined on S, where $P^*(S)$ stands for the collection of all non-empty subsets of S. The pair $(S, *)$ is named a hypergroupoid.

In this elucidation, if P and R are two non-empty subsets of S and $s \in S$, we describe:

i) $P * R = \cup_{\substack{p\in P \\ r\in R}} p * r$,

ii) $s * P = \{s\} * P$ and $P * s = P * \{s\}$.

Definition 2.2. [5] A hypergroupoid $(S, *)$ can be categorized as:

- Semihypergroup: Providing for every $\rho, \sigma, \varsigma \in S$, the equation $\rho * (\sigma * \varsigma) = (\rho * \sigma) * \varsigma$ provides,
- Quasihypergroup: Providing for every $\rho \in S$, this is what we get $\rho * S = S = S * \rho$ (This requirement is named as reproduction property),
- Hypergroup: Providing that $(S, *)$ satisfies both semihypergroup and quasihypergroup properties,

- H_v-group: Providing that $(S,*)$ be a quasihypergroup and for every $\rho,\sigma,\varsigma \in S$ intersection of $\rho*(\sigma*\varsigma)$ and $(\rho*\sigma)*\varsigma$ is non-empty.

Definition 2.3. [5] Consider a hypergroup (H_v-group) represented by $(S,*)$, and let K be a subset of S. $(K,*)$ is recognized as a subhypergroup (H_v-subgroup) of $(S,*)$ if for every $k \in K$, we get $k*K = K = K*k$.

Definition 2.4. [16] A fuzzy set characterized on a universe of discourse S is explained by a membership function $\lambda_F(s)$, which appoints a grade of membership to each element s in F. The fuzzy set can be symbolized as follows:

$$F = \{(s, \lambda_F(s)) : s \in S\}$$

, where $\lambda_F(s)$ belongs to the interval $[0,1]$.

Definition 2.5. [7] Consider the universe of discourse S. Representation of bipolar fuzzy set B in S is characterized as follows:

$$B = \{(s, \lambda_B^+(s), \lambda_B^-(s)) : s \in S\}$$

where

$$\lambda_B^+(s) : S \to [0,1]$$
$$\lambda_B^-(s) : S \to [-1,0]$$

As a positive membership function $\lambda_B^+(s)$ signifies the rate of satisfaction of an element s with a attribute corresponding to the bipolar fuzzy set B and as a negative membership function $\lambda_B^-(s)$ indicates the rate of satisfaction of an element s with some implicit counter attribute to B.

Definition 2.6. [7] Suppose that P and R are two bipolar fuzzy sets represented together

$$P = \{(s, \lambda_P^+(s), \lambda_P^-(s)) : s \in S\}$$

and

$$R = \{(s, \lambda_R^+(s), \lambda_R^-(s)) : s \in S\}$$

the set operations of the bipolar fuzzy set are characterized:

(1) $P \cup R = \{(s, \lambda_{P\cup R}^+(s), \lambda_{P\cup R}^-(s)) : s \in S\}$ such that $\lambda_{P\cup R}^+(s) = \max\{\lambda_P^+(s), \lambda_R^+(s)\}$ and $\lambda_{P\cup R}^-(s) = \min\{\lambda_P^-(s), \lambda_R^-(s)\}$.

(2) $P \cap R = \{(s, \lambda_{P\cap R}^+(s), \lambda_{P\cap R}^-(s)) : s \in S\}$ such that $\lambda_{P\cap R}^+(s) = \min\{\lambda_P^+(s), \lambda_R^+(s)\}$ and $\lambda_{P\cap R}^-(s) = \max\{\lambda_P^-(s), \lambda_R^-(s)\}$.

(3) $P^C = \{(s, \lambda^+_{PC}(s), \lambda^-_{PC}(s)) : s \in S\}$ such that $\lambda^+_{PC}(s) = 1 - \lambda^+_P(s)$ and $\lambda^-_{PC}(s) = -1 - \lambda^-_P(s)$.

Definition 2.7. [6] Suppose that $(S, *)$ be a hypergroup (H_v−group), B be a bipolar fuzzy subset of S together positive membership function $\lambda^+_B(s) \in [0, 1]$ and negative membership function $\lambda^-_B(s) \in [-1, 0]$. Providing the following conditions hold, B is called a bipolar fuzzy subhypergroup (H_v−subgroup) of S :

(1) $\min\{\lambda^+_B(\rho), \lambda^+_B(\sigma)\} \leq \inf\{\lambda^+_B(\varsigma) : \varsigma \in \rho * \sigma\}$ for every $\rho, \sigma \in S$,
(2) $\max\{\lambda^-_B(\rho), \lambda^-_B(\sigma)\} \geq \sup\{\lambda^-_B(\varsigma) : \varsigma \in \rho * \sigma\}$ for every $\rho, \sigma \in S$,
(3) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\min\{\lambda^+_B(d), \lambda^+_B(\rho)\} \leq \lambda^+_B(\gamma)$,
(4) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\max\{\lambda^-_B(d), \lambda^-_B(\rho)\} \geq \lambda^-_B(\gamma)$,
(5) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\min\{\lambda^+_B(d), \lambda^+_B(\rho)\} \leq \lambda^+_B(\delta)$,
(6) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\max\{\lambda^-_B(d), \lambda^-_B(x)(\rho)\} \geq \lambda^-_B(\delta)$.

Lemma 2.8. [6] *Assume that $(S, *)$ be a hypergroup (H_v−group), $B = \{(s, \lambda^+_B(s), \lambda^-_B(s)) : s \in S\}$ be a bipolar fuzzy subhypergroup (H_v−subgroup) of S. For every $g_1, g_2, ..., g_n \in S$*

$$\min\{\lambda^+_B(g_1), \lambda^+_B(g_2), ..., \lambda^+_B(g_n)\} \leq \inf\{\lambda^+_B(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

and

$$\max\{\lambda^-_B(g_1), \lambda^-_B(g_2), ..., \lambda^-_B(g_n)\} \geq \sup\{\lambda^-_B(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}.$$

Definition 2.9. [6] Suppose that $(S, *)$ is a hypergroup (H_v−group), B is a bipolar fuzzy subset of S together $\lambda^+_B(s) \in [0, 1]$ and negative membership function $\lambda^-_B(s) \in [-1, 0]$. Providing the following conditions hold, B is called a bipolar anti-fuzzy subhypergroup (H_v−subgroup) of S :

(1) $\sup\{\lambda^+_B(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{\lambda^+_B(\rho), \lambda^+_B(\sigma)\}$ for every $\rho, \sigma \in S$,
(2) $\inf\{\lambda^-_B(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{\lambda^-_B(\rho), \lambda^-_B(\sigma)\}$ for every $\rho, \sigma \in S$,
(3) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\lambda^+_B(\gamma) \leq \max\{\lambda^+_B(d), \lambda^+_B(\rho)\}$,
(4) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\lambda^-_B(\gamma) \geq \min\{\lambda^-_B(d), \lambda^-_B(\rho)\}$,
(5) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\lambda^+_B(\delta) \leq \max\{\lambda^+_B(d), \lambda^+_B(\rho)\}$,

(6) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\lambda_B^-(\delta) \geq \min\{\lambda_B^-(d), \lambda_B^-(\rho)\}$.

Lemma 2.10. [6] *Assume that $(S, *)$ is a hypergroup (H_v-group), $B = \{(s, \lambda_B^+(s), \lambda_B^-(s)) : s \in S\}$ be a bipolar fuzzy subhypergroup (H_v-subgroup) of S. For every $g_1, g_2, ..., g_n \in S$*

$$\max\{\lambda_B^+(g_1), \lambda_B^+(g_2), ..., \lambda_B^+(g_n)\} \geq \sup\{\lambda_B^+(x) : x \in g_1*(g_2*(...)*g_n)...)\}$$

and

$$\min\{\lambda_B^-(g_1), \lambda_B^-(g_2), ..., \lambda_B^-(g_n)\} \leq \inf\{\lambda_B^-(x) : x \in g_1*(g_2*(...)*g_n)...)\}.$$

Theorem 2.11. [6] *Assume that $(S, *)$ is a hypergroup (H_v-group), $B = \{(s, \lambda_B^+(s), \lambda_B^-(s)) : s \in S\}$ is a bipolar fuzzy subset of S. B is a bipolar fuzzy subhypergroup (H_v-subgroup) of S if and only if B^C is a bipolar anti-fuzzy subhypergroup (H_v-subgroup) of S.*

3. Bipolar Complex Fuzzy H_v-subgroups

In this part, we will employ the notion of bipolar complex fuzzy subsets to clarify and analyze bipolar complex fuzzy subhypergroups (H_v-subgroups). Besides, we will delve into their algebraic properties and investigate key theorems.

Definition 3.1. Suppose that $B = \{(s, \lambda_B^+(s), \lambda_B^-(s)) : s \in S\}$ is a bipolar fuzzy set. Bipolar $\pi-$ fuzzy set B_π is characterized as $B_\pi = \{(s, \lambda_{B_\pi}^+(s), \lambda_{B_\pi}^-(s)) : s \in S\}$, where $\lambda_{B_\pi}^+(s) = 2\pi\lambda_B^+(s)$ and $\lambda_{B_\pi}^-(s) = -2\pi\lambda_B^-(s)$ indicate the degree of positive belongingness and negative belongingness of an element s of S, respectively.

Proposition 3.2. *Consider $(S, *)$ be a hypergroup (H_v-group). B be a bipolar fuzzy subhypergroup (H_v-subgroup) of S if and only if a bipolar $\pi-$ fuzzy set B_π be a bipolar $\pi-$ fuzzy subhypergroup (H_v-subgroup) of S.*

Proof. Evidence is explicit. □

Definition 3.3. A bipolar complex fuzzy set B is characterized its positive membership function $\lambda_B^+(s)$ and negative membership function $\lambda_B^-(s)$, both assigned to elements s in a universe of discourse S. These functions indicate a complex-valued rate for both positive and negative memberships in B. According to the definition, $\lambda_B^+(s)$ and $\lambda_B^-(s)$ can take values within the unit disks in the complex plane and are expressed as

$$\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)} \text{ for positive membership function in } B$$
and
$$\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)} \text{ for negative membership function in } B$$

, where $r_B^+(s)$ and $w_B^+(s)$ are in the range $[0,1]$ and $r_B^-(s)$ and $w_B^-(s)$ are in the range $[-1,0]$. Parameter α varies within $[0,2\pi]$. Consequently, we represent the bipolar complex fuzzy set B as:

$$B = \{(s, \lambda_B^+(s), \lambda_B^-(s)) : s \in S\}$$
$$\lambda_B^+(s) : S \to [0,1]e^{i\alpha[0,1]}$$
$$\lambda_B^-(s) : S \to [-1,0]e^{i\alpha[-1,0]}$$

Definition 3.4. Suppose that P and R are two bipolar complex fuzzy sets on the same universe S such that $P = \{(s, \lambda_P^+(s), \lambda_P^-(s)) : s \in S\}$ and $R = \{(s, \lambda_R^+(s), \lambda_R^-(s)) : s \in S\}$. Union of P and R represented by $P \cup R$ is characterized as follows:

$$P \cup R = \{(s, \lambda_{P\cup R}^+(s), \lambda_{P\cup R}^-(s)) : s \in S\}$$
$$\lambda_{P\cup R}^+(s) = \max\{r_P^+(s), r_R^+(s)\}e^{i\alpha \max\{w_P^+(s), w_R^+(s)\}}$$
$$\lambda_{P\cup R}^-(s) = \min\{r_P^-(s), r_R^-(s)\}e^{i\alpha \min\{w_P^-(s), w_R^-(s)\}} \text{ for every } s \in S.$$

Definition 3.5. Suppose that P and R are two bipolar complex fuzzy sets on the same universe S such that $P = \{(s, \lambda_P^+(s), \lambda_P^-(s)) : s \in S\}$ and $R = \{(s, \lambda_R^+(s), \lambda_R^-(s)) : s \in S\}$.
Intersection of P and R represented by $P \cap R$ is characterized as follows:

$$P \cap R = \{(s, \lambda_{P\cap R}^+(s), \lambda_{P\cap R}^-(s)) : s \in S\}$$
$$\lambda_{P\cap R}^+(s) = \min\{r_P^+(s), r_R^+(s)\}e^{i\alpha \min\{w_P^+(s), w_R^+(s)\}}$$
$$\lambda_{P\cap R}^-(s) = \max\{r_P^-(s), r_R^-(s)\}e^{i\alpha \max\{w_P^-(s), w_R^-(s)\}} \text{ for every } s \in S.$$

Definition 3.6. Let B be a bipolar complex fuzzy set,where B is yielded as:

$$B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$$

The complement of B is described:

$$B^C = \{(s, (1 - r_B^+(s))e^{i\alpha(1-w_B^+(s))}, (-1 - r_B^-(s))e^{i\alpha(-1-w_B^-(s))}) : s \in S\}$$

Definition 3.7. Suppose that $P = \{(s, \lambda_P^+(s), \lambda_P^-(s)) : s \in S\}$ and $R = \{(s, \lambda_R^+(s), \lambda_R^-(s)) : s \in S\}$ are two bipolar complex fuzzy subsets of a non-empty set S together the positive membership functions $\lambda_P^+(s) = r_P^+(s)e^{i\alpha w_P^+(s)}$ and $\lambda_R^+(s) = r_R^+(s)e^{i\alpha w_R^+(s)}$ respectively and negative membership functions $\lambda_P^-(s) = r_P^-(s)e^{i\alpha w_P^-(s)}$ and $\lambda_R^-(s) = r_R^-(s)e^{i\alpha w_R^-(s)}$ respectively.

(1) A bipolar complex fuzzy subset P is called homogeneous providing for every $\gamma, \delta \in S$ we get
$r_P^+(\gamma) \leq r_P^+(\delta)$ if and only if $w_P^+(\gamma) \leq w_P^+(\delta)$
and
$r_P^-(\gamma) \geq r_P^-(\delta)$ if and only if $w_P^-(\gamma) \geq w_P^-(\delta)$.

(2) A bipolar complex fuzzy subset P is called homogeneous together R providing for every $\gamma, \delta \in S$ we get
$r_P^+(\gamma) \leq r_R^+(\delta)$ if and only if $w_P^+(\gamma) \leq w_R^+(\delta)$
and
$r_P^-(\gamma) \geq r_R^-(\delta)$ if and only if $w_P^-(\gamma) \geq w_R^-(\delta)$.

Remark 3.8. Suppose that $P = \{(s, \lambda_P^+(s), \lambda_P^-(s)) : s \in S\}$ and $R = \{(s, \lambda_R^+(s), \lambda_R^-(s)) : s \in S\}$ are two bipolar complex fuzzy subsets of a non-empty set S together the membership functions $\lambda_P^+(s) = r_P^+(s)e^{i\alpha w_P^+(s)}$ and $\lambda_R^+(s) = r_R^+(s)e^{i\alpha w_R^+(s)}$ respectively and negative membership functions $\lambda_P^-(s) = r_P^-(s)e^{i\alpha w_P^-(s)}$ and $\lambda_R^-(s) = r_R^-(s)e^{i\alpha w_R^-(s)}$ respectively.

- With $\lambda_P^+(s) \leq \lambda_R^+(s)$, it refers that $r_P^+(s) \leq r_R^+(s)$ and $w_P^+(s) \leq w_R^+(s)$.
- With $\lambda_P^-(s) \geq \lambda_R^-(s)$, it refers that $r_P^-(s) \geq r_R^-(s)$ and $w_P^-(s) \geq w_R^-(s)$.

Throughout work, whole bipolar complex fuzzy sets take into account homogeneous.

Definition 3.9. Suppose that $(S, *)$ is a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subset of S together positive membership function $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and negative membership function $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. Providing that the following requirements are provided, B is named a bipolar complex fuzzy subhypergroup (H_v−subgroup) of S:

(1) $\min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\} \leq \inf\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\}$ for every $\rho, \sigma \in S$,
(2) $\max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\} \geq \sup\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\}$ for every $\rho, \sigma \in S$,
(3) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\min\{\lambda_B^+(d), \lambda_B^+(\rho)\} \leq \lambda_B^+(\gamma)$,
(4) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\max\{\lambda_B^-(d), \lambda_B^-(\rho)\} \geq \lambda_B^-(\gamma)$,
(5) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\min\{\lambda_B^+(d), \lambda_B^+(\rho)\} \leq \lambda_B^+(\delta)$,
(6) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\max\{\lambda_B^-(d), \lambda_B^-(\rho)\} \geq \lambda_B^-(\delta)$.

Example 3.10. Presume that $S = \{\rho, \sigma\}$ and describe as a hypergroup $(S, *)$ by next table:

$*$	ρ	σ
ρ	ρ	S
σ	S	σ

We describe a bipolar complex fuzzy subset B of S as:
$\lambda_B^+(\rho) = 0.3e^{i0}$ and $\lambda_B^-(\rho) = -0.2e^{-\pi/2i}$
$\lambda_B^+(\sigma) = 0.5e^{i\pi/3}$ and $\lambda_B^-(\sigma) = -0.4e^{-\pi i}$

In that case, B is homogeneous bipolar complex fuzzy subhypergroup of S.

Example 3.11. Presume that $S = \{\rho, \sigma, \varsigma\}$ and describe the hypergroup $(S, *)$ by next table:

$*$	ρ	σ	ς
ρ	$\{\rho\}$	$\{\sigma\}$	$\{\varsigma\}$
σ	$\{\sigma\}$	$\{\rho\}$	$\{\sigma, \varsigma\}$
ς	$\{\varsigma\}$	$\{\sigma, \varsigma\}$	$\{\rho\}$

We describe a bipolar complex fuzzy subset B of S as:
$\lambda_B^+(\rho) = 0.5e^{i\pi/2}$ and $\lambda_B^-(\rho) = -0.2e^{-\pi/2i}$
$\lambda_B^+(\sigma) = 0.6e^{i\pi}$ and $\lambda_B^-(\sigma) = -0.3e^{-3\pi/4i}$
$\lambda_B^+(\varsigma) = 0.6e^{i3\pi/2}$ and $\lambda_B^-(\varsigma) = -0.4e^{-\pi i}$

In that case, B is homogeneous bipolar complex fuzzy subhypergroup of S.

Theorem 3.12. *Suppose that $(S,*)$ is a hypergroup (H_v-group), B is a (homogeneous) bipolar complex fuzzy subset of S together positive membership function $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and negative membership function $\lambda_B^-(x) = r_B^-(s)e^{i\alpha w_B^-(s)}$. B is a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S if and only if r_B^+ and r_B^- are bipolar fuzzy subhypergroups (H_v-subgroups) of S and w_B^+ and w_B^- are bipolar $\pi-$fuzzy subhypergroups (H_v-subgroups) of S.*

Proof. ($\Longrightarrow$) : Assume B be a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S. We wish to indicate requirements of description of bipolar fuzzy subhypergroups are yielded for r_B^+ and r_B^-, w_B^+ and w_B^-. For every $\rho, \sigma \in S$, we get

$$\min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\} \leq \inf\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\},$$
$$\max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\} \geq \sup\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\}.$$

Remark 3.8 refers

$$\inf\{r_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{r_B^+(\rho), r_B^+(\sigma)\},$$
$$\sup\{r_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{r_B^-(\rho), r_B^-(\sigma)\},$$

and

$$\inf\{w_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{w_B^+(\rho), w_B^+(\sigma)\},$$
$$\sup\{w_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{w_B^-(\rho), w_B^-(\sigma)\}.$$

Assume $\rho, d \in S$. There exist $\gamma, \delta \in S$ such that $d \in \rho*\gamma$ and $d \in \delta*\rho$. Then,

$$\min\{\lambda_B^+(\rho), \lambda_B^+(d)\} \leq \lambda_B^+(\gamma)$$
$$\max\{\lambda_B^-(\rho), \lambda_B^-(d)\} \geq \lambda_B^-(\gamma)$$

and

$$\min\{\lambda_B^+(\rho), \lambda_B^+(d)\} \leq \lambda_B^+(\delta)$$
$$\max\{\lambda_B^-(\rho), \lambda_B^-(d)\} \geq \lambda_B^-(\delta).$$

Remark 3.8 means the conditions $3,4$ and $5,6$ of description of bipolar fuzzy subhypergroups are yielded for both r_B^+ and r_B^-, w_B^+ and w_B^-.

($\Longleftarrow$) : Let r_B^+ and r_B^- are bipolar fuzzy subhypergroups (H_v-subgroups) of S and w_B^+ and w_B^- are bipolar $\pi-$ fuzzy subhypergroups (H_v-subgroups) of S. We wish to indicate requirements of bipolar complex fuzzy subhypergroups are provided. For every $\rho, \sigma \in S$, we get

$$\inf\{r_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{r_B^+(\rho), r_B^+(\sigma)\},$$
$$\sup\{r_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{r_B^-(\rho), r_B^-(\sigma)\},$$

and

$$\inf\{w_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{w_B^+(\rho), w_B^+(\sigma)\},$$
$$\sup\{w_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{w_B^-(\rho), w_B^-(\sigma)\}.$$

Remark 3.8 refers

$$\min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\} \leq \inf\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\},$$
$$\max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\} \geq \sup\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\}.$$

Presume $\rho, d \in S$. There exist $\gamma, \delta \in S$ such that $d \in \rho*\gamma$ and $d \in \delta*\rho$

$$\min\{r_B^+(\rho), r_B^+(d)\} \leq r_B^+(\gamma) \text{ and } \min\{w_B^+(\rho), w_B^+(d)\} \leq w_B^+(\gamma),$$
$$\max\{r_B^-(\rho), r_B^-(d)\} \geq r_B^-(\gamma) \text{ and } \max\{w_B^-(\rho), w_B^-(d)\} \geq w_B^-(\gamma),$$

and

$$\min\{r_B^+(\rho), r_B^+(d)\} \leq r_B^+(\delta) \text{ and } \min\{w_B^+(\rho), w_B^+(d)\} \leq w_B^+(\delta),$$
$$\max\{r_B^-(\rho), r_B^-(d)\} \geq r_B^-(\delta) \text{ and } \max\{w_B^-(\rho), w_B^-(d)\} \geq w_B^-(\delta).$$

Remark 3.8 means the conditions $3, 4$ and $5, 6$ of description of bipolar complex fuzzy subhypergroups are yielded for B. □

Lemma 3.13. *Suppose that* $(S, *)$ *be a hypergroup* (H_v−*group*), $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ *is (homogeneous) bipolar complex fuzzy subhypergroup* (H_v−*subgroup*) *of* S. *For every* $g_1, g_2, ..., g_n \in S$

$$\min\{\lambda_B^+(g_1), \lambda_B^+(g_2), ..., \lambda_B^+(g_n)\} \leq \inf\{\lambda_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

and

$$\max\{\lambda_B^-(g_1), \lambda_B^-(g_2), ..., \lambda_B^-(g_n)\} \geq \sup\{\lambda_B^-(x) : x \in g_1*(g_2*(...)*g_n)...)\}.$$

Proof. Assume that $g_1, g_2, ..., g_n \in S$ and as a positive membership function $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$, negative membership function $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. To evidence the lemma, it is adequate to indicate

$$\min\{r_B^+(g_1), r_B^+(g_2), ..., r_B^+(g_n)\} \leq \inf\{r_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$
$$\min\{w_B^+(g_1), w_B^+(g_2), ..., w_B^+(g_n)\} \leq \inf\{w_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

and

$$\max\{r_B^-(g_1), r_B^-(g_2), ..., r_B^-(g_n)\} \geq \sup\{r_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$
$$\max\{w_B^-(g_1), w_B^-(g_2), ...w_B^-(g_n)\} \geq \sup\{w_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}.$$

Because B is homogeneous, it is adequate to indicate

$$\min\{r_B^+(g_1), r_B^+(g_2), ..., r_B^+(g_n)\} \leq \inf\{r_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$
$$\max\{r_B^-(g_1), r_B^-(g_2), ..., r_B^-(g_n)\} \geq \sup\{r_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}.$$

Theorem 3.12 claims that r_B^+ and r_B^- are bipolar fuzzy subhypergroups (H_v-subgroups) of S. Using Lemma 2.8 end the proof. □

Definition 3.14. Let

$$B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$$

is (homogeneous) a bipolar complex fuzzy subset of a non-empty set S. Level subset $B_{(r,t)}$ of S is described as $B_{(r,t)} = \{s \in S : \lambda_B^+(s) \geq r$ and $\lambda_B^-(s) \leq t\}$, where $r = me^{i\varphi}, t = ne^{i\psi}$ such that $m \in [0,1], n \in [-1,0]$ and $\varphi \in [0, 2\pi], \psi \in [-2\pi, 0]$.

Remark 3.15. $\lambda_B^+(s) \geq r$ means $s \in \lambda_r^+$. Similarly, $\lambda_B^-(s) \leq t$ means that $s \in \lambda_t^-$.

Remark 3.16. Let $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ is (homogeneous) a bipolar complex fuzzy subset of a non-empty set S. Followings are true:

(1) Provided that $r_1 \leq r_2$, then $\lambda_{r_2}^+ \subseteq \lambda_{r_1}^+$,
(2) $\lambda_{0e^{0i}}^+ = S$,
(3) Provided that $t_1 \leq t_2$, then $\lambda_{t_1}^- \subseteq \lambda_{t_2}^-$,
(4) $\lambda_{0e^{0i}}^- = S$.

Theorem 3.17. *Presume that $(S, *)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subset of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. B is a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S if and only if for every $r = ke^{i\psi}$, $t = me^{i\varphi}$ such that $k \in [0,1], m \in [-1,0]$ and $\psi \in [0, 2\pi], \varphi \in [-2\pi, 0]$, $B_{(r,t)} \neq \emptyset$ is a subhypergroup (H_v-subgroup) of S.*

Proof. Consider that B be a bipolar complex fuzzy subhypergroup (H_v−subgroup) of S and $\rho, \sigma \in B_{(r,t)} \neq \emptyset$. For every $\varsigma \in \rho * \sigma$, we get

$$\lambda_B^+(\varsigma) \geq \min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\} \geq r,$$
$$\lambda_B^-(\varsigma) \leq \max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\} \leq t.$$

Hence, $\varsigma \in \rho*\sigma \subseteq B_{(r,t)}$ and for every $\varsigma \in B_{(r,t)}$ we get $\varsigma*B_{(r,t)} \subseteq B_{(r,t)}$.

Moreover, let $\rho \in B_{(r,t)}$, using description of bipolar complex fuzzy subhypergroup, there exists $\gamma \in S$ such that $\rho \in \varsigma * \gamma$ and

$$\lambda_B^+(\gamma) \geq \min\{\lambda_B^+(\varsigma), \lambda_B^+(\rho)\} \geq r,$$
$$\lambda_B^-(\gamma) \leq \max\{\lambda_B^-(\varsigma), \lambda_B^-(\rho)\} \leq t.$$

Therefore, this means $\gamma \in B_{(r,t)}$. We can use of description of bipolar complex fuzzy subhypergroup, then we can find $B_{(r,t)} * \varsigma \subseteq B_{(r,t)}$.

Conversely, suppose for every $r = ke^{i\psi}, t = me^{i\varphi}$ such that $k \in [0,1], m \in [-1,0]$ and $\psi \in [0,2\pi], \varphi \in [-2\pi,0]$ and $B_{(r,t)} \neq \emptyset$ is a subhypergroup (H_v−subgroup) of S. Presume

$$r_0 = k_0e^{i\psi_0} = \min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\},$$
$$t_0 = m_0e^{i\varphi_0} = \max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\}.$$

Then,

$$k_0 = \min\{r_B^+(\rho), r_B^+(\sigma)\}$$
$$\psi_0 = \min\{w_B^+(\rho), w_B^+(\sigma)\}$$

and

$$m_0 = \max\{r_B^-(\rho), r_B^-(\sigma)\}$$
$$\varphi_0 = \max\{w_B^-(\rho), w_B^-(\sigma)\}.$$

Because $\rho, \sigma \in B_{(r_0,t_0)}$ and $B_{(r_0,t_0)}$ is a a subhypergroup (H_v−subgroup) of S, it is like this $\rho * \sigma \subseteq B_{(r_0,t_0)}$. Thus, for all $\varsigma \in \rho * \sigma$ we get

$$\lambda_B^+(\varsigma) \geq r_0 = \min\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\}$$
$$\lambda_B^-(\varsigma) \leq t_0 = \max\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\}$$

thus requirements $1,2$ of the description of bipolar complex fuzzy subhypergroup are yielded. Similarly, we will prove conditions $3,4$ and $5,6$ are obtained. For every $\varsigma,\rho\in S$, setting by

$$r_1=k_1e^{i\psi_1}=\min\{\lambda_B^+(\rho),\lambda_B^+(\varsigma)\}$$
$$t_1=m_1e^{i\varphi_1}=\max\{\lambda_B^-(\rho),\lambda_B^-(\varsigma)\}.$$

then $\rho,\varsigma\in B_{(r_1,t_1)}$. With $B_{(r_1,t_1)}$ is a subhypergroup (H_v-subgroup) of S refers $\varsigma * B_{(r_1,t_1)}=B_{(r_1,t_1)}$. The latter demonstrates there exists $\sigma\in B_{(r_1,t_1)}$ such that $\rho\in\varsigma*\sigma$. Hence,

$$\lambda_B^+(\sigma)\geq r_1=\ \min\{\lambda_B^+(\rho),\lambda_B^+(\varsigma)\}$$
$$\lambda_B^-(\sigma)\leq t_1=\max\{\lambda_B^-(\rho),\lambda_B^-(\varsigma)\}.$$

□

Corollary 3.18. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subhypergroup (H_v-subgroup) of S together $\lambda_B^+(s)=r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s)=r_B^-(s)e^{i\alpha w_B^-(s)}$.*

Provided that $0e^{0i}\leq r_1=s_1e^{i\theta_1}<r_2=s_2e^{i\theta_2}\leq 1e^{2\pi i}$, $\lambda_{r_1}^+=\lambda_{r_2}^+$ if and only if there is no $\vartheta\in S$ such that $r_1\leq\lambda_B^+(\vartheta)<r_2$ and similarly if $0e^{0i}\geq m_1=n_1e^{i\varphi_1}>m_2=n_2e^{i\varphi_2}\geq -1e^{-2\pi i}$, $\lambda_{m_1}^-=\lambda_{m_2}^-$ if and only if there is no $\vartheta\in S$ such that $m_1\geq\lambda_B^-(\vartheta)>m_2$.

Proof. Suppose that $0e^{0i}\leq r_1=s_1e^{i\theta_1}<r_2=s_2e^{i\theta_2}\leq 1e^{2\pi i}$ such that $\lambda_{r_1}^+=\lambda_{r_2}^+$. Consider there exists $\vartheta\in S$ such that $r_1\leq\lambda_B^+(\vartheta)<r_2$. We get $\vartheta\in\lambda_{r_1}^+=\lambda_{r_2}^+$. This means $\lambda_B^+(\vartheta)\geq r_2$ and it is a contradiction. Similarly, let $0e^{0i}\geq m_1=n_1e^{i\varphi_1}>m_2=n_2e^{i\varphi_2}\geq -1e^{-2\pi i}$ such that $\lambda_{m_1}^-=\lambda_{m_2}^-$. Think there exists $\vartheta\in S$ such that $m_1\geq\lambda_B^-(\vartheta)>m_2$. We get $\vartheta\in\lambda_{m_1}^-=\lambda_{m_2}^-$. This means $\lambda_B^-(\vartheta)\leq m_2$ and it is a contradiction.

Because of $0e^{0i}\leq r_1=s_1e^{i\theta_1}<r_2=s_2e^{i\theta_2}\leq 1e^{2\pi i}$, using previous Remark 3.16 $\lambda_{r_2}^+\subseteq\lambda_{r_1}^+$. To indicate $\lambda_{r_1}^+\subseteq\lambda_{r_2}^+$, let $\vartheta\in\lambda_{r_1}^+$. We get $\lambda_B^+(\vartheta)\geq r_1$. Because there is no $\vartheta\in S$ such that $r_1\leq\lambda_B^+(\vartheta)<r_2$, it is like this $\lambda_B^+(\vartheta)\geq r_2$. Thus, $\vartheta\in\lambda_{r_2}^+$ and $\lambda_{r_1}^+\subseteq\lambda_{r_2}^+$. Similarly, since $0e^{0i}\geq m_1=n_1e^{i\varphi_1}>m_2=n_2e^{i\varphi_2}\geq -1e^{-2\pi i}$,using previous Remark 3.16 that $\lambda_{m_2}^-\subseteq\lambda_{m_1}^-$. To indicate $\lambda_{m_1}^-\subseteq\lambda_{m_2}^-$, let $\vartheta\in\lambda_{m_1}^-$.We get $\lambda_B^-(\vartheta)\leq m_1$. Because there is no $\vartheta\in S$ such that $m_1\geq\lambda_B^-(\vartheta)>m_2$, it is like this $\lambda_B^-(\vartheta)\leq m_2$. Thus, $\vartheta\in\lambda_{m_2}^-$ and $\lambda_{m_1}^-\subseteq\lambda_{m_2}^-$. □

Corollary 3.19. *Assume that* $(S,*)$ *be a hypergroup* (H_v*−group), B be a (homogeneous) bipolar complex fuzzy subhypergroup* (H_v*−subgroup) of S together* $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ *and* $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$.

Providing that the range of λ_B^+ *is finite set* $\{r_1, r_2, ..., r_n\}$ *and* λ_B^- *is finite set* $\{m_1, m_2, ..., m_n\}$*, then the sets* $\{\lambda_{r_i}^+ : i = 1, 2, ..., n\}$ *and* $\{\lambda_{m_i}^- : i = 1, 2, ..., n\}$ *comprises whole the level subhypergroups* (H_v*−subgroups) of S. Besides of this, providing that* $r_1 \geq r_2 \geq ... \geq r_n$*, whole the level subhypergroups of S create the chain* $\lambda_{r_1}^+ \subseteq \lambda_{r_2}^+ \subseteq ... \subseteq \lambda_{r_n}^+$*. Similarly, providing that* $m_1 \leq m_2 \leq ... \leq m_n$*, whole the level subhypergroups of S create the chain* $\lambda_{m_1}^- \subseteq \lambda_{m_2}^- \subseteq ... \subseteq \lambda_{m_n}^-$.

Proof. Assume that $\lambda_q^+, \lambda_q^- \neq \emptyset$ are level subhypergroups (H_v−subgroups) of S such that $\lambda_q^+ \neq \lambda_{r_i}^+$ and $\lambda_q^- \neq \lambda_{m_i}^-$ for every $1 \leq i \leq n$. Suppose that r_z and m_z are closest complex numbers to q. There are two cases : $q < r_z$ and $q > m_z$, $q > r_z$ and $q < m_z$. We think the first case, the second case is like that of the first case. Because the ranges of λ_B^+ and λ_B^- are finite sets $\{r_1, r_2, ..., r_n\}$ and $\{m_1, m_2, ..., m_n\}$ respectively, we get there is no $\epsilon \in S$ such that $q \leq \lambda_B^+(\epsilon) < r_z$ and $q \geq \lambda_B^-(\epsilon) > m_z$. Using previous Corollary 3.18, we get a contradiction. □

Proposition 3.20. *Assume that* $(S,*)$ *be the biset hypergroup like* $\rho * \sigma = \{\rho, \sigma\}$ *for every* $\rho, \sigma \in S$ *and B be any homogeneous bipolar complex fuzzy subset of S. B is a bipolar complex fuzzy subhypergroup of S.*

Proof. Suppose that $r = me^{i\psi}$and $t = ne^{i\varphi}$ such that $m \in [0,1], n \in [-1,0]$ and $\psi \in [0, 2\pi], \varphi \in [-2\pi, 0]$. Using Theorem 3.17, it is adequate to demonstrate $B_{(r,t)} \neq \emptyset$ is a subhypergroup of S. We get $B_{(r,t)} \subseteq \upsilon * B_{(r,t)}$ as for every $\tau \in B_{(r,t)}$, $\tau \in \upsilon * \tau = \{\upsilon, \tau\}$. Besides of this, $\upsilon * B_{(r,t)} = B_{(r,t)} * \upsilon = \{\tau * \upsilon : \tau \in B_{(r,t)}\} = \{\tau, \upsilon\} \subseteq B_{(r,t)}$ for every $\upsilon \in B_{(r,t)}$. Therefore, $B_{(r,t)}$ is a subhypergroup of S. □

Proposition 3.21. *Assume that* $(S,*)$ *be the total hypergroup like* $\rho * \sigma = S$ *for every* $\rho, \sigma \in S$ *and B be any homogeneous bipolar complex fuzzy subset of S. B is a bipolar complex fuzzy subhypergroup of S if and only if* λ_B^+ *and* λ_B^- *are stable complex functions.*

Proof. It is explicit if λ_B^+ and λ_B^- are stable complex functions, B is a bipolar complex fuzzy subhypergroup of S. Assume that B be a bipolar complex fuzzy subhypergroup of S and λ_B^+ is not a stable complex function. We may find $\rho, \sigma \in S$, $r = se^{i\theta}$ such that $s \in [0,1]$ and $\theta \in [0, 2\pi]$ such that $\lambda_B^+(\rho) < \lambda_B^+(\sigma) = r$. It is explicit $\rho \notin \lambda_r^+$ and $\sigma \in \lambda_r^+$. Because

$B_{(r,t)} \neq \emptyset$ is a subhypergroup of S, it is like this $S = \sigma * \sigma \subseteq B_{(r,t)}$. Similarly, let B be a bipolar complex fuzzy subhypergroup of S and λ_B^- is not a stable complex function. We may find $\rho, \sigma \in S$, $m = ke^{i\varphi}$ such that $k \in [-1, 0]$ and $\varphi \in [-2\pi, 0]$ such that $\lambda_B^-(\rho) > \lambda_B^-(\sigma) = m$. It is explicit that $\rho \notin \lambda_m^-$ and $\sigma \in \lambda_m^-$. Because $B_{(r,t)} \neq \emptyset$ is a subhypergroup of S, it is like this $S = \sigma * \sigma \subseteq B_{(r,t)}$. These are contradictions. Hence, λ_B^+ and λ_B^- are stable complex functions. □

Proposition 3.22. *Assume that $(S, *)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subset of S. B is a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S if and only if for every $r = ke^{i\theta}$ and $t = me^{i\varphi}$ such that $k \in [0, 1], m \in [-1, 0]$ and $\theta \in [0, 2\pi], \varphi \in [-2\pi, 0]$, the subsequent conditions are provided:*

(1) $B_{(r,t)} * B_{(r,t)} \subseteq B_{(r,t)}$,
(2) $\upsilon * (S - B_{(r,t)}) - (S - B_{(r,t)}) \subseteq \upsilon * B_{(r,t)}$, *for every* $\upsilon \in B_{(r,t)}$,
(3) $(S - B_{(r,t)}) * \upsilon - (S - B_{(r,t)}) \subseteq B_{(r,t)} * \upsilon$, *for every* $\upsilon \in B_{(r,t)}$.

Proof. Imagine B be a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S. $B_{(r,t)}$ is a subhypergroup (H_v-subgroup) of S as $\upsilon * B_{(r,t)} = B_{(r,t)}$ for every $\upsilon \in B_{(r,t)}$. Hence, $B_{(r,t)} * B_{(r,t)} \subseteq B_{(r,t)}$. We wish to demostrate $\upsilon * (S - B_{(r,t)}) - (S - B_{(r,t)}) \subseteq \upsilon * B_{(r,t)}$. Suppose that $\delta \in \upsilon * (S - B_{(r,t)}) - (S - B_{(r,t)})$. We get δ is not an element in $(S - B_{(r,t)})$. This refers $\delta \in B_{(r,t)} = \upsilon * B_{(r,t)}$. Similarly, Condition 3 may be displayed.

Conversely, assume the conditions 1, 2 are provided. Using Theorem 3.17 it is adequate to indicate $B_{(r,t)}$ is a subhypergroup (H_v-subgroup) of S as $\upsilon * B_{(r,t)} = B_{(r,t)} * \upsilon = B_{(r,t)}$ for every $\upsilon \in B_{(r,t)}$. Consider there exists $\gamma \in B_{(r,t)}$ such that γ is not an element in $\upsilon * B_{(r,t)}$. Using reproduction property of $(S, *)$, there exists $\epsilon \in S$ such that $\gamma \in \upsilon * \epsilon$. There are two cases for ϵ :

Case 1 : $\epsilon \in B_{(r,t)}$. We get $\gamma \in \upsilon * \epsilon \subseteq \upsilon * B_{(r,t)}$. This is a contradiction.

Case 2 : $\epsilon \notin B_{(r,t)}$. We get $\epsilon \in (S - B_{(r,t)})$. $\gamma \in \upsilon * \epsilon$ means $\gamma \in \upsilon * (S - B_{(r,t)})$. Because $\gamma \in B_{(r,t)}$, it is like this γ is not in $(S - B_{(r,t)})$. Hence, using assumption $\gamma \in \upsilon * (S - B_{(r,t)}) - (S - B_{(r,t)}) \subseteq \upsilon * B_{(r,t)}$. This is a contradiction. Similarly, using condition 3, we can display $B_{(r,t)} * \upsilon = B_{(r,t)}$. □

Proposition 3.23. *Assume that $(S, *)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subhypergroup (H_v-subgroup)* *of S together* $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ *and* $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. *Describe* $\overline{\lambda}^+$ *and* $\overline{\lambda}^-$:

$$\overline{\lambda}^+ = \{s \in S : \lambda_B^+(s) = 1e^{2\pi i}\} \text{ and } \overline{\lambda}^- = \{s \in S : \lambda_B^-(s) = -1e^{-2\pi i}\}$$

$\overline{\lambda}^+$ *and* $\overline{\lambda}^-$ *are empty or subhypergroups (H_v−subgroups) of S.*

Proof. We wish to demonstrate $\tau * \overline{\lambda}^+ = \overline{\lambda}^+ = \overline{\lambda}^+ * \tau$ for every $\tau \in \overline{\lambda}^+$. Let $d \in \overline{\lambda}^+$and $\gamma \in \tau * d$. We get $\lambda_B^+(\gamma) \geq \min\{\lambda_B^+(\tau), \lambda_B^+(d)\} = 1e^{2\pi i}$ indicates $\lambda_B^+(\gamma) = 1e^{2\pi i}$. Hence, $\gamma \in \tau * d \subseteq \overline{\lambda}^+$. For every $\tau, d \in \overline{\lambda}^+$, there exists $\delta \in S$ such that $d \in \tau * \delta$ and $\lambda_B^+(\delta) \geq \min\{\lambda_B^+(\tau), \lambda_B^+(d)\} = 1e^{2\pi i}$. This implies $\lambda_B^+(\delta) = 1e^{2\pi i}$ and $\delta \in \overline{\lambda}^+$.

Similarly, we wish to demonstrate $\nu * \overline{\lambda}^- = \overline{\lambda}^- = \overline{\lambda}^- * \upsilon$ for every $\upsilon \in \overline{\lambda}^-$. Let $d \in \overline{\lambda}^-$ and $\gamma \in \upsilon * d$. We get $\lambda_B^-(\gamma) \leq \max\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} = -1e^{-2\pi i}$ indicates $\lambda_B^-(\gamma) = -1e^{-2\pi i}$. Hence, $\gamma \in \upsilon * d \subseteq \overline{\lambda}^-$. For every $\upsilon, d \in \overline{\lambda}^-$, there exists $\delta \in S$ such that $d \in \upsilon * \delta$ and $\lambda_B^-(\delta) \leq \max\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} = -1e^{-2\pi i}$. This implies $\lambda_B^-(\delta) = -1e^{-2\pi i}$ and $\delta \in \overline{\lambda}^-$. □

Proposition 3.24. *Assume that $(S, *)$ be a hypergroup (H_v−group), B be a (homogeneous) bipolar complex fuzzy subhypergroup (H_v−subgroup) of S together* $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ *and* $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. *Describe* $supp(\lambda^+)$ *and* $supp(\lambda^-)$:

$$supp(\lambda^+) = \{s \in S : \lambda_B^+(s) > 0e^{0i}\} \text{ and } supp(\lambda^-) = \{s \in S : \lambda_B^-(s) < 0e^{0i}\}$$

Then, $supp(\lambda^+)$ and $supp(\lambda^-)$ are empty or subhypergroups (H_v−subgroups) of S.

Proof. We wish to demonstrate $\tau * supp(\lambda^+) = supp(\lambda^+) = supp(\lambda^+) * \tau$ for every $\tau \in supp(\lambda^+)$. Let $d \in supp(\lambda^+)$ and $\gamma \in \tau * d$. We get $\lambda_B^+(\gamma) \geq \min\{\lambda_B^+(\tau), \lambda_B^+(d)\} > 0e^{0i}$ indicates $\lambda_B^+(\gamma) > 0e^{0i}$. Hence, $\gamma \in \tau * d \subseteq supp(\lambda^+)$. For every $\tau, d \in supp(\lambda^+)$, there exists $\delta \in S$ such that $d \in \tau * \delta$ and $\lambda_B^+(\delta) \geq \min\{\lambda_B^+(\tau), \lambda_B^+(d)\} > 0e^{0i}$. This implies $\lambda_B^+(\delta) > 0e^{0i}$ and $\delta \in supp(\lambda^+)$.

Similarly, we wish to demonstrate $\upsilon * supp(\lambda^-) = supp(\lambda^-) = supp(\lambda^-) * \upsilon$ for every $\upsilon \in supp(\lambda^-)$. Let $d \in supp(\lambda^-)$ and $\gamma \in \upsilon * d$. We get $\lambda_B^-(\gamma) \leq \max\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} < 0e^{0i}$ indicates $\lambda_B^-(\gamma) < 0e^{0i}$. Hence, $\gamma \in \upsilon * d \subseteq supp(\lambda^-)$. For every $\upsilon, d \in supp(\lambda^-)$, there exists $\delta \in S$

such that $d \in \upsilon * \delta$ and $\lambda_B^-(\delta) \leq \max\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} < 0e^{0i}$. This refers $\lambda_B^-(\delta) < 0e^{0i}$ and $\delta \in supp(\lambda^-)$. □

Remark 3.25. Let $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ is a homogeneous bipolar complex fuzzy subset of a non-empty set S. We clarified the complement of the bipolar complex fuzzy subset B of S :

$$B^C = \{(s, \lambda_{B^C}^+(s) = (1 - r_B^+(s))e^{i\alpha(2\pi - w_B^+(s))}, \lambda_{B^C}^-(s) \\ = (-1 - r_B^-(s))e^{i\alpha(-2\pi - w_B^-(s))}) : s \in S\}$$

Example 3.26. Assume that $S = \{\rho, \sigma\}$ and we define the hypergroup $(S, *)$ by the next table:

$*$	ρ	σ
ρ	ρ	S
σ	S	σ

We describe a bipolar complex fuzzy subset B of S :

$$\lambda_B^+(\rho) = 0.4e^{i0} \text{ and } \lambda_B^-(\rho) = -0.1e^{-i\pi/2},$$
$$\lambda_B^+(\sigma) = 0.5e^{i\pi} \text{ and } \lambda_B^-(\sigma) = -0.3e^{-i\pi}.$$

We can take B^C as follows:

$$\lambda_{B^C}^+(\rho) = 0.6e^{i2\pi} \text{ and } \lambda_{B^C}^-(\rho) = -0.9e^{-i3\pi/2}$$
$$\lambda_{B^C}^+(\sigma) = 0.5e^{i\pi} \text{ and } \lambda_{B^C}^-(\sigma) = -0.7e^{-i\pi}.$$

In that case, B and $B^C$are homogeneous bipolar complex fuzzy subhypergroups of S.

Example 3.27. Suppose that $(S, *)$ be any hypergroup (H_v-group) together bipolar complex fuzzy subset B of S, which is characterized as: $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$,where $r_B^+(s), w_B^+(s) \in [0, 1]$ and $r_B^-(s), w_B^-(s) \in [-1, 0], \alpha \in [0, 2\pi]$ are stable real numbers. Therefore, B and B^C are homogeneous bipolar complex fuzzy subhypergroups (H_v-subgroups) of S.

Remark 3.28. Assume that $(S, *)$ be a hypergroup (H_v-group), B is a (homogeneous) bipolar complex fuzzy subhypergroup (H_v-subgroup) of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. B^C is not requisite a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S.

We will sample above situation with the subsequent exemplary.

Example 3.29. Presume that $S = \{\rho, \sigma, \varsigma\}$ and H_v−group $(S, *)$ is described as:

$*$	ρ	σ	ς
ρ	ρ	$\{\sigma, \varsigma\}$	ς
σ	$\{\sigma, \varsigma\}$	ς	ρ
ς	ς	ρ	σ

Describe bipolar complex fuzzy subset B of S, i.e.,

$$\lambda_B^+(\rho) = 0.6e^{i\pi} \text{ and } \lambda_B^-(\rho) = -0.4e^{-i3\pi/2}$$
$$\lambda_B^+(\sigma) = \lambda_B^+(\varsigma) = 0.4e^{i\pi/2} \text{ and } \lambda_B^-(\sigma) = \lambda_B^-(\varsigma) = -0.2e^{-i\pi}.$$

We get,

$$\lambda_t^+ = \left\{ \begin{array}{ll} S, & \text{if } t \leq 0.4e^{i\pi/2} \\ \{\rho\}, & \text{if } \ 0.4e^{i\pi/2} < t \leq 0.6e^{i\pi} \\ \emptyset, & \text{otherwise} \end{array} \right\}$$

and

$$\lambda_s^- = \left\{ \begin{array}{ll} S, & \text{if } s \geq -0.2e^{-i\pi} \\ \{\rho\}, & \text{if } \ -0.2e^{-i\pi} > s \geq -0.4e^{-i3\pi/2} \\ \emptyset, & \text{otherwise} \end{array} \right\}$$

either empty sets or subhypergroups of S, which refers B is homogeneous bipolar complex fuzzy subhypergroup of S.
But

$$0.4e^{i\pi} = \lambda_{B^C}^+(\rho) = \lambda_{B^C}^+(\sigma * \varsigma) < \min\{\lambda_{B^C}^+(\sigma), \lambda_{B^C}^+(\varsigma)\} = 0.6e^{i3\pi/2}$$
$$-0.6e^{-i\pi/2} = \lambda_{B^C}^-(\rho) = \lambda_{B^C}^-(\sigma * \xi) > \max\{\lambda_{B^C}^-(\sigma), \lambda_{B^C}^-(\varsigma)\} = -0.8e^{-i\pi}.$$

It is a contradiction. Hence, B^C is not bipolar complex fuzzy H_v−subgroup of S.

4. Bipolar Complex Anti-Fuzzy H_v−subgroups

In this part, we will explore bipolar complex anti-fuzzy subhypergroups (H_v−subgroups) and delve into their characteristics. Additionally, we will establish connections between bipolar complex fuzzy subhypergroups (H_v−subgroups) and bipolar complex anti-fuzzy subhypergroups (H_v−subgroups).

Definition 4.1. Suppose that $(S, *)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subset of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. Providing that the following requirements are provided, B is named a bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S:

(1) $\sup\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\}$ for every $\rho, \sigma \in S$,
(2) $\inf\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\}$ for every $\rho, \sigma \in S$,
(3) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\lambda_B^+(\gamma) \leq \max\{\lambda_B^+(d), \lambda_B^+(\rho)\}$,
(4) For every $d, \rho \in S$, there exists $\gamma \in S$ such that $d \in \rho * \gamma$ and $\lambda_B^-(\gamma) \geq \min\{\lambda_B^-(d), \lambda_B^-(\rho)\}$,
(5) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\lambda_B^+(\delta) \leq \max\{\lambda_B^+(d), \lambda_B^+(\rho)\}$,
(6) For every $d, \rho \in S$, there exists $\delta \in S$ such that $d \in \delta * \rho$ and $\lambda_B^-(\delta) \geq \min\{\lambda_B^-(d), \lambda_B^-(\rho)\}$.

Now, we will give some examples about bipolar complex anti-fuzzy H_v-subgroups.

Example 4.2. Assume that $S = \{\rho, \sigma\}$ and we describe the hypergroup $(S, *)$ by the next table:

$*$	ρ	σ
ρ	ρ	S
σ	S	σ

We describe a bipolar complex fuzzy subset B of S :
$\lambda_B^+(\rho) = 0.5e^{i2\pi/3}$ and $\lambda_B^-(\rho) = -0.4e^{-i3\pi/2}$
$\lambda_B^+(\sigma) = 0.3e^{i\pi/2}$ and $\lambda_B^-(\sigma) = -0.2e^{-i\pi}$

In that case, B is homogeneous bipolar complex anti-fuzzy subhypergroup of S.

Example 4.3. Assume that $S = \{\rho, \sigma, \varsigma\}$ and we describe the hypergroup $(S, *)$ by the next table:

$*$	ρ	σ	ς
ρ	$\{\rho\}$	$\{\sigma\}$	$\{\varsigma\}$
σ	$\{\sigma\}$	$\{\sigma, \varsigma\}$	S
ς	$\{\varsigma\}$	S	$\{\sigma, \varsigma\}$

We describe a bipolar complex fuzzy subset B of S :
$\lambda_B^+(\rho) = 0.6e^{i3\pi/2}$ and $\lambda_B^-(\rho) = -0.8e^{-i\pi}$

$\lambda_B^+(\sigma) = 0.5e^{i\pi}$ and $\lambda_B^-(\sigma) = -0.5e^{-i\pi/2}$
$\lambda_B^+(\varsigma) = 0.4e^{i\pi/2}$ and $\lambda_B^-(\varsigma) = -0.3e^{-i\pi/3}$

In that case, B be homogeneous bipolar complex anti-fuzzy subhypergroup of S.

Example 4.4. Suppose that $(S, *)$ be any hypergroup (H_v−group) together the bipolar complex fuzzy subset B of S, which is characterized as: $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$, where $r_B^+(s), w_B^+(s) \in [0,1]$ and $r_B^-(s), w_B^-(s) \in [-1,0], \alpha \in [0, 2\pi]$ are stable real numbers. Therefore, B is homogeneous bipolar complex anti-fuzzy subhypergroup (H_v−subgroup) of S.

Proposition 4.5. *Suppose that $(S, *)$ be a hypergroup (H_v−group). B is a bipolar anti-fuzzy subhypergroup (H_v−subgroup) of S if and only if a bipolar $\pi-$ fuzzy set B_π is a bipolar $\pi-$ anti-fuzzy subhypergroup (H_v-subgroup) of S.*

Proof. Evidence is explicit. □

Theorem 4.6. *Suppose that $(S, *)$ be a hypergroup (H_v−group), B be a (homogeneous) bipolar complex fuzzy subset of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. B is a bipolar complex anti-fuzzy subhypergroup (H_v−subgroup) of S if and only if r_B^+ and r_B^- are bipolar anti-fuzzy subhypergroups (H_v−subgroups) of S and w_B^+ and w_B^- are bipolar $\pi-$anti-fuzzy subhypergroups (H_v−subgroups) of S.*

Proof. ($\Longrightarrow$) : Presume that B be a bipolar complex anti-fuzzy subhypergroup (H_v−subgroup) of S. We wish to demonstrate requirements of description of bipolar anti-fuzzy subhypergroup are yielded for r_B^+ and r_B^- and w_B^+ and w_B^-. For every $\rho, \sigma \in S$, we get

$$\sup\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \le \max\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\}$$
$$\inf\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \ge \min\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\}.$$

Remark 3.8 refers

$$\sup\{r_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \le \max\{r_B^+(\rho), r_B^+(\sigma)\}$$
$$\inf\{r_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \ge \min\{r_B^-(\rho), r_B^-(\sigma)\}$$

and

$$\sup\{w_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \le \max\{w_B^+(\rho), w_B^+(\sigma)\}$$
$$\inf\{w_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \ge \min\{w_B^-(\rho), w_B^-(\sigma)\}.$$

Consider $\rho, d \in S$. There exist $\gamma, \delta \in S$ such that $d \in \rho * \gamma$ and $d \in \delta * \rho$. Then,

$$\max\{\lambda_B^+(\rho), \lambda_B^+(d)\} \geq \lambda_B^+(\gamma)$$
$$\min\{\lambda_B^-(\rho), \lambda_B^-(d)\} \leq \lambda_B^-(\gamma)$$

and

$$\max\{\lambda_B^+(\rho), \lambda_B^+(d)\} \geq \lambda_B^+(\delta)$$
$$\min\{\lambda_B^-(\rho), \lambda_B^-(d)\} \leq \lambda_B^-(\delta).$$

Remark 3.8 indicates the requirements $3, 4$ and $5, 6$ of description of bipolar anti-fuzzy subhypergroup are yielded for both r_B^+ and r_B^- and w_B^+ and w_B^-.

$(\Longleftarrow)$: Presume that r_B^+ and r_B^- are bipolar anti-fuzzy subhypergroups (H_v-subgroups) of S and w_B^+ and w_B^- are bipolar $\pi-$anti-fuzzy subhypergroups (H_v-subgroups) of S. We wish to indicate requirements of bipolar complex anti-fuzzy subhypergroup are yielded. For every $\rho, \sigma \in S$, we get

$$\sup\{r_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{r_B^+(\rho), r_B^+(\sigma)\}$$
$$\inf\{r_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{r_B^-(\rho), r_B^-(\sigma)\}$$

and

$$\sup\{w_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{w_B^+(\rho), w_B^+(\sigma)\}$$
$$\inf\{w_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{w_B^-(\rho), w_B^-(\sigma)\}.$$

Remark 3.8 refers

$$\sup\{\lambda_B^+(\varsigma) : \varsigma \in \rho * \sigma\} \leq \max\{\lambda_B^+(\rho), \lambda_B^+(\sigma)\}$$
$$\inf\{\lambda_B^-(\varsigma) : \varsigma \in \rho * \sigma\} \geq \min\{\lambda_B^-(\rho), \lambda_B^-(\sigma)\}.$$

Consider $\rho, d \in S$. There exist $\gamma, \delta \in S$ such that $d \in \rho * \gamma$, $d \in \delta * \rho$ and

$$\max\{r_B^+(\rho), r_B^+(d)\} \geq r_B^+(\gamma) \text{ and } \max\{w_B^+(\rho), w_B^+(d)\} \geq w_B^+(\gamma)$$
$$\min\{r_B^-(\rho), r_B^-(d)\} \leq r_B^-(\gamma) \text{ and } \min\{w_B^-(\rho), w_B^-(d)\} \leq w_B^-(\gamma),$$

and

$$\max\{r_B^+(\rho), r_B^+(d) \geq r_B^+(\delta)\} \text{ and } \max\{w_B^+(\rho), w_B^+(d)\} \geq w_B^+(\delta)$$
$$\min\{r_B^-(\rho), r_B^-(d) \leq r_B^-(\delta)\} \text{ and } \min\{w_B^-(\rho), w_B^-(d)\} \leq w_B^-(\delta).$$

Remark 3.8 refers the requirements $3, 4$ and $5, 6$ of description of bipolar complex anti-fuzzy subhypergroup are yielded for B. □

Lemma 4.7. *Suppose that $(S, *)$ be a hypergroup (H_v-group), $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ be (homogeneous) bipolar complex anti-fuzzy subhypergroup (H_v-subgroups) of S. For every $g_1, g_2, ..., g_n \in S$*

$$\max\{\lambda_B^+(g_1), \lambda_B^+(g_2), ..., \lambda_B^+(g_n)\} \geq \sup\{\lambda_B^+(x) : x \in g_1*(g_2*(...)*g_n)...)\}$$

and

$$\min\{\lambda_B^-(g_1), \lambda_B^-(g_2), ..., \lambda_B^-(g_n)\} \leq \inf\{\lambda_B^-(x) : x \in g_1*(g_2*(...)*g_n)...)\}.$$

Proof. Suppose that $g_1, g_2, ..., g_n \in S$ and $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. To demonstrate the lemma, it is adequate to indicate

$$\max\{r_B^+(g_1), r_B^+(g_2), ..., r_B^+(g_n)\} \geq \sup\{r_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

$$\max\{w_B^+(g_1), w_B^+(g_2), ..., w_B^+(g_n)\} \geq \sup\{w_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

and

$$\min\{r_B^-(g_1), r_B^-(g_2), ..., r_B^-(g_n)\} \leq \inf\{r_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

$$\min\{w_B^-(g_1), w_B^-(g_2), ...w_B^-(g_n)\} \leq \inf\{w_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}.$$

Because B is homogeneous, it is adequate to indicate

$$\max\{r_B^+(g_1), r_B^+(g_2), ..., r_B^+(g_n)\} \geq \sup\{r_B^+(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}$$

$$\min\{r_B^-(g_1), r_B^-(g_2), ..., r_B^-(g_n)\} \leq \inf\{r_B^-(x) : x \in g_1 * (g_2 * (...) * g_n)...)\}.$$

Previous theorem claims r_B^+ and r_B^- are bipolar anti-fuzzy subhypergroups (H_v-subgroups) of S. Using Lemma 2.10 completes the proof. □

Definition 4.8. Let $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ be a (homogeneous) bipolar complex fuzzy subset of a non-empty set S. Lower level subset $\overline{B}_{(r,t)}$ of S is described as $\overline{B}_{(r,t)} = \{s \in S : \lambda_B^+(s) \leq r$ and $\lambda_B^-(s) \geq t\}$, where $r = me^{i\varphi}, t = ne^{i\psi}$ such that $m \in [0, 1], n \in [-1, 0]$ and $\varphi \in [0, 2\pi], \psi \in [-2\pi, 0]$.

Remark 4.9. Let $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ be a (homogeneous) bipolar complex fuzzy subset of a non-empty set S. Followings are true:

(1) Provided that $r_1 \leq r_2$, then $\overline{\lambda}_{r_1}^+ \subseteq \overline{\lambda}_{r_2}^+$,

(2) $\overline{\lambda}^{+}_{1e^{2\pi i}} = S$,
(3) Provided that $t_1 \leq t_2$, then $\overline{\lambda}^{-}_{t_2} \subseteq \overline{\lambda}^{-}_{t_1}$,
(4) $\overline{\lambda}^{-}_{-1e^{-2\pi i}} = S$.

Theorem 4.10. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex fuzzy subset of S together $\lambda^+_B(s) = r^+_B(s)e^{i\alpha w^+_B(s)}$ and $\lambda^-_B(s) = r^-_B(s)e^{i\alpha w^-_B(s)}$. B is a bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S if and only if for every $r = ke^{i\psi}$, $t = me^{i\varphi}$ such that $k \in [0,1], m \in [-1,0]$ and $\psi \in [0,2\pi], \varphi \in [-2\pi, 0]$, $\overline{B}_{(r,t)} \neq \emptyset$ is a subhypergroup (H_v-subgroup) of S.*

Proof. Evidence is alike that of Theorem 3.17. □

Corollary 4.11. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S together $\lambda^+_B(s) = r^+_B(s)e^{i\alpha w^+_B(s)}$ and $\lambda^-_B(s) = r^-_B(s)e^{i\alpha w^-_B(s)}$. Provided that $0e^{0i} \leq r_1 = m_1e^{i\varphi_1} < r_2 = m_2e^{i\varphi_2} \leq 1e^{2\pi i}$, then $\overline{\lambda}^{+}_{r_1} = \overline{\lambda}^{+}_{r_2}$ if and only if there is no $\vartheta \in S$ such that $r_1 < \lambda^+_B(\vartheta) \leq r_2$ and similarly provided that $0e^{0i} \geq s_1 = n_1e^{i\psi_1} > s_2 = n_2e^{i\psi_2} \geq -1e^{-2\pi i}$, then $\overline{\lambda}^{-}_{s_1} = \overline{\lambda}^{-}_{s_2}$ if and only if there is no $\vartheta \in S$ such that $s_1 > \lambda^-_B(\vartheta) \geq s_2$.*

Proof. Suppose that $0e^{0i} \leq r_1 = m_1e^{i\varphi_1} < r_2 = m_2e^{i\varphi_2} \leq 1e^{2\pi i}$ such that $\overline{\lambda}^{+}_{r_1} = \overline{\lambda}^{+}_{r_2}$. Consider there exists $\vartheta \in S$ such that $r_1 < \lambda^+_B(\vartheta) \leq r_2$. We get, $\vartheta \in \overline{\lambda}^{+}_{r_2} = \overline{\lambda}^{+}_{r_1}$. This refers $\lambda^+_B(\vartheta) \leq r_1$ and it is a contradiction. Similarly, consider $0e^{0i} \geq s_1 = n_1e^{i\psi_1} > s_2 = n_2e^{i\psi_2} \geq -1e^{-2\pi i}$, such that $\overline{\lambda}^{-}_{s_1} = \overline{\lambda}^{-}_{s_2}$. Imagine that there exists $\vartheta \in S$ such that $s_1 > \lambda^-_B(\vartheta) \geq s_2$. Then, $\vartheta \in \overline{\lambda}^{-}_{s_2} = \overline{\lambda}^{-}_{s_1}$. This refers $\lambda^-_B(\vartheta) \geq s_1$ and it is a contradiction.

Because of $0e^{0i} \leq r_1 = m_1e^{i\varphi_1} < r_2 = m_2e^{i\varphi_2} \leq 1e^{2\pi i}$, with previous Remark 4.9 $\overline{\lambda}^{+}_{r_1} \subseteq \overline{\lambda}^{+}_{r_2}$. To demonstrate $\overline{\lambda}^{+}_{r_2} \subseteq \overline{\lambda}^{+}_{r_1}$, consider $\vartheta \in \overline{\lambda}^{+}_{r_2}$. Then, $\lambda^+_B(\vartheta) \leq r_2$. Because there is no $\vartheta \in S$ such that $r_1 < \lambda^+_B(\vartheta) \leq r_2$, we get $\lambda^+_B(\vartheta) \leq r_1$. Hence, $\vartheta \in \overline{\lambda}^{+}_{r_1}$ and $\overline{\lambda}^{+}_{r_2} \subseteq \overline{\lambda}^{+}_{r_1}$. Similarly, since $0e^{0i} \geq s_1 = n_1e^{i\psi_1} > s_2 = n_2e^{i\psi_2} \geq -1e^{-2\pi i}$, with previous Remark 4.9 $\overline{\lambda}^{-}_{s_1} \subseteq \overline{\lambda}^{-}_{s_2}$. To demonstrate $\overline{\lambda}^{-}_{s_2} \subseteq \overline{\lambda}^{-}_{s_1}$, consider $\vartheta \in \overline{\lambda}^{-}_{s_2}$. Then, $\lambda^-_B(\vartheta) \geq s_2$. Because there is no $\vartheta \in S$ such that $s_1 > \lambda^-_B(\vartheta) \geq s_2$, we get $\lambda^-_B(\vartheta) \geq s_1$. Hence, $\vartheta \in \overline{\lambda}^{-}_{s_1}$ and $\overline{\lambda}^{-}_{s_2} \subseteq \overline{\lambda}^{-}_{s_1}$. □

Corollary 4.12. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S together $\lambda^+_B(s) = r^+_B(s)e^{i\alpha w^+_B(s)}$ and $\lambda^-_B(s) = r^-_B(s)e^{i\alpha w^-_B(s)}$.*

Providing that the range of λ_B^+ *is finite set* $\{r_1, r_2, ..., r_n\}$ *and* λ_B^- *is finite set* $\{s_1, s_2, ..., s_n\}$*, then the sets* $\{\overline{\lambda}_{r_i}^+ : i = 1, 2, ..., n\}$ *and* $\{\overline{\lambda}_{s_i}^- : i = 1, 2, ..., n\}$ *comprises whole the lower level subhypergroups (H_v−subgroups) of* S*. Besides of this, providing that* $r_1 \leq r_2 \leq ... \leq r_n$*, whole the lower level subhypergroups of* S *create the chain* $\overline{\lambda}_{r_1}^+ \subseteq \overline{\lambda}_{r_2}^+ \subseteq ... \subseteq \overline{\lambda}_{r_n}^+$*. Similarly, providing that* $s_1 \geq s_2 \geq ... \geq s_n$*, whole the lower level subhypergroups of* S *create the chain* $\overline{\lambda}_{s_1}^- \subseteq \overline{\lambda}_{s_2}^- \subseteq ... \subseteq \overline{\lambda}_{s_n}^-$*.*

Proof. Assume that $\overline{\lambda}_m^+$ and $\overline{\lambda}_m^- \neq \emptyset$ are lower level subhypergroups (H_v-subgroups) of S such that $\overline{\lambda}_m^+ \neq \overline{\lambda}_{r_i}^+$ and $\overline{\lambda}_m^- \neq \overline{\lambda}_{s_i}^-$ for every $1 \leq i \leq n$. Consider r_q and s_q are closest complex numbers to m. There are two cases : $m < r_q$ and $m > s_q$, $m > r_q$ and $m < s_q$. We think the first case, second case is like that of the first case. Because the ranges of λ_B^+ and λ_B^- are finite sets $\{r_1, r_2, ..., r_n\}$ and $\{s_1, s_2, ..., s_n\}$ respectively, we get there is no $\epsilon \in S$ such that $m < \lambda_B^+(\epsilon) \leq r_q$ and $m > \lambda_B^-(\epsilon) \geq s_q$. Using previous Corollary, we get a contradiction. □

Proposition 4.13. *Presume that* $(S, *)$ *be the biset hypergroup,* B *be any homogeneous bipolar complex fuzzy subset of* S*.* B *is a bipolar complex anti-fuzzy subhypergroup of* S*.*

Proof. Evidence is alike proof of Proposition 3.20. □

Proposition 4.14. *Presume that* $(S, *)$ *be the total hypergroup,* B *be any homogeneous bipolar complex fuzzy subset of* S*.* B *is a bipolar complex anti-fuzzy subhypergroup of* S *if and only if* λ_B^+ *and* λ_B^- *are stable complex functions.*

Proof. Evidence is alike proof of Proposition 3.21. □

Proposition 4.15. *Assume that* $(S, *)$ *be a hypergroup (H_v−group),* B *be a (homogeneous) bipolar complex fuzzy subset of* S*.* B *is a bipolar complex anti-fuzzy subhypergroup (H_v−subgroup) of* S *if and only if for every* $r = ke^{i\theta}$ *and* $t = me^{i\varphi}$ *such that* $k \in [0, 1], m \in [-1, 0]$ *and* $\theta \in [0, 2\pi], \varphi \in [-2\pi, 0]$*,, the subsequent conditions are provided:*

(1) $\overline{B}_{(r,t)} * \overline{B}_{(r,t)} \subseteq \overline{B}_{(r,t)}$,
(2) $\upsilon * (S - \overline{B}_{(r,t)}) - (S - \overline{B}_{(r,t)}) \subseteq \upsilon * \overline{B}_{(r,t)}$, *for every* $\upsilon \in \overline{B}_{(r,t)}$,
(3) $(S - \overline{B}_{(r,t)}) * \upsilon - (S - \overline{B}_{(r,t)}) \subseteq \overline{B}_{(r,t)} * \upsilon$, *for every* $\upsilon \in \overline{B}_{(r,t)}$.

Proof. Imagine B be a bipolar complex anti-fuzzy subhypergroup (H_v−subgroup) of S. $\overline{B}_{(r,t)}$ is a subhypergroup (H_v−subgroup) of S as $\upsilon * \overline{B}_{(r,t)} = \overline{B}_{(r,t)}$ for every $\upsilon \in \overline{B}_{(r,t)}$. Hence, $\overline{B}_{(r,t)} * \overline{B}_{(r,t)} \subseteq \overline{B}_{(r,t)}$. We wish to demostrate $\upsilon * (S - \overline{B}_{(r,t)}) - (S - \overline{B}_{(r,t)}) \subseteq \upsilon * \overline{B}_{(r,t)}$. Suppose that $\delta \in \upsilon * (S - \overline{B}_{(r,t)}) - (S - \overline{B}_{(r,t)})$. We get δ is not an element in $(S - \overline{B}_{(r,t)})$. This refers $\delta \in \overline{B}_{(r,t)} = \upsilon * \overline{B}_{(r,t)}$. Similarly, Condition 3 may be displayed.

Conversely, assume the conditions $1, 2$ are provided. Using Theorem 4.10 it is adequate to indicate $\overline{B}_{(r,t)}$ is a subhypergroup (H_v-subgroup) of S as $\upsilon * \overline{B}_{(r,t)} = \overline{B}_{(r,t)} * \upsilon = \overline{B}_{(r,t)}$ for every $\upsilon \in \overline{B}_{(r,t)}$. Consider there exists $\gamma \in \overline{B}_{(r,t)}$ such that γ is not an element in $\upsilon * \overline{B}_{(r,t)}$. Using reproduction property of $(S, *)$, there exists $\epsilon \in S$ such that $\gamma \in \upsilon * \epsilon$. There are two cases for ϵ :

Case 1 : $\epsilon \in \overline{B}_{(r,t)}$. We get $\gamma \in \upsilon * \epsilon \subseteq \upsilon * \overline{B}_{(r,t)}$. This is a contradiction.

Case 2 : $\epsilon \notin \overline{B}_{(r,t)}$. We get $\epsilon \in (S - \overline{B}_{(r,t)})$. $\gamma \in \upsilon * \epsilon$ means $\gamma \in \upsilon * (S - \overline{B}_{(r,t)})$. Because $\gamma \in \overline{B}_{(r,t)}$, it is like this γ is not in $(S - \overline{B}_{(r,t)})$. Hence, using assumption $\gamma \in \upsilon * (S - \overline{B}_{(r,t)}) - (S - \overline{B}_{(r,t)}) \subseteq \upsilon * \overline{B}_{(r,t)}$. This is a contradiction. Similarly, using condition 3, we can display $\overline{B}_{(r,t)} * \upsilon = \overline{B}_{(r,t)}$. □

Proposition 4.16. *Presume that $(S, *)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. Describe $\overline{\overline{\lambda}}^+$ and $\overline{\overline{\lambda}}$:*

$$\overline{\overline{\lambda}}^+ = \{s \in S : \lambda_B^+(s) = 0e^{0i}\} \text{ and } \overline{\overline{\lambda}}^- = \{s \in S : \lambda_B^-(s) = 0e^{0i}\}$$

$\overline{\overline{\lambda}}^+$ and $\overline{\overline{\lambda}}^-$ are empty or subhypergroups (H_v-subgroups) of S.

Proof. We wish to demonstrate $\tau * \overline{\overline{\lambda}}^+ = \overline{\overline{\lambda}}^+ = \overline{\overline{\lambda}}^+ * \tau$ for every $\tau \in \overline{\overline{\lambda}}^+$. Let $d \in \overline{\overline{\lambda}}^+$ and $\gamma \in \tau * d$. We get $\lambda_B^+(\gamma) \leq \max\{\lambda_B^+(\tau), \lambda_B^+(d)\} = 0e^{0i}$ indicates $\lambda_B^+(\gamma) = 0e^{0i}$. Hence, $\gamma \in \tau * d \subseteq \overline{\overline{\lambda}}^+$. For every $\tau, d \in \overline{\overline{\lambda}}^+$, there exists $\delta \in S$ such that $d \in \tau * \delta$ and $\lambda_B^+(\delta) \leq \max\{\lambda_B^+(\tau), \lambda_B^+(d)\} = 0e^{0i}$. This implies $\lambda_B^+(\delta) = 0e^{0i}$ and $\delta \in \overline{\overline{\lambda}}^+$.

Similarly, we wish to demonstrate $\nu * \overline{\overline{\lambda}}^- = \overline{\overline{\lambda}}^- = \overline{\overline{\lambda}}^- * \upsilon$ for every $\upsilon \in \overline{\overline{\lambda}}^-$. Let $d \in \overline{\overline{\lambda}}^-$ and $\gamma \in \upsilon * d$. We get $\lambda_B^-(\gamma) \geq \min\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} = 0e^{0i}$ indicates $\lambda_B^-(\gamma) = 0e^{0i}$. Hence, $\gamma \in \upsilon * d \subseteq \overline{\overline{\lambda}}^-$. For every $\upsilon, d \in \overline{\overline{\lambda}}^-$, there exists $\delta \in S$ such that $d \in \upsilon * \delta$ and $\lambda_B^-(\delta) \geq \min\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} = 0e^{0i}$. This implies $\lambda_B^-(\delta) = 0e^{0i}$ and $\delta \in \overline{\overline{\lambda}}^-$. □

Proposition 4.17. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a (homogeneous) bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. Describe $\overline{supp}(\lambda^+)$ and $\overline{supp}(\lambda^-)$:*

$$\overline{supp}(\lambda^+) = \{s \in S : \lambda_B^+(s) < 1e^{2\pi i}\}$$

and

$$\overline{supp}(\lambda^-) = \{s \in S : \lambda_B^-(s) > -1e^{-2\pi i}\}$$

$\overline{supp}(\lambda^+)$ and $\overline{supp}(\lambda^-)$ are empty or subhypergroups (H_v-subgroups) of S.

Proof. We wish to demonstrate $\tau * \overline{supp}(\lambda^+) = \overline{supp}(\lambda^+) = \overline{supp}(\lambda^+) * \tau$ for every $\tau \in \overline{supp}(\lambda^+)$. Let $d \in \overline{supp}(\lambda^+)$ and $\gamma \in \tau * d$. We get $\lambda_B^+(\gamma) \leq \max\{\lambda_B^+(\tau), \lambda_B^+(d)\} < 1e^{2\pi i}$ indicates $\lambda_B^+(\gamma) < 1e^{2\pi i}$. Hence, $\gamma \in \tau * d \subseteq \overline{supp}(\lambda^+)$. For every $\tau, d \in \overline{supp}(\lambda^+)$, there exists $\delta \in S$ such that $d \in \tau * \delta$ and $\lambda_B^+(\delta) \leq \max\{\lambda_B^+(\tau), \lambda_B^+(d)\} < 1e^{2\pi i}$. This implies $\lambda_B^+(\delta) < 1e^{2\pi i}$ and $\delta \in \overline{supp}(\lambda^+)$.

Similarly, we wish to demonstrate $\upsilon * \overline{supp}(\lambda^-) = \overline{supp}(\lambda^-) = \overline{supp}(\lambda^-) * \upsilon$ for every $\upsilon \in \overline{supp}(\lambda^-)$. Let $d \in \overline{supp}(\lambda^-)$ and $\gamma \in \upsilon * d$. We get $\lambda_B^-(\gamma) \geq \min\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} > -1e^{-2\pi i}$ indicates $\lambda_B^-(\gamma) > -1e^{-2\pi i}$. Hence, $\gamma \in \upsilon * d \subseteq \overline{supp}(\lambda^-)$. For every $\upsilon, d \in \overline{supp}(\lambda^-)$, there exists $\delta \in S$ such that $d \in \upsilon * \delta$ and $\lambda_B^-(\delta) \geq \min\{\lambda_B^-(\upsilon), \lambda_B^-(d)\} > -1e^{-2\pi i}$. This refers $\lambda_B^-(\delta) > -1e^{-2\pi i}$ and $\delta \in \overline{supp}(\lambda^-)$. □

Theorem 4.18. *Assume that $(S,*)$ be a hypergroup (H_v-group), B be a homogeneous bipolar complex fuzzy subset of S together $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$. B is a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S if and only if B^C is a bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S.*

Proof. $(\Rightarrow)$: Presume that B be a bipolar complex fuzzy subhypergroup (H_v-subgroup) of S. Using Theorem 3.12, r_B^+ and r_B^- are bipolar fuzzy subhypergroups (H_v-subgroups) of S and w_B^+ and w_B^- are bipolar $\pi-$fuzzy subhypergroups (H_v-subgroups) of S. Using Theorem 2.11, $r_{B^C}^+$ and $r_{B^C}^-$ are bipolar anti-fuzzy subhypergroups (H_v-subgroups) of S and $w_{B^C}^+$ and $w_{B^C}^-$ are bipolar $\pi-$anti- fuzzy subhypergroups (H_v-subgroups) of S. Therefore, B^C be a bipolar complex anti-fuzzy subhypergroup (H_v-subgroup) of S.

$(\Leftarrow)$: Evidence is alike to previous part. □

Corollary 4.19. *Assume that* $(S, *)$ *be a hypergroup* (H_v-*group), B be a homogeneous bipolar complex fuzzy subset of S together* $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ *and* $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$*. B is a bipolar complex fuzzy and anti-fuzzy subhypergroup* (H_v-*subgroup) of S if and only if* B^C *is a bipolar complex fuzzy and anti-fuzzy subhypergroup* (H_v-*subgroup) of S.*

Proof. Evidence is explicit. □

Example 4.20. Assume that $(S, *)$ be a biset hypergroup, $B = \{(s, \lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}, \lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}) : s \in S\}$ be any homogeneous bipolar complex fuzzy subset of S. Using Proposition 3.20 and 4.13, B and B^C are bipolar complex fuzzy and anti-fuzzy subhypergroups of S.

Example 4.21. Assume that $(S, *)$ be any hypergroup (H_v-group) together the bipolar complex fuzzy subset B of S, which is described as: $\lambda_B^+(s) = r_B^+(s)e^{i\alpha w_B^+(s)}$ and $\lambda_B^-(s) = r_B^-(s)e^{i\alpha w_B^-(s)}$,where $r_B^+(s), w_B^+(s) \in [0, 1]$ and $r_B^-(s), w_B^-(s) \in [-1, 0], \alpha \in [0, 2\pi]$ are stable real numbers. Therefore, B and B^C are homogeneous bipolar complex fuzzy and anti-fuzzy subhypergroups of S.

5. Conclusion

Article subscribes to exploration of complex subhyperstructures enlightening the notions of bipolar complex fuzzy and anti-fuzzy subhypergroups. The work delves into the view of bipolar complex fuzzy subhypergroups using bipolar complex fuzzy sets and hyperstructures, discussing their properties. It also characterizes bipolar complex anti-fuzzy subhypergroups and investigates the transition between bipolar complex fuzzy subhypergroups and bipolar complex anti-fuzzy subhypergroups. Furthermore, we suggest the relationship between both traditional bipolar fuzzy subhypergroups, traditional bipolar anti-fuzzy subhypergroups and bipolar complex fuzzy subhypergroups, bipolar complex anti-fuzzy subhypergroups.

Based on our findings, we recommended that researchers may examine subhypergroups (or H_v-subgroups) using other complex sets just as we defined subhypergroups (or H_v-subgroups) with the help of bipolar complex fuzzy sets and hyperstructures and clarified their properties.

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Sanem Yavuz
Department of Mathematics, Yildiz Technical University, P.O.Box 34220, Istanbul,Trkiye
Email: ssanemy@gmail.com

Serkan Onar
Department of Mathematical Engineering, Yildiz Technical University, P.O.Box 34220, Istanbul,Trkiye
Email: serkan10ar@gmail.com

Bayram Ali Ersoy
Department of Mathematics, Yildiz Technical University, P.O.Box 34220, Istanbul,Trkiye
Email: ersoya@gmail.com