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INTUITIONISTIC FUZZY WEAKLY PRIME IDEALS

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ABSTRACT. In this study, the fundamental definitions and theorems regarding intuitionistic fuzzy sets and intuitionistic fuzzy ideals of commutative ring R with identity have been given as preliminaries. After the preliminaries, we introduce the notions of intuitionistic fuzzy weakly prime ideals, intuitionistic fuzzy partial weakly prime ideals, intuitionistic fuzzy weakly semiprime ideals of R. Also, we give some relations between intuitionistic fuzzy weakly prime ideals and weakly prime ideals of R.

Key Words: Intuitionistic fuzzy weakly prime ideal, Intuitionistic fuzzy partial weakly prime ideal, Intuitionistic fuzzy weakly semiprime ideal.
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1. Introduction

A fuzzy set μ which is introduced by Zadeh in 1965 [10], is a (membership)function defined from the nonempty set X to the interval $[0, 1]$. In 1986, Atanassov introduced intuitionistic fuzzy sets as a generalization of fuzzy sets [2]. The intuitionistic fuzzy sets are defined on a nonempty set X as

$$A = \{\langle x, \mu(x), \nu(x)\rangle | x \in X\}$$

where the functions $\mu : X \to [0, 1]$ and $\nu : X \to [0, 1]$ denote the degrees of membership and of non-membership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu(x) + \nu(x) \leq 1$ for all $x \in X$. We use the symbol $\langle \mu, \nu\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu(x), \nu(x)\rangle | x \in X\}$[2].

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Throughout in this paper, we study on commutative rings R with identity. A proper ideal I is called a weakly prime ideal if $a \in I$ or $b \in I$ whenever $0 \neq ab \in I$ where $a, b \in R$ [1]. A proper ideal I is called a weakly semiprime ideal if $a \in I$ whenever $0 \neq a^2 \in I$ where $a \in R$ [3].

Let $P = \langle \mu_P, \nu_P \rangle$ be a nonconstant intuitionistic fuzzy ideal of R. If $(0,1)_R \neq AB \subseteq P$ implies $A \subseteq P$ or $A \subseteq P$ where $A = \langle \mu_A, \nu_A \rangle$, $B = \langle \mu_B, \nu_B \rangle$ intuitionistic fuzzy ideals of R, then P is called an intuitionistic fuzzy weakly prime ideal of R. If $P(xy) = P(x)$ or $P(xy) = P(y)$ for $xy \neq 0$, then P is called an intuitionistic fuzzy partial weakly prime ideal of R. A nonconstant an intuitionistic fuzzy ideal P is called an intuitionistic fuzzy weakly semiprime ideal of R if $(0,1)_R \neq B^2 \subseteq P$ implies $B \subseteq P$ where B is an intuitionistic fuzzy ideal of R.

2. Preliminaries

Definition 2.1. [2] Let X be a nonempty set and let $A = \langle \mu_A, \nu_A \rangle$ and $B = \langle \mu_B, \nu_B \rangle$ be intuitionistic fuzzy sets in X. Then,

(1) $A \subset B$ iff $\mu_A \leq \mu_B$ and $\nu_A \geq \nu_B$,
(2) $A = B$ iff $A \subset B$ and $B \subset A$,
(3) $A^c = \langle \nu_A, \mu_A \rangle$,
(4) $A \cap B = (\mu_A \wedge \mu_B, \nu_A \vee \nu_B)$,
(5) $A \cup B = (\mu_A \vee \mu_B, \nu_A \wedge \nu_B)$,
(6) $[\,]A = (\mu_A, 1 - \mu_A)$, $\langle\rangle A = (1 - \nu_A, \nu_A)$.

Definition 2.2. [6] Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy set in X and $(a, b) \in [0, 1] \times [0, 1]$ with $a + b \leq 1$. Then the set

$$A_{(a,b)} = \{x \in X : \mu_A(x) \geq a \text{ and } \nu_A(x) \leq b\}$$

is called (a, b)-level subset of A.

Definition 2.3. Let $A = \langle \mu_A, \nu_A \rangle$ be anintuitionistic fuzzy set in X. Then the set

$$A_* = \{x \in X | \mu_A(x) = \mu_A(0) \text{ and } \nu_A(x) = \nu_A(0)\}.$$

Definition 2.4. [7] An intuitionistic fuzzy point $x_{(a,b)}$ is defined to be an intuitionistic fuzzy subset of R, given by

$$x_{(a,b)}(y) = \begin{cases} (a, b), & x = y \\ (0, 1), & x \neq y. \end{cases}$$

Definition 2.5. [7] An intuitionistic fuzzy point $x_{(a,b)}$ is said to be belong in intuitionistic fuzzy set $A = \langle \mu_A, \nu_A \rangle$ denoted by $x_{(a,b)} \in A$ if $\mu_A(x) \geq a$ and $\nu_A(x) \leq b$ and we have for $x, y \in R$

$$x_{(a,b)} + y_{(c,d)} = (x + y)_{(a \wedge c, b \vee d)}$$
$$x_{(a,b)} \cdot y_{(c,d)} = (x \cdot y)_{(a \wedge c, b \vee d)}.$$

Definition 2.6. [4] An intuitionistic fuzzy set $A = \langle \mu_A, \nu_A \rangle$ of R is said to be an intuitionistic fuzzy subring of R if for all $x, y \in R$

(1) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$,
(2) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$.

Definition 2.7. [4] An intuitionistic fuzzy set $A = \langle \mu_A, \nu_A \rangle$ of R is said to be an intuitionistic fuzzy ideal of R if for all $x, y \in R$

(1) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$,
(2) $\mu_A(xy) \geq \mu_A(x) \vee \mu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$.

Lemma 2.8. [5] *Let A and B be an intuitionistic fuzzy ideals of R. Then $A \cap B$ is intuitionistic fuzzy ideal of R.*

Definition 2.9. [5] Let $P = \langle \mu_P, \nu_P \rangle$ be a nonconstant function. If $A \cdot B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ where $A = \langle \mu_A, \nu_A \rangle$, $B = \langle \mu_B, \nu_B \rangle$ intuitionistic fuzzy ideals of R, then P is called an intuitionistic fuzzy prime ideal of R.

Let A and B be intuitionistic fuzzy prime ideals of R. The ideal $A \cap B$ need not to be an intuitionistic fuzzy prime ideal of R.

Example 2.10. Let $R = \mathbb{Z}_6$,

$$A(x) = \begin{cases} (0.7, 0.3), & x \in 2\mathbb{Z}_6 \\ (0.1, 0.8), & \text{otherwise} \end{cases}$$

and

$$B(x) = \begin{cases} (0.6, 0.3), & x \in 3\mathbb{Z}_6 \\ (0.2, 0.8), & \text{otherwise.} \end{cases}$$

It is clear that A and B are intuitionistic fuzzy prime ideals, but $A \cap B$ is not an intuitionistic fuzzy prime ideal of because $\bar{0}_{(0.6,0.3)} = \bar{2}_{(0.6,0.3)} \cdot \bar{3}_{(0.6,0.3)} \in A \cap B$ but $\bar{2}_{(0.6,0.3)}$, $\bar{3}_{(0.6,0.3)} \notin A \cap B$.

3. Intuitionistic Fuzzy Weakly Prime Ideals

Definition 3.1. Let $P = \langle \mu_P, \nu_P \rangle$ be a nonconstant function. If $(0, 1)_R \neq A \cdot B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$ where $A = \langle \mu_A, \nu_A \rangle$,

$B = \langle \mu_B, \nu_B \rangle$ intuitionistic fuzzy ideals of R, then P is called an intuitionistic fuzzy weakly prime ideal of R.

Example 3.2. Let $R = \mathbb{Z}_6$ and $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy ideal of R defined by

$$\mu_A(x) = \begin{cases} 0.6, & x \in \{\bar{0}, \bar{3}\} \\ 0, & \text{otherwise} \end{cases} \text{ and } \nu_A(x) = \begin{cases} 0.3, & x \in \{\bar{0}, \bar{3}\} \\ 1, & \text{otherwise.} \end{cases}$$

Then, A is an intuitionistic fuzzy prime ideal, and also intuitionistic fuzzy weakly prime ideal.

Theorem 3.3. *P is an intuitionistic fuzzy weakly prime ideal of R if and only if $(0,1)_R \neq x_{(a,b)} \cdot y_{(c,d)} \in P$ implies $x_{(a,b)} \in P$ or $y_{(c,d)} \in P$ where $x_{(a,b)}, y_{(c,d)}$ are intuitionistic fuzzy points of R.*

Proof. Suppose that $(0,1)_R \neq x_{(a,b)} \cdot y_{(c,d)} \in P$. Let A and B be intuitionistic fuzzy sets defined by $A = \langle \mu_A, \nu_A \rangle$, $B = \langle \mu_B, \nu_B \rangle$ such that

$$\mu_A(r) = \begin{cases} a, & r \in \langle x \rangle \\ 0, & \text{otherwise} \end{cases} \text{ and } \nu_A(r) = \begin{cases} b, & r \in \langle x \rangle \\ 1, & \text{otherwise} \end{cases},$$

$$\mu_B(s) = \begin{cases} c, & s \in \langle y \rangle \\ 0, & \text{otherwise} \end{cases} \text{ and } \nu_B(s) = \begin{cases} d, & s \in \langle y \rangle \\ 1, & \text{otherwise.} \end{cases}$$

for $r, s \in R$. It is clear that A and B intuitionistic fuzzy ideals of R. Also, for all $z \in R$,

$$A \cdot B(z) = \{\vee\{\mu_A(z_1) \wedge \{\mu_B(z_2)\}, \wedge\{\nu_A(z_1) \vee \nu_B(z_2)\}\}.$$

If $z = xy$, then

$$A \cdot B(z) = (a \wedge c, b \vee d) \leq (\mu_P(xy), \nu_P(xy) = P(z), \text{ and so } A \cdot B \subseteq P.$$

If $z \neq xy$, then $A \cdot B(z) = (0,1) \leq P(z)$, and we get again $A \cdot B \subseteq P$. Further, $a \wedge c \neq 0$ and $b \vee d \neq 1$ since $(0,1) \neq (xy)_{(a \wedge c, b \vee d)}(xy)$ and $(xy)_{(a \wedge c, b \vee d)}(xy) = (a \wedge c, b \vee d)$. Then $A \cdot B(xy) = (a \wedge c, b \vee d) \neq (0,1)$, and so $A \cdot B \neq (0,1)_R$. Hence, $A \subseteq P$ or $B \subseteq P$ since $(0,1)_R \neq A \cdot B \subseteq P$ and P is an intuitionistic fuzzy weakly prime ideal of R, so we get $a = \mu_A(x) \leq \mu_P(x)$ and $b = \nu_A(x) \geq \nu_P(x)$, or $c = \mu_B(y) \leq \mu_P(y)$ and $d = \nu_B(y) \geq \nu_P(y)$. This implies that $x_{(a,b)} \in P$ or $y_{(c,d)} \in P$.

Conversely, let $(0,1)_R \neq A \cdot B \subseteq P$. Suppose that $A \nsubseteq P$ and $B \nsubseteq P$. Then,

$$0 \neq a = \mu_A(x) > \mu_P(x),\ 1 \neq b = \nu_A(x) < \nu_P(x) \text{ and}$$

$$0 \neq c = \mu_B(y) > \mu_P(y),\ 1 \neq d = \nu_B(y) < \nu_P(y),$$

so $x_a \notin \mu_P$, $x_b \notin \nu_P$ and $y_c \notin \mu_P$, $y_d \notin \nu_P$. Hence, $x_{(a,b)} \notin P$ and $y_{(c,d)} \notin P$ since $(a \wedge c, b \vee d) \neq (0,1)$. Further,

$$A \cdot B(xy) = (\vee\{\mu_A(x) \wedge \mu_B(x)\}, \wedge\{\nu_A(y) \vee \nu_B(y)\}) \geq (a \wedge c, b \vee d) \neq (0,1)$$

and $(a \wedge c, b \vee d) = xy_{a \wedge c, b \vee d}(xy) \leq P(xy)$. From here, $x_{(a,b)} \in P$ or $y_{(c,d)} \in P$, and so

$$\mu_P(x) \geq a \text{ and } \nu_P(x) \leq b, \text{ or}$$
$$\mu_P(y) \geq c \text{ and } \nu_P(y) \leq d.$$

This contradicts with assumption. Hence, $A \subseteq P$ or $B \subseteq P$. □

Note that an intuitionistic fuzzy weakly prime ideal need not to be an intuitionistic fuzzy prime ideal. Consider the ring $\mathbb{Z}_6$ and its intuitionistic fuzzy ideal

$$A(x) = \begin{cases} (0.7, 0.1), & x \neq \bar{0} \\ (0,1), & \text{otherwise.} \end{cases}$$

A is an intuitionistic fuzzy weakly prime ideal but not an intuitionistic fuzzy prime ideal.

Theorem 3.4. *If A is an intuitionistic fuzzy weakly prime ideal, then $A_{(a,b)}$ is a weakly prime ideal of R for all $a, b \in [0,1]$.*

Proof. Take $0 \neq xy \in A_{(a,b)}$. Then $\mu_A(xy) \geq a$ and $\nu_A(xy) \leq b$, so $xy_{(a,b)} \in A$. If $x \in A_{(a,b)}$ or $y \in A_{(a,b)}$, we are done. Suppose that $x \notin A_{(a,b)}$ and $y \notin A_{(a,b)}$. Then $\mu_A(x) < a$ or $\nu_A(x) > b$, and $\mu_A(y) < a$ or $\nu_A(y) > b$. This implies that $a \neq 0$ or $b \neq 1$, so $(a,b) \neq (0,1)$. From here, $xy_{(a,b)}(xy) = (a,b) \neq (0,1)$ and $xy_{(a,b)} \neq (0,1)_R$. Hence, $x_{(a,b)} \in A$ or $y_{(a,b)} \in A$ since A is intuitionistic fuzzy weakly prime ideal. We get $\mu_A(x) \geq a$ and $\nu_A(x) \leq b$, or $\mu_A(y) \geq a$ and $\nu_A(y) \leq b$. Therefore, $x \in A_{(a,b)}$ or $y \in A_{(a,b)}$ is obtained. □

The converse of Theorem 3.4 need not to be true.

Example 3.5. Let $R = \mathbb{Z}_4$ and $A = \langle \mu_A, \nu_A \rangle$ intuitionistic fuzzy ideal of R defined by

$$\mu_A(x) = \begin{cases} 0.6, & x \in \{\bar{0}, \bar{2}\} \\ 0, & \text{otherwise} \end{cases} \text{ and } \nu_A(x) = \begin{cases} 0.3, & x \in \{\bar{0}, \bar{2}\} \\ 1, & \text{otherwise.} \end{cases}$$

$A_{(a,b)} = \{\bar{0}, \bar{2}\}$ is a weakly prime, but A is not an intuitionistic fuzzy weakly prime ideal of R.

Theorem 3.6. *Let A be a nonconstant intuitionistic fuzzy ideal of R and $A(0) = (1, 0)$. If A is an intuitionistic fuzzy weakly prime ideal of R, then A_* is a weakly prime ideal of R and $|ImA| = 2$.*

Proof. Let $0 \neq xy \in A_*$ and suppose that $x, y \notin A_*$. Then, we have $(a, b) \leq A(xy) = A(0) = (1, 0)$ for all $(a, b) \in (0, 1] \times (0, 1]$. Hence, $(0, 1)_R \neq (xy)_{(a,b)} \in A$, and so $x_{(a,b)} \in A$ or $y_{(a,b)} \in A$ since A is an intuitionistic fuzzy weakly prime ideal. For all $(a, b) \in (0, 1] \times (0, 1]$, $x_{(a,b)}(x) = (a, b) \leq A(x)$ or $y_{(a,b)}(y) = (a, b) \leq A(y)$. In particular if we take $(a, b) = (1, 0)$, we get $A(x) = (1, 0)$ or $A(y) = (1, 0)$, so $x \in A_*$ or $y \in A_*$. But this contradicts our assumption. Further, $|ImA| \neq 1$ since A is nonconstant. Assume that $|ImA| \geq 3$. Let $A(1) = (a, b)$, $A(0) = (1, 0)$.Then, we have $r \in R$ such that $A(r) = (s, t)$ and $a < s < 1$, $0 < t < b$. Then, $r_{(s,t)}(r) = (s, t) \neq (0, 1)$ and $r_{(1,0)} \cdot 1_{(s,t)} = r_{s,t} \neq (0, 1)$. From here, we get $r_{(1,0)} \in A$ or $1_{(s,t)} \in A$. But this contradicts our assumptions, and so $|ImA| = 2$. □

4. Intuitionistic Fuzzy Partial Weakly Prime Ideals

Definition 4.1. Let A be a nonconstant intuitionistic fuzzy ideal of R. If

$$A(xy) = \langle \mu(xy), \nu(xy) \rangle = \langle \mu(x), \nu(x) \rangle = A(x) \text{ or}$$
$$A(xy) = \langle \mu(xy), \nu(xy) \rangle = \langle \mu(y), \nu(y) \rangle = A(y)$$

for $xy \neq 0$, then A is called an intuitionistic fuzzy partial weakly prime ideal.

Theorem 4.2. *If A is an intuitionistic fuzzy weakly prime ideal, then A is an intuitionistic fuzzy partial weakly prime ideal.*

Proof. Let $xy \neq 0$ and A be an intuitionistic fuzzy weakly prime ideal of R. If $A(xy) = (0, 1)$, then $A(xy) = A(x)$ or $A(xy) = A(y)$ since A is an intuitionistic fuzzy ideal of R. Let $(0, 1) \neq A(xy) = (a, b)$. Then $xy_{(a,b)} \neq (0, 1)_R$ since $xy_{(a,b)}(xy) = (a, b) \neq (0, 1)$. We get from here, $x_{(a,b)} \in A$ or $y_{(a,b)} \in A$ since A is intuitionistic fuzzy weakly prime ideal of R. Hence, $A(x) \geq (a, b) = A(xy)$ or $A(y) \geq (a, b) = A(xy)$, and so $A(xy) = A(x)$ or $A(xy) = A(y)$. □

Theorem 4.3. *Let A be a nonconstant intuitionistic fuzzy ideal of R. Then, A is intuitionistic fuzzy partial weakly prime ideal if and only if $A_{(a,b)}$ is a weakly prime ideal of R for all $a, b \in [0, 1]$.*

Proof. Suppose that A is an intuitionistic fuzzy partial weakly prime ideal of R and $0 \neq xy \in A_{(a,b)}$. Then $A(xy) \geq (a,b)$ and $A(xy) = A(x)$ or $A(xy) = A(y)$. We get $A(x) \geq (a,b)$ or $A(y) \geq (a,b)$, and so $A_{(a,b)}$ is a weakly prime ideal of R.

Conversely, take $0 \neq xy \in A$. Suppose that $A(xy) = (r,s)$. Then $0 \neq xy \in A_{(r,s)}$, and so $x \in A_{(r,s)}$ or $y \in A_{(r,s)}$. We get from here, $A(x) \geq (r,s) = A(xy)$ or $A(y) \geq (r,s) = A(xy)$. Hence, $A(x) = A(xy)$ or $A(x) = A(xy)$. □

Theorem 4.4. *Let $f : R \to S$ be an injective ring homomorphism. If B is an intuitionistic fuzzy partial weakly prime ideal of S, then $f^{-1}(B)$ is an intuitionistic fuzzy partial weakly prime ideal of R.*

Proof. Let $xy \neq 0$ where $x, y \in R$. We have

$$f^{-1}(B)(xy) = B(f(xy)) = B(f(x)f(y)).$$

Since f is injective, $f(xy) \neq 0$ and also,

$$B(f(xy)) = B(f(x)f(y)) = B(f(x)) \text{ or}$$
$$B(f(xy)) = B(f(x)f(y)) = B(f(y))$$

since B is an intuitionistic fuzzy partial weakly prime ideal of S. We get from here,

$$f^{-1}(B)(xy) = f^{-1}(B)(x) \text{ or } f^{-1}(B)(xy) = f^{-1}(B)(y),$$

and so $f^{-1}(B)$ is an intuitionistic fuzzy partial weakly prime ideal of R. □

Theorem 4.5. *Let $f : R \to S$ be a surjective ring homomorphism. If A is an intuitionistic fuzzy partial weakly prime ideal of R which is constant on $Kerf$, then $f(A)$ is an intuitionistic fuzzy partial weakly prime ideal of S.*

Proof. Let $uv \neq 0$ where $u, v \in S$. There exist $x, y \in R$ such that $f(x) = u$, $f(y) = v$ because f is an epimorphism. We have

$$f(A)(uv) = f(A)(f(x)f(y)) = f(A)(f(xy)) = A(xy)$$

since f is constant on $Kerf$. If $xy = 0$, then $f(0) = f(xy) = f(x)f(y)$, and so $uv = 0$, but this contradicts our assumption. Then, $xy \neq 0$ and,

$$f(A)(uv) = A(xy) = A(x) = f(A)(f(x)) = f(A)(u) \text{ or}$$
$$f(A)(uv) = A(xy) = A(y) = f(A)(f(y)) = f(A)(v)$$

since A is an intuitionistic fuzzy partial weakly prime ideal of R. Hence, $f(A)$ is an is intuitionistic fuzzy partial weakly prime ideal of S. $\square$

5. Intuitionistic Fuzzy Weakly Semiprime Ideals

Definition 5.1. A nonconstant intuitionistic fuzzy ideal A is called an intuitionistic fuzzy weakly semiprime ideal of R if $(0,1) \neq B^2 \subseteq A$ implies $B \subseteq A$ where B is an intuitionistic fuzzy ideal.

Theorem 5.2. *Let A be an intuitionistic fuzzy weakly semiprime ideal of R. Then A_* is a weakly semiprime ideal of R.*

Proof. Take $0 \neq x^2 \in A_*$. Then $A(x^2) = A(0)$ and $A(0) \neq (0,1)$ since A is nonconstant. Let $A(x^2) = (a,b) \neq (0,1)$. Since $x^2_{(a,b)}(x^2) = (a,b)$, $x^2_{(a,b)} \neq (0,1)_R$. Then, we have $x_{(a,b)} \in A$ since A is an intuitionistic fuzzy weakly semiprime ideal of R. Hence, $A(x) \geq (a,b) = A(0)$, and so $A(x) = A(0)$. In conclusion we get $x \in A_*$ and A_* is a weakly semiprime ideal of R. $\square$

Theorem 5.3. *A is an intuitionistic fuzzy weakly semiprime ideal of R if and only if for all $x \in R$, $a,b \in (0,1]$, $(0,1)_R \neq x^2_{(a,b)} \in A$ implies that*

Proof. Suppose that A is an intuitionistic fuzzy weakly semiprime ideal of R. Let $(0,1)_R \neq x^2_{(a,b)} \in A$. Since $(0,1)_R \neq x^2_{(a,b)}$, $(a,b) \neq (0,1)$. Let define an intuitionistic fuzzy subset of R as follows:

$$I(r) = \begin{cases} (a,b), & r \in \langle x \rangle \\ (0,1), & r \notin \langle x \rangle. \end{cases}$$

It is clear that I is an intuitionistic fuzzy ideal and $I \neq (0,1)_R$. Then, $I^2(r) = (a,b) \leq A(r)$ if $x^2 = r$, $I^2(r) = (0,1) \leq A(r)$ if $x^2 \neq r$, so $I^2 \subseteq A$. We get from here, $I \subseteq A$ since A is an intuitionistic fuzzy weakly semiprime ideal. Then, $I(x) = (a,b) \leq A(x)$. Hence, $x_{(a,b)} \in A$.

Conversely, let $(0,1) \neq B^2 \subseteq A$ and suppose that $B \nsubseteq A$. Then, there exists $x \in R$ such that $(0,1) \neq (a,b) = B(x) > A(x)$. Then, $x_{(a,b)} \notin A$. Also, we have $B^2(x^2) \geq (a,b) \neq (0,1)$, so $B^2 \neq (0,1)_R$. Further, $A(x^2) \geq B^2(x^2) \geq (a,b) = x^2_{(a,b)}(x^2)$, so $(0,1)_R \neq x^2_{(a,b)} \in A$. This implies that $x_{(a,b)} \in A$. Then, $A(x) \geq (a,b)$. But this is contradiction. Hence, $B \subseteq A$ is obtained. $\square$

Theorem 5.4. *Let A be an intuitionistic fuzzy weakly semiprime ideal of R iff $(0,1)_R \neq A(x^2)$ implies that $A(x^2) = A(x)$, for all $x \in R$.*

Proof. Suppose that A is an intuitionistic fuzzy weakly semiprime ideal of R and let $A(x^2) = (a, b) \neq (0, 1)$.Then $x^2_{(a,b)} \in A$ and $x^2_{(a,b)}(x^2) = (a, b)$. This implies that $x^2_{(a,b)} \neq (0, 1)_R$, and so $x_{(a,b)} \in A$ since A is an intuitionistic fuzzy weakly semiprime ideal of R. From here, we get $A(x) \geq (a, b) = A(x^2)$. Thus, $A(x) = A(x^2)$ is obtained.
Conversely, take $(0, 1)_R \neq x^2_{(a,b)} \in A$. Then $A(x^2) \geq (a, b) \neq (0, 1)$. From assumption we have $A(x^2) = A(x)$ and $A(x) \geq (a, b)$. Hence, $x_{(a,b)} \in A$. □

6. Conclusion

We introduced the definitions of intuitionistic fuzzy weakly prime ideals, intuitionistic fuzzy partial weakly prime ideals and intuitionistic fuzzy weakly semiprime ideals of a commutative ring with identity. Also, we gave some theorems and examples about them.

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