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PRIME AND SEMIPRIME L-FUZZY SOFT BI-HYPERIDEALS

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Abstract. In this paper, the conception of prime (semiprime) L-fuzzy soft bi-hyperideals, strongly prime L-fuzzy soft bi-hyperideals, irreducible (strongly irreducible) L-fuzzy soft bi-hyperideals of a semihypergroup S is introduced, where L is a complete bounded distributive lattice. Using the properties of these L-fuzzy soft bi-hyperideals some characterizations of regular and intra-regular semihypergroups are given.

Key Words: Prime (Semiprime) L-fuzzy soft bi-hyperideal, Strongly prime L-fuzzy soft bi-hyperideal, Irreducible (Strongly irreducible) L-fuzzy soft bi-hyperideal.
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1. Introduction

The hyperstructure theory was rose in 1934 by a French mathematician Marty [24] at the 8th congress of Scandinavian mathematicians where Marty introduced the concept of hypergroups as a generalized notion of groups and displayed its efficacy in groups, algebraic functions and rational fractions. Later on, hypercompositional structures widely studied by Kuntuzman, Krasner, Eaton, Grifth, Utumi, Dresher and Ore for their applications in both pure and applied mathematics. Koskas [18] introduced the notion of semihypergroups as an application of hyperstructure theory in pure mathematics. Corsini and Leoreanu [7] presented a book entitled by applications of hyperstructure theorey which contains wealth of applications of semihypergroups in automata, fuzzy

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set theory, rough set theory, codes, cryptography, geometry, graphs, hypergraphs, lattices, binary relations, artificial intelligence and probability theory. Hasankhani [16] introduced the notion of hyperideals in semihypergroups and Green's relations. Davvaz et al. [9], Mahmood et al. [8], Naz et al. [26], Onipchuk [27] and many other scientists studied semihypergroups in their interesting research areas and found its applications in both applied and pure mathematics.

On the other hand, soft set theory was born in 1999 by Molodtsov [25] to deal with uncertainty which occurs in engineering, medical sciences, economics and social sciences. Maji et al. [23] defined several basic operations of soft sets. Ali et al. [3] improved the work of Maji et al. [23] and introduced some new operations of soft sets. Sezgin, Atagun [31] and Ali et al. [4] also studied the algebraic structures of soft sets. Aktas and Cagman, Pei and Miao, Feng et al., Chen et al. studied the soft sets (see [1, 28, 12, 6]) and found the applications of soft sets in their respective research areas as well.

Fuzzy set was introduced by Zadeh [37]. Later on fuzzy sets studied by a large number of mathematicians and thousands of papers have been written on it. Fuzzy sets have many applications in algebraic structures like fuzzy hypernearrings, fuzzy groups, fuzzy intuintionistic hyperideals in semihypergroups, fuzzy semiprime ideals in semigroups and fuzzy ideals in semirings (see [38, 29, 17, 19, 10]). Goguen [15] introduced L-fuzzy sets as an extension of Zadeh's fuzzy sets [37] and defined basic operations of L-fuzzy sets. L-fuzzy sets are proved more useful for solving problems in optimization theory.

Maji et al. [22] initiated the study of soft sets combined with fuzzy sets. Feng et al. [11, 13], Tanay and Kandemir [35] and Yang et al. [36] studied fuzzy soft sets to boot and enhanced the work [22]. Li et al. [20] inaugurated L-fuzzy soft sets as a generalization of fuzzy soft sets based on complete boolean lattice L. Ali et al. [5], Shabir and Ghafoor [32], Shabir and Kanwal [34] and Shabir et al. [33, 21] investigated most important applications of L-fuzzy soft sets to semirings, semigroups, nearrings, semihypergroups and regular, intra-regular semihypergroups, respectively.

In this paper, the notion of prime L-fuzzy soft bi-hyperideals, strongly prime L-fuzzy soft bi-hyperideals, semiprime L-fuzzy soft bi-hyperideals, irreducible L-fuzzy soft bi-hyperideals and strongly irreducible L-fuzzy soft bi-hyperideals of a semihypergroup S over an initial universe U based on complete bounded distributive lattice L are introduced and their fundamental properties are investigated. Some counter examples have also been constructed. Moreover, some characterizations of regular and intra-regular semihypergroups are given by using these notions. Throughout this paper L denotes a complete bounded distributive lattice and S a semihypergroup.

2. Preliminaries

A non-empty set S together with a hyperoperation $\circ$ is called a hypergroupoid and denoted by $(S, \circ)$, where $\circ : S \times S \longrightarrow P^*(S)$ and $P^*(S)$ is the set of all non-empty subsets of S (c. f. [7]). We shall write

$$\circ(a, b) = a \circ b \text{ for all } a, b \in S.$$

A hypergroupoid $(S, \circ)$ is called a semihypergroup if the associative property with respect to hyperoperation $\circ$ holds, that is

$$a \circ (b \circ c) = (a \circ b) \circ c \text{ for all } a, b, c \in S.$$

Let A, B be non-empty subsets of a semihypergroup $(S, \circ)$. Then the hyperproduct of A and B is defined by

$$A \circ B = \cup_{x \in A, y \in B} (x \circ y).$$

We'll use $x \circ A$ for $\{x\} \circ A$ and $A \circ x$ for $A \circ \{x\}$.

Let $(S, \circ)$ be a semihypergroup and $\emptyset \neq H \subseteq S$. Then H is called a subsemihypergroup of S if $H \circ H \subseteq H$.

An element e of a semihypergroup $(S, \circ)$ is called the identity of S, if $x \in e \circ x = x \circ e$ for all $x \in S$.

Definition 2.1. [16] A non-empty subset A of a semihypergroup S is called:

(1) a *left hyperideal* of S, if for all $a \in A \Longrightarrow b \circ a \subseteq A$ for all $b \in S$.
(2) a *right hyperideal* of S, if for all $a \in A \Longrightarrow a \circ b \subseteq A$ for all $b \in S$.
(3) a *hyperideal* of S, if it is both a left as well as a right hyperideal of S.

Definition 2.2. [9] Let x be an element of a semihypergroup S. Then the left (right) hyperideal of S generated by x is denoted by $< x >_l$ $(< x >_r)$, where $< x >_l = (S \circ x) \cup \{x\}$ $(< x >_r = (x \circ S) \cup \{x\})$ and $< x > = (S \circ x \circ S) \cup S \circ x \cup x \circ S \cup \{x\}$.

If S is a semihypergroup with identity element, say e then $< x >_l = S \circ x$ $(< x >_r = x \circ S)$ and $< x > = S \circ x \circ S$.

Definition 2.3. [8] A non-empty subset Q of a semihypergroup S is called a Qausi-hyperideal of S if $Q \circ S \cap S \circ Q \subseteq Q$.

Definition 2.4. [8] A subsemihypergroup B of a semihypergroup S is called a bi-hyperideal of S if $B \circ S \circ B \subseteq B$.

Definition 2.5. [8] A non-empty subset G of a semihypergroup S is called a *generalized bi-hyperideal* of S if $G \circ S \circ G \subseteq G$.

Definition 2.6. [8] A bi-hyperideal B of a semihypergroup S is called prime (semiprime) if $B_1 \circ B_2 \subseteq B\,(B_1 \circ B_1 \subseteq B)$ implies that either $B_1 \subseteq B$ or $B_2 \subseteq B\,(B_1 \subseteq B)$ for any bi-hyperideals B_1 and B_2 of S.

Definition 2.7. [8] A bi-hyperideal B of a semihypergroup S is called strongly prime if $B_1 \circ B_2 \cap B_2 \circ B_1 \subseteq B$ implies that either $B_1 \subseteq B$ or $B_2 \subseteq B$ for any bi-hyperideals B_1 and B_2 of S.

Definition 2.8. [8] A bi-hyperideal B of a semihypergroup S is called irreducible (strongly irreducible) if $B_1 \cap B_2 = B\,(B_1 \cap B_2 \subseteq B)$ implies that either $B_1 = B$ or $B_2 = B\,(B_1 \subseteq B$ or $B_2 \subseteq B)$ for any bi-hyperideals B_1 and B_2 of S.

Definition 2.9. [8] A semihypergroup S is called regular if for all $a \in S$ there exists $s \in S$ such that $a \in a \circ s \circ a$.

Definition 2.10. [8] A semihypergroup S is called intra-regular if for all $a \in S$ there exist $x, y \in S$ such that $a \in x \circ a \circ a \circ y$.

Definition 2.11. [20] Let $(L, \leq)$ be a partailly ordered set. Then L is called a *lattice* if

$$a \vee b \in L, a \wedge b \in L \quad \text{for all } a, b \in L.$$

Definition 2.12. [20] A lattice $(L, \leq)$ is called:

(1) a *complete lattice,* if $\vee N \in L, \wedge N \in L$ for every subset N of L.
(2) a *bounded lattice,* if a top element $1_L \in L$ as well as a lower element $0_L \in L$.
(3) a *distributive lattice,* if

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)\,, a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

for all $a, b, c \in L$.

Definition 2.13. [15] Let $U \neq \emptyset$ and L be a complete distributive lattice with 0_L and 1_L. Then an L-fuzzy set A in U is a function $A : U \longrightarrow L$ which maps each element of U to a unique element of L. The set of all L-fuzzy sets in U is denoted by L^U.

Definition 2.14. [20] Let E be the set of parameters, U be an initial universe and L be a complete boolean lattice. Then an L-fuzzy soft set f_A over U is defined as $f_A : E \longrightarrow L^U$ such that $f_A(e) = \widehat{0}$ for all $e \notin A$, where $\widehat{0}$ is the L-fuzzy set which maps each element of U to $0 \in L$ and $A \subseteq E$.

Definition 2.15. [20] Some fundamental operations of L-fuzzy soft sets are given as follows:

(1) Let f_A and g_B be two L-fuzzy soft sets over U. Then f_A is said to be a *subset* of g_B if $f_A(x) \subseteq g_B(x)$ for all $x \in E$ and is denoted by $f_A \widetilde{\subseteq} g_B$.
(2) The *union* of two L-fuzzy soft sets f_A and g_B over U is denoted by $f_A \widetilde{\cup} g_B \widetilde{=} h_{A\cup B}$, where $h_{A\cup B}(x) = f_A(x) \cup g_B(x)$ for all $x \in E$.
(3) The *intersection* of two L-fuzzy soft sets f_A and g_B over U is denoted by $f_A \widetilde{\cap} g_B \widetilde{=} h_{A\cap B}$, where $h_{A\cap B}(x) = f_A(x) \cap g_B(x)$ for all $x \in E$.
(4) Two L-fuzzy soft sets f_A and g_B over U are said to be *equal* if $f_A(x) = g_B(x)$ for all $x \in E$ and is denoted by $f_A \widetilde{=} g_B$.

Definition 2.16. [33] Let S be a semihypergroup, U be an initial universe, L be a complete bounded distributive lattice and $A \subseteq S$. Then an *L-fuzzy soft set f_A of a semihypergroup S over U* is a function $f_A : S \longrightarrow L^U$ such that $f_A(s) = \widehat{0}$ for all $s \notin A$.

Let f_A and g_B be two L-fuzzy soft sets of a semihypergroup S over U. Then the product of f_A and g_B is an L-fuzzy soft set defined by

$$(f_A \circledast g_B)(s) = \begin{cases} \cup_{s\in a\circ b} \{f_A(a) \cap g_B(b)\}, & \text{if } \exists\, a, b \in S \text{ such that } s \in a \circ b \\ \widehat{0}, & \text{otherwise.} \end{cases}$$

for all $s \in S$.

Definition 2.17. [33] Let S be a semihypergroup and $\emptyset \neq A \subseteq S$. Then an L-fuzzy soft set $C_A : S \longrightarrow L^U$ defined by

$$C_A(x) = \begin{cases} \widehat{1}, \text{ if } x \in A \\ \widehat{0}, \text{ if } x \notin A. \end{cases}$$

for all $x \in S$, *called the L-fuzzy soft characteristic function of A over U.*

Proposition 2.18. [33] *Let S be a semihypergroup and A, $B \subseteq S$ such that $A \neq \emptyset$, $B \neq \emptyset$. Then*

(1) $A \subseteq B$ if and only if $C_A \widetilde{\subseteq} C_B$.
(2) $C_A \widetilde{\cap} C_B \widetilde{=} C_{A\cap B}$ and $C_A \widetilde{\cup} C_B \widetilde{=} C_{A\cup B}$.
(3) $C_A \circledast C_B \widetilde{=} C_{A\circ B}$.

Definition 2.19. [33] Let S be a semihypergroup. Then an L-fuzzy soft set f_A of S over U is called an L-fuzzy soft subsemihypergroup of S over U if for all $a \in x \circ y$, we have $\cap_{a\in x\circ y} \{f_A(a)\} \supseteq f_A(x) \cap f_A(y)$ for all $x, y \in S$.

Proposition 2.20. [33] *An L-fuzzy soft set f_A of a semihypergroup S over U is an L-fuzzy soft subsemihypergroup of S if and only if $f_A \circledast f_A \widetilde{\subseteq} f_A$.*

Definition 2.21. [33] Let $\alpha \in L^U$ and f_A be an L-fuzzy soft set of a semihypergroup S over U. Then $\alpha-$cut of f_A is denoted and defined by $f_A^{\alpha} = \{x \in S : f_A(x) \supseteq \alpha\}$.

Definition 2.22. [33] Let S be a semihypergroup and f_A be an L-fuzzy soft set of S over U. Then f_A is called

(1) an *L-fuzzy soft left hyperideal* of S over U if for each $a, b \in S$, we have $\cap_{x\in a\circ b} f_A(x) \supseteq f_A(b)$.
(2) an *L-fuzzy soft right hyperideal* of S over U if for each $a, b \in S$, we have $\cap_{x\in a\circ b} f_A(x) \supseteq f_A(a)$.
(3) an *L-fuzzy soft hyperideal* of S over U if it is both an L-fuzzy soft left hyperideal and an L-fuzzy soft right hyperideal of S over U.

Proposition 2.23. [33] *Let S be a semihypergroup and $\emptyset \neq A \subseteq S$. Then A is a left (right) hyperideal of S if and only if the L-fuzzy soft characteristic function C_A of A is an L-fuzzy soft left (right) hyperideal of S over U.*

Definition 2.24. [21] An L-fuzzy soft set f_A of a semihypergroup S over U is called an L-fuzzy soft qausi-hyperideal of S over U if

$$\left(f_A \circledast \widehat{1}\right) \widetilde{\cap} \left(\widehat{1} \circledast f_A\right) \widetilde{\subseteq} f_A.$$

Definition 2.25. [21] An L-fuzzy soft set f_G of a semihypergroup S over U is called an L-fuzzy soft generalized bi-hyperideal of S over U if $\cap_{a\in x\circ y\circ z} \{f_G(a)\} \supseteq f_G(x) \cap f_G(z)$ for all $x, y, z \in S$.

Proposition 2.26. [21] *An L-fuzzy soft set f_A of a semihypergroup S over U is an L-fuzzy soft bi-hyperideal of S over U if and only if*

(1) $f_A \circledast f_A \widetilde{\subseteq} f_A$.
(2) $f_A \circledast \widehat{1} \circledast f_A \widetilde{\subseteq} f_A$.

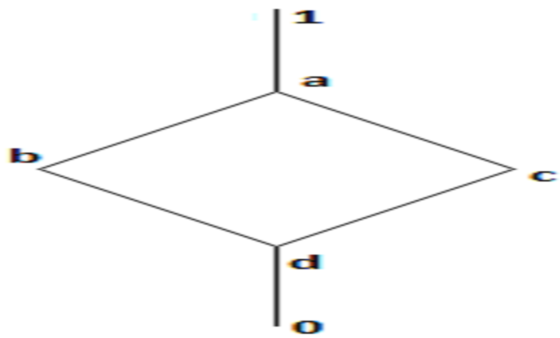
Theorem 2.27. [21] *The following assertions are equivalent for a semihypergroup S:*

(1) S is regular and intra-regular.
(2) Every L-fuzzy soft qausi-hyperideal of S over U is idempotent.
(3) Every L-fuzzy soft bi-hyperideal of S over U is idempotent.
(4) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft qausi-hyperideal f_A and g_B of S over U.
(5) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
(6) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft qausi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
(7) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
(8) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft bi-hyperideal f_A and g_B of S over U.
(9) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft bi-hyperideal f_A and every L-fuzzy soft generalized bi-hyperideal g_B of S over U.
(10) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft qausi-hyperideal g_B of S over U.
(11) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and every L-fuzzy soft bi-hyperideal g_B of S over U.
(12) $f_A \widetilde{\cap} g_B \widetilde{\subseteq} f_A \circledast g_B$ for every L-fuzzy soft generalized bi-hyperideal f_A and g_B of S over U.

3. Prime and Semiprime L-fuzzy soft Bi-hyperideals

In this Section, we introduce prime (semiprime) L-fuzzy soft bi-hyperideal and strongly prime L-fuzzy soft bi-hyperideal of a semihypergroup S over U. We also introduce the notions of irreducible (strongly irreducile) L-fuzzy soft bi-hyperideal of S over U. We prove some significant results about these notions. Later on, we characterize regular and intra-regular semihypergroups by the properties of these L-fuzzy soft bi-hyperideals.

Definition 3.1. Let f_B be an L-fuzzy soft bi-hyperideal of a semihypergroup S over U. Then we say that f_B is prime (semiprime) L-fuzzy soft bi-hyperideal if for all L-fuzzy soft bi-hyperideals f_{B_1} and f_{B_2} of S



over U, we have

$$f_{B_1} \circledast f_{B_2} \widetilde{\subseteq} f_B \left(f_{B_1} \circledast f_{B_1} \widetilde{\subseteq} f_B \right) \Rightarrow f_{B_1} \widetilde{\subseteq} f_B \text{ or } f_{B_2} \widetilde{\subseteq} f_B \left(f_{B_1} \widetilde{\subseteq} f_B \right).$$

Definition 3.2. Let f_B be an L-fuzzy soft bi-hyperideal of a semihypergroup S over U. Then we say that f_B is a strongly prime L-fuzzy soft bi-hyperideal of S over U if for all L-fuzzy soft bi-hyperideals f_{B_1} and f_{B_2} of S over U, we have

$$(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_B \text{ implies that either } f_{B_1} \widetilde{\subseteq} f_B \text{ or } f_{B_1} \widetilde{\subseteq} f_B.$$

Remark 3.3. Every strongly prime L-fuzzy soft bi-hyperideal of a semihypergroup S over U is a prime L-fuzzy soft bi-hyperideal but the converse is not true in general.

Example 3.4. Consider a semihypergroup $S = \{e, x, y, z\}$ with hyperoperation $\circ$ defined in the following table:

$\circ$	e	x	y	z
e	$\{e\}$	$\{e\}$	$\{e\}$	$\{e\}$
x	$\{e\}$	$\{x\}$	$\{x\}$	$\{x\}$
y	$\{e\}$	$\{y\}$	$\{y\}$	$\{y\}$
z	$\{e\}$	$\{z\}$	$\{z\}$	$\{z\}$

Then every subset of S containing e is a bi-hyperideal of S. Let the initial universe be $U = \{p, q\}$ and $L = \{0, a, b, c, d, 1\}$ be the complete bounded distributive lattice shown in the above figure:

Let $A = \{e\}$ be the subset of S. Define an L-fuzzy soft set $f_A : S \longrightarrow L^U$ by $f_A(e) = \left\{\frac{p}{1}, \frac{q}{1}\right\} = \widehat{1}$, $f_A(x) = f_A(y) = f_A(z) = \left\{\frac{p}{0}, \frac{q}{0}\right\}$. Then f_A is an L-fuzzy soft bi-hyperideal of S over U. Let B, D be subsets of S and g_B and h_D be any L-fuzzy soft bi-hyperideals of S over U such that $h_D \circledast g_B \widetilde{\subseteq} f_A$. Simple calculations show that either $h_D \widetilde{\subseteq} f_A$ or

$g_B \widetilde{\subseteq} f_A$. This shows that f_A is prime. Let us define two L-fuzzy soft bi-hyperideals of S over U by $f_B(e) = \left\{\frac{p}{1}, \frac{q}{1}\right\} = \widehat{1}$, $f_B(x) = f_B(y) = \left\{\frac{p}{a}, \frac{q}{1}\right\}$, $f_B(z) = \left\{\frac{p}{0}, \frac{q}{0}\right\} = \widehat{0}$ and $f_D(e) = \left\{\frac{p}{1}, \frac{q}{1}\right\} = \widehat{1}$, $f_D(x) = f_D(y) = \left\{\frac{p}{0}, \frac{q}{0}\right\} = \widehat{0}$, $f_B(z) = \left\{\frac{p}{a}, \frac{q}{1}\right\}$. Then simple calculations show that $(f_B \circledast f_D) \widetilde{\cap} (f_D \circledast f_B) \widetilde{=} f_B \widetilde{\cap} f_D \widetilde{=} f_A$. But neither $f_B \widetilde{\nsubseteq} f_A$ nor $f_D \widetilde{\nsubseteq} f_A$. Hence f_A is not strongly prime L-fuzzy soft bi-hyperideal of S over U.

Proposition 3.5. *Let $\{f_{A_i} : i \in I\}$ be a family of prime L-fuzzy soft bi-hyperideals of a semihypergroup S over U. Then $\widetilde{\cap}_{i \in I} f_{A_i}$ is a semiprime L-fuzzy soft bi-hyperideal of S over U.*

Proof. Straightforward. □

Definition 3.6. Let f_B be an L-fuzzy soft bi-hyperideal of a semihypergroup S over U. Then we say f_B is an irreducible (strongly irreducible) L-fuzzy soft bi-hyperideal of S over U if for any L-fuzzy soft bi-hyperideals f_{B_1} and f_{B_2} of S over U, we have $f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{=} f_B \left(f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{\subseteq} f_B \right)$ implies $f_{B_1} \widetilde{=} f_B$ or $f_{B_2} \widetilde{=} f_B \left(f_{B_1} \widetilde{\subseteq} f_B \text{ or } f_{B_2} \widetilde{\subseteq} f_B \right)$.

Proposition 3.7. *Every strongly irreducible semiprime L-fuzzy soft bi-hyperideal of a semihypergroup S over U is a strongly prime L-fuzzy soft bi-hyperideal of S over U.*

Proof. Let f_B be an strongly irreducible semiprime L-fuzzy soft bi-hyperideal of S over U. Let f_{B_1} and f_{B_2} be two L-fuzzy soft bi-hyperideals of S over U such that $(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_B$. Since $f_{B_1} \widetilde{\cap} f_{B_2}$ is an L-fuzzy soft bi-hyperideal of S over U and

$$\left(f_{B_1} \widetilde{\cap} f_{B_2}\right) \circledast \left(f_{B_2} \widetilde{\cap} f_{B_1}\right) \widetilde{\subseteq} f_{B_1} \circledast f_{B_2}$$
$$\text{also } \left(f_{B_1} \widetilde{\cap} f_{B_2}\right) \circledast \left(f_{B_2} \widetilde{\cap} f_{B_1}\right) \widetilde{\subseteq} f_{B_2} \circledast f_{B_1}.$$

Therefore $\left(f_{B_1} \widetilde{\cap} f_{B_2}\right) \circledast \left(f_{B_2} \widetilde{\cap} f_{B_1}\right) \widetilde{\subseteq} (f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_B$.

Since f_B is a semiprime L-fuzzy soft bi-hyperideal of S over U, so $f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{\subseteq} f_B$ and f_B is strongly irreducible L-fuzzy soft bi-hyperideal of S over U, so either $f_{B_1} \widetilde{\subseteq} f_B$ or $f_{B_2} \widetilde{\subseteq} f_B$. Thus f_B is a strongly prime L-fuzzy soft bi-hyperideal of S over U. □

Theorem 3.8. *Let f_A be an L-fuzzy soft bi-hyperideal of a semihypergroup S over U with $f_A(s) = \alpha$, where $s \in S$ and $\alpha \in L^U$. Then there exists an irreducible L-fuzzy soft bi-hyperideal g_B of S over U such that $f_A \widetilde{\subseteq} g_B$ and $g_B(s) = \alpha$.*

Proof. Let

$X=\left\{h_C : h_C \text{ is L-fuzzy soft bi-hyperideal of } S \text{ over } U,\ h_C(s)=\alpha \text{ and } f_A\widetilde{\subseteq}h_C\right\}.$

Then $X\neq\emptyset$, because $f_A\in X$. The collection X is partially ordered set under inclusion. Let Y be any totally ordered subset of X, say $Y=\{h_{C_i}: i\in I\}$. Let $x,y,z\in S$. Then for any $a\in x\circ y$,

$$\begin{aligned}\cap_{a\in x\circ y}\left(\widetilde{\cup}_{i\in I}h_{C_i}\right)(a) &= \cup_{i\in I}\left(\cap_{a\in x\circ y}h_{C_i}(a)\right)\\ &\supseteq \cup_{i\in I}\left(h_{C_i}(x)\cap h_{C_i}(y)\right)\\ &= \left(\cup_{i\in I}h_{C_i}(x)\right)\cap\left(\cup_{i\in I}h_{C_i}(y)\right)\\ &= \left\{\widetilde{\cup}_{i\in I}h_{C_i}\right\}(x)\cap\left\{\widetilde{\cup}_{i\in I}h_{C_i}\right\}(y).\end{aligned}$$

Which shows that $\widetilde{\cup}_{i\in I}h_{C_i}$ is an L-fuzzy soft subsemihypergroup of S over U. Also for each $b\in x\circ y\circ z$,

$$\begin{aligned}\cap_{b\in x\circ y\circ z}\left(\widetilde{\cup}_{i\in I}h_{C_i}\right)(b) &= \cup_{i\in I}\left(\cap_{b\in x\circ y\circ z}h_{C_i}(b)\right)\\ &\supseteq \cup_{i\in I}\left(h_{C_i}(x)\cap h_{C_i}(z)\right)\\ &= \left(\cup_{i\in I}h_{C_i}(x)\right)\cap\left(\cup_{i\in I}h_{C_i}(z)\right)\\ &= \left\{\widetilde{\cup}_{i\in I}h_{C_i}\right\}(x)\cap\left\{\widetilde{\cup}_{i\in I}h_{C_i}\right\}(z).\end{aligned}$$

Hence $\widetilde{\cup}_{i\in I}h_{C_i}$ is an L-fuzzy soft bi-hyperideal of S over U. As $f_A\widetilde{\subseteq}h_{C_i}$ for each $i\in I$, so $f_A\widetilde{\subseteq}\widetilde{\cup}_{i\in I}h_{C_i}$. Also $\left(\widetilde{\cup}_{i\in I}h_{C_i}\right)(s)=\widetilde{\cup}_{i\in I}\left(h_{C_i}(s)\right)=\alpha$. Thus $\widetilde{\cup}_{i\in I}h_{C_i}$ is the least upper bound of Y. Hence by Zorn's Lemma, there exists an L-fuzzy soft bi-hyperideal g_B of S over U with respect to the property that $f_A\widetilde{\subseteq}g_B$ and $g_B(s)=\alpha$.

Now we show that g_B is an irreducible L-fuzzy soft bi-hyperideal of S over U. Suppose $g_B\widetilde{=}t_D\widetilde{\cap}l_F$, where t_D and l_F are L-fuzzy soft bi-hyperideals of S over U. Then $g_B\widetilde{\subseteq}t_D$ and $g_B\widetilde{\subseteq}l_F$. We claim that $g_B\widetilde{=}t_D$ or $g_B\widetilde{=}l_F$. Suppose on contrary that $g_B\not\cong t_D$ and $g_B\not\cong l_F$. Since g_B is maximal with respect to the property that $g_B(s)=\alpha$ but $g_B\not\cong t_D$ and $g_B\not\cong l_F$. It follows that $t_D(s)\neq\alpha$ and $l_F(s)\neq\alpha$, this implies that $g_B(s)=\left(t_D\widetilde{\cap}l_F\right)(s)=t_D(s)\cap l_F(s)\neq\alpha$. Which is a contradiction. Hence either $g_B\widetilde{=}t_D$ or $g_B\widetilde{=}l_F$. Thus g_B is an irreducible L-fuzzy soft bi-hyperideal of S over U. □

Theorem 3.9. *Let S be a semihypergroup. Then the following statements are equivalent:*

(1) S is both regular and intra-regular.
(2) $f_B\circledast f_B\widetilde{=}f_B$ for each L-fuzzy soft bi-hyperideal f_B of S over U.

(3) $f_{B_1}\widetilde{\cap} f_{B_2} \widetilde{=} (f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1})$ for each L-fuzzy soft bi-hyperideal f_{B_1} and f_{B_2} of S over U.
(4) Each L-fuzzy soft bi-hyperideal of S over U is semiprime.
(5) Each proper L-fuzzy soft bi-hyperideal of S over U is the intersection of all irreducible L-fuzzy soft bi-hyperideals of S over U which contain it.

Proof. (1) $\Leftrightarrow$ (2) Follows from Theorem 2.27.

(2) $\Longrightarrow$ (3) Let f_{B_1} and f_{B_2} be any two L-fuzzy soft bi-hyperideals of S over U. Since $f_{B_1}\widetilde{\cap} f_{B_2}$ is an L-fuzzy soft bi-hyperideal of S over U, so by hypothesis

$$f_{B_1}\widetilde{\cap} f_{B_2} \widetilde{=} \left(f_{B_1}\widetilde{\cap} f_{B_2}\right) \circledast \left(f_{B_1}\widetilde{\cap} f_{B_2}\right) \widetilde{\subseteq} f_{B_1} \circledast f_{B_2}$$
$$\text{and } f_{B_1}\widetilde{\cap} f_{B_2} \widetilde{=} \left(f_{B_1}\widetilde{\cap} f_{B_2}\right) \circledast \left(f_{B_1}\widetilde{\cap} f_{B_2}\right) \widetilde{\subseteq} f_{B_2} \circledast f_{B_1}.$$

This implies that $f_{B_1}\widetilde{\cap} f_{B_2} \widetilde{\subseteq} (f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1})$.

Since $f_{B_1} \circledast f_{B_2}$ and $f_{B_2} \circledast f_{B_1}$ are L-fuzzy soft bi-hyperideals of S over U, so by hypothesis we have

$$\begin{aligned}
&(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \\
&\widetilde{=} \left\{(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1})\right\} \circledast \left\{(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1})\right\} \\
&\widetilde{\subseteq} (f_{B_1} \circledast f_{B_2}) \circledast (f_{B_2} \circledast f_{B_1}) \\
&\widetilde{\subseteq} \left(f_{B_1} \circledast \widehat{1}\right) \circledast \left(\widehat{1} \circledast f_{B_1}\right) \\
&\widetilde{\subseteq} f_{B_1} \circledast \left(\widehat{1} \circledast \widehat{1}\right) \circledast f_{B_1} \\
&\widetilde{\subseteq} f_{B_1} \circledast \widehat{1} \circledast f_{B_1} \widetilde{\subseteq} f_{B_1}.
\end{aligned}$$

Similarly, we can show that $(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_{B_2}$. This implies that $(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_{B_1}\widetilde{\cap} f_{B_2}$. Hence

$$(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{=} f_{B_1}\widetilde{\cap} f_{B_2}.$$

(3) $\Longrightarrow$ (4) Let f_A and g_B be L-fuzzy soft bi-hyperideals of S over U such that $f_A \circledast f_A \widetilde{\subseteq} g_B$. By hypothesis,

$$f_A \widetilde{=} f_A \widetilde{\cap} f_A \widetilde{=} (f_A \circledast f_A) \widetilde{\cap} (f_A \circledast f_A) \widetilde{=} f_A \circledast f_A.$$

Thus $f_A \widetilde{\subseteq} g_B$. Hence every L-fuzzy soft bi-hyperideal of S over U is semiprime.

(4) $\Longrightarrow$ (5) Let f_A be a proper L-fuzzy soft bi-hyperideal of S over U and $\{f_{A_i} : i \in I\}$ be the collection of all irreducible L-fuzzy soft bi-hyperideals of S over U which contain f_A. By Proposition 3.7, this collection is non-empty. Hence $f_A \widetilde{\subseteq} \widetilde{\cap}_{i\in I} f_{A_i}$. Let $s \in S$. Then by Proposition 3.7, there exists an irreducible L-fuzzy soft bi-hyperideal f_{A_α} of S over

U such that $f_A \widetilde{\subseteq} f_{A_\alpha}$ and $f_A(s) = f_{A_\alpha}(s)$. Thus $f_{A_\alpha} \in \{f_{A_i} : i \in I\}$. Hence $\widetilde{\cap}_{i\in I} f_{A_i} \widetilde{\subseteq} f_{A_\alpha}$, which shows that $\cap_{i\in I} f_{A_i}(s) \subseteq f_{A_\alpha}(s) = f_A(s)$. Thus $\widetilde{\cap}_{i\in I} f_{A_i} \widetilde{\subseteq} f_{A_\alpha}$. Consequently $\widetilde{\cap}_{i\in I} f_{A_i} \widetilde{=} f_A$. By hypothesis, each L-fuzzy soft bi-hyperideal of S over U is semiprime. Thus each L-fuzzy soft bi-hyperideal of S over U is the intersection of all irreducible semiprime L-fuzzy soft bi-hyperideals of S over U which contain it.

(5) $\Longrightarrow$ (2) Let f_B be an L-fuzzy soft bi-hyperideal of S over U. Then $f_B \circledast f_B$ is also an L-fuzzy soft bi-hyperideal of S over U. Since f_B is an L-fuzzy soft subsemihypergroup of S over U, so by Proposition 2.20, $f_B \circledast f_B \widetilde{\subseteq} f_B$. By hypothesis

$$f_B \circledast f_B \widetilde{=} \widetilde{\cap}_{i\in I} \left\{ \begin{array}{c} f_{A_i} : f_{A_i} \text{ is irreducible semiprime L-fuzzy soft bi-hyperideals} \\ \text{which contains } f_B \circledast f_B \end{array} \right\}.$$

Thus $f_B \circledast f_B \widetilde{\subseteq} f_{A_i}$ for all $i \in I$. Since each f_{A_i} is semiprime, so $f_B \widetilde{\subseteq} f_{A_i}$ for all $i \in I$. Thus $f_B \widetilde{\subseteq} \widetilde{\cap}_{i\in I} f_{A_i} \widetilde{=} f_B \circledast f_B$. Hence $f_B \circledast f_B \widetilde{=} f_B$. □

Remark 3.10. Every prime L-fuzzy soft bi-hyperideal of a semihypergroup S over U is a semiprime L-fuzzy soft bi-hyperideal of S over U. But the converse is not true in general.

Example 3.11. Consider the semihypergroup $S = \{x, y\}$ with hyperoperation defined in the following table:

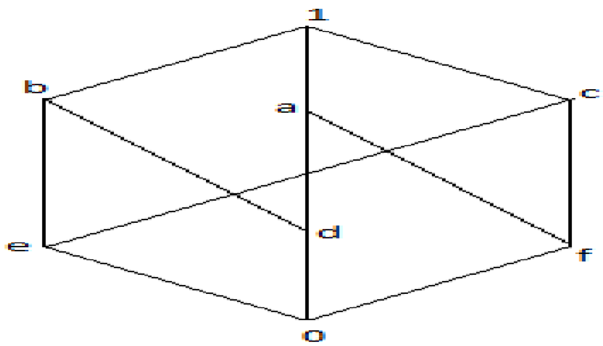
$\circ$	x	y
x	$\{x\}$	$\{x\}$
y	$\{y\}$	$\{y\}$

This semihypergroup S is both regular and intra-regular. Here the bi-hyperideals are $\{x\}$, $\{y\}$ and S. Let l_D be an arbitrary L-fuzzy soft set of S over U. Then it can easily be see that

$$\begin{aligned} (l_D \circledast l_D)(x) &= l_D(x) \\ (l_D \circledast l_D)(y) &= l_D(y) \end{aligned}$$

This shows that $l_D \circledast l_D \widetilde{\subseteq} l_D$. By Proposition 2.20 l_D is an L-fuzzy soft subsemihypergroup of S over U. Thus each L-fuzzy soft set of S over U is an L-fuzzy soft subsemihypergroup of S over U. Also

$$\begin{aligned} \left(l_D \circledast \widehat{1} \circledast l_D\right)(x) &= l_D(x) \\ \left(l_D \circledast \widehat{1} \circledast l_D\right)(y) &= l_D(y) \end{aligned}$$



Which shows that $l_D \circledast \widehat{1} \circledast l_D \widetilde{\subseteq} l_D$. Hence by Proposition 2.26 l_D is an L-fuzzy soft subsemihypergroup of S over U. Hence each L-fuzzy soft set of S over U is an L-fuzzy soft bi-hyperideal. Since S is regular as well as intra-regular, so by Theorem 3.9 every $L-$fuzzy soft bi-hyperideal is semiprime.

Now let $U = \{l, m, n\}$ and $L = \{0, a, b, c, d, e, f, 1\}$ depicted in the above figure.

Let $A = \{x\}$, $B = \{y\}$ and $C = \{x, y\} = S$. Define L-fuzzy soft sets $f_A : S \longrightarrow L^U$ as $f_A(x) = \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{d}\right\}$, $f_A(y) = \widehat{0}$, $g_B : S \longrightarrow L^U$ as $g_B(x) = \widehat{0}$, $g_B(y) = \left\{\frac{l}{1}, \frac{m}{1}, \frac{n}{f}\right\}$ and $h_C : S \longrightarrow L^U$ as $h_C(x) = h_C(y) = \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{0}\right\}$. Then we obtain $f_A \circledast g_B$ after simple calculations as

$$\begin{aligned}
(f_A \circledast g_B)(x) &= \cup_{x \in u \circ v} \{f_A(u) \cap g_B(v)\} \\
&= \cup \{f_A(x) \cap g_B(x), f_A(x) \cap g_B(y)\} \\
&= \cup \left\{\left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{d}\right\} \cap \widehat{0}, \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{d}\right\} \cap \left\{\frac{l}{1}, \frac{m}{1}, \frac{n}{f}\right\}\right\} \\
&= \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{0}\right\} = h_C(x).
\end{aligned}$$

$$\begin{aligned}
(f_A \circledast g_B)(y) &= \cup_{y \in u \circ v} \{f_A(u) \cap g_B(v)\} \\
&= \cup \{f_A(y) \cap g_B(x), f_A(y) \cap g_B(y)\} \\
&= \cup \left\{\widehat{0} \cap \widehat{0}, \widehat{0} \cap \left\{\frac{l}{1}, \frac{m}{1}, \frac{n}{f}\right\}\right\} \\
&= \widehat{0} \subseteq \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{0}\right\} = h_C(y).
\end{aligned}$$

Clearly, $f_A \circledast g_B \widetilde{\subseteq} h_C$ but $f_A \widetilde{\nsubseteq} h_C$ not $g_B \widetilde{\nsubseteq} h_C$ because $f_A(x) = \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{d}\right\} \nsubseteq \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{0}\right\}$, since $d \nleq 0$ and similarly, $g_B(y) = \left\{\frac{l}{1}, \frac{m}{1}, \frac{n}{f}\right\} \nsubseteq \left\{\frac{l}{1}, \frac{m}{a}, \frac{n}{0}\right\}$, since$1 \nleq a$ and $f \nleq 0$.

Proposition 3.12. *Let S be both regular and intra-regular semihypergroup. Then the following statements are equivalent:*

(1) Every L-fuzzy soft bi-hyperideal of S over U is strongly irreducible.
(2) Every L-fuzzy soft bi-hyperideal of S over U is strongly prime.

Proof. (1) $\Longrightarrow$ (2) Suppose that f_A is a strongly irreducible L-fuzzy soft bi-hyperideal of S over U and g_B, h_C be any L-fuzzy soft bi-hyperideals of S over U such that $(g_B \circledast h_C) \widetilde{\cap} (h_C \circledast g_B) \widetilde{\subseteq} f_A$. Since S is both regular and intra-regular semihypergroup, so by Theorem 3.9, $(g_B \circledast h_C) \widetilde{\cap} (h_C \circledast g_B) \widetilde{=} g_B \widetilde{\cap} h_C$. This implies

$g_B \widetilde{\cap} h_C \widetilde{\subseteq} f_A$. Since f_A is strongly irreducible, so either $g_B \widetilde{\subseteq} f_A$ or $h_C \widetilde{\subseteq} f_A$. Thus f_A is strongly prime L-fuzzy soft bi-hyperideal of S over U.

(2) $\Longrightarrow$ (1) Assume that f_A is a strongly prime L-fuzzy soft bi-hyperideal of S over U and g_B, h_C be L-fuzzy soft bi-hyperideals of S over U such that $g_B \widetilde{\cap} h_C \widetilde{\subseteq} f_A$. Since $(g_B \circledast h_C) \widetilde{\cap} (h_C \circledast g_B) \widetilde{\subseteq} g_B \widetilde{\cap} h_C \widetilde{\subseteq} f_A$ and f_A is strongly prime, so either $g_B \widetilde{\subseteq} f_A$ or $h_C \widetilde{\subseteq} f_A$. Thus f_A is strongly irreducible L-fuzzy soft bi-hyperideal of S over U. □

Theorem 3.13. *Each L-fuzzy soft bi-hyperideal of a semihypergroup S over U is strongly prime if and only if S is regular, intra-regular and the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered by inclusion.*

Proof. Suppose that each L-fuzzy soft bi-hyperideal of a semihypergroup S over U is strongly prime. Then each L-fuzzy soft bi-hyperideal of a semihypergroup S over U is semiprime. Thus by Theorem 3.9, S is both regular and intra-regular. Let f_{B_1} and f_{B_2} be any two L-fuzzy soft bi-hyperideals of S over U. Then by Theorem 3.9,

$$f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{=} (f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}).$$

By hypothesis $f_{B_1} \widetilde{\cap} h_{B_2}$ is stronlgy prime. This implies that either $f_{B_1} \widetilde{\subseteq} f_{B_1} \widetilde{\cap} f_{B_2}$ or $f_{B_2} \widetilde{\subseteq} f_{B_1} \widetilde{\cap} f_{B_2}$. If $f_{B_1} \widetilde{\subseteq} f_{B_1} \widetilde{\cap} f_{B_2}$, then $f_{B_1} \widetilde{\subseteq} f_{B_2}$. If $f_{B_2} \widetilde{\subseteq} f_{B_1} \widetilde{\cap} f_{B_2}$, then $f_{B_2} \widetilde{\subseteq} f_{B_1}$.

Conversely, assume that S is regular, intra-regular and the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered by inclusion. Let f_B be an arbitrary L-fuzzy soft bi-hyperideal of S over U and f_{B_1}, f_{B_2} be L-fuzzy soft bi-hyperideals of S over U such that $(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{\subseteq} f_B$. Since S is both regular and intra-regular, so by Theorem 3.9, $(f_{B_1} \circledast f_{B_2}) \widetilde{\cap} (f_{B_2} \circledast f_{B_1}) \widetilde{=} f_{B_1} \widetilde{\cap} f_{B_2}$. Thus $f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{\subseteq} f_B$. Since the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered, so either $f_{B_1} \widetilde{\subseteq} f_{B_2}$ or $f_{B_2} \widetilde{\subseteq} f_{B_1}$, that is either $f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{=} f_{B_1}$ or $f_{B_1} \widetilde{\cap} f_{B_2} \widetilde{=} f_{B_2}$. Thus either $f_{B_1} \widetilde{\subseteq} f_B$ or $f_{B_2} \widetilde{\subseteq} f_B$. Hence f_B is strongly prime L-fuzzy soft bi-hyperideal of S over U. □

Theorem 3.14. *Let S be a semihypergroup and the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered under inclusion. Then S is both regular and intra-regular if and only if each L-fuzzy soft bi-hyperideal of S over U is prime.*

Proof. Suppose that S is both regular and intra-regular. Let f_B, f_{B_1} and f_{B_2} be L-fuzzy soft bi-hyperideals of S over U such that $f_{B_1} \circledast f_{B_2} \widetilde{\subseteq} f_B$. Since the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered under inclusion, therefore either $f_{B_1} \widetilde{\subseteq} f_{B_2}$ or $f_{B_2} \widetilde{\subseteq} f_{B_1}$. Suppose $f_{B_1} \widetilde{\subseteq} f_{B_2}$, then $f_{B_1} \circledast f_{B_1} \widetilde{\subseteq} f_{B_1} \circledast f_{B_2} \widetilde{\subseteq} f_B$. Since S is both regular and intra-regular, so by Theorem 3.9, f_A is semiprime, so $f_{B_1} \widetilde{\subseteq} f_B$. Hence f_B is prime L-fuzzy soft bi-hyperideal of S over U.

Conversely, assume that each L-fuzzy soft bi-hyperideal of S over U is prime. So each L-fuzzy soft bi-hyperideal of S over U is semiprime. Thus by Theorem 3.9, S is both regular and intra-regular. □

Theorem 3.15. *Let S be a semihypergroup. Then the following assertions are equivalent:*

(1) The set of L-fuzzy soft bi-hyperideals of S over U is totally ordered under inclusion.
(2) Each L-fuzzy soft bi-hyperideal of S over U is strongly irreducible.
(3) Each L-fuzzy soft bi-hyperideal of S over U is irreducible.

Proof. (1) $\Longrightarrow$ (2) Let f_A, g_B and h_C be L-fuzzy soft bi-hyperideals of S over U such that $g_B \widetilde{\cap} h_C \widetilde{\subseteq} f_A$. By hypothesis we have either $g_B \widetilde{\subseteq} h_C$ or $h_C \widetilde{\subseteq} g_B$. If $g_B \widetilde{\subseteq} h_C$, then $g_B \widetilde{\cap} h_C \widetilde{=} g_B$. If $h_C \widetilde{\subseteq} g_B$, then $g_B \widetilde{\cap} h_C \widetilde{=} h_C$. Hence $g_B \widetilde{\cap} h_C \widetilde{\subseteq} f_A$ implies that either $g_B \widetilde{\subseteq} f_A$ or $h_C \widetilde{\subseteq} f_A$. Thus f_A is strongly irreducible L-fuzzy soft bi-hyperideal of S over U.

(2) $\Longrightarrow$ (3) Let f_A be an arbitrary L-fuzzy soft bi-hyperideal of S over U and g_B, h_C be L-fuzzy soft bi-hyperideals of S over U such that $g_B \widetilde{\cap} h_C \widetilde{=} f_A$. Then $f_A \widetilde{\subseteq} g_B$ and $f_A \widetilde{\subseteq} h_C$. By hypothesis, we have either $g_B \widetilde{\subseteq} f_A$ or $h_C \widetilde{\subseteq} f_A$. Thus either $g_B \widetilde{=} f_A$ or $h_C \widetilde{=} f_A$. Hence f_A is irreducible L-fuzzy soft bi-hyperideal of S over U.

(3) $\Longrightarrow$ (1) Let g_B and h_C be any two L-fuzzy soft bi-hyperideals of S over U. Then $g_B \widetilde{\cap} h_C$ is an L-fuzzy soft bi-hyperideal of S over U. Since $g_B \widetilde{\cap} h_C \widetilde{=} h_C \widetilde{\cap} g_B$, so either $g_B \widetilde{=} g_B \widetilde{\cap} h_C$ or $h_C \widetilde{=} g_B \widetilde{\cap} h_C$, that is either $g_B \widetilde{\subseteq} h_C$ or $h_C \widetilde{\subseteq} g_B$. Hence the set of L-fuzzy soft bi-hyperideals of S over U is totally ordered under inclusion. □

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