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ON DOMINATION IN AN EDGE PRODUCT HYPERGRAPHS

KISHOR F. PAWAR AND MEGHA M. JADHAV

Abstract. In this paper, we study domination in an edge product hypergraphs and found some results on it. It is proved that the unit edge in a unit edge product hypergraph is a dominating set of hypergraph $\mathcal{H}$. Later, we obtained some results which are relatives of the Nordhaus-Gaddum theorem, regarding the sums and products of domination parameters in an edge product hypergraph and their compliments.

Key Words: Edge Product Hypergraph, Unit Edge Product Hypergraph, Nordhaus-Gaddum Theorem, Domination.
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1. Introduction

The concept of domination in graphs was originated with the queen problem during 1850. The problem states, What is the minimum number of queens needed on an 8×8 chessboard such that all squares are either occupied or can be attacked by a queen. The rule of chess tells us that a queen can move horizontally, vertically or diagonally on the chessboard as long as there are no other pieces in its way. The correct answer to the above problem is five and the minimum number in this problem is nothing but the domination number of that graph G formed from 8×8 chessboard. The set of queens who dominate all the squares gives one dominating set of G. Claude Berge [3] in 1958 and Oystein

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∗Address correspondence to Kishor F. Pawar; E-mail: kfpawar@nmu.ac.in.
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Ore [12] in 1962 introduced the idea of domination in graphs. Berge named domination as an external stability and domination number as a coefficient of external stability while Ore used the words domination and domination number for the same idea.

The domination in graphs caught much attention in graph theory with its applications in many fields like design and analysis of communication networks, optimization, social sciences, linear algebra and military surveillance etc. For more details about domination and its related parameters, reader may refer to the books [7], [8] written by Haynes et. al . A huge body of literature has developed around domination in graphs, however much less is done about domination in hypergraphs. The domination in hypergraphs was introduced by Acharya in [1] and [2]. Hypergraph is a generalization of a graph in which any subset of a given set may be an edge rather than two element subset. In [9] Jadhav and Pawar introduced a special kind of hypergraph, called an edge product hypergraphs. A hypergraph $\mathcal{H}$ is said to be an edge product hypergraph if the edges of hypergraph can be labeled with distinct positive integers such that the product of all the labels of the edges incident to a vertex is again an edge label of $\mathcal{H}$ and if the product of any collection of edges is a label of an edge in $\mathcal{H}$ then they are incident to a vertex.

In the present paper, we study domination in an edge product hypergraphs. Also several important properties are studied and some results are found. Further, we obtained some Nordhaus-Gaddum [10], [11] type results for a unit edge product hypergraphs and edge product hypergraphs.

2. Preliminaries

We begin with recalling some basic definitions from [4]-[6] and some results from [9] required for our purpose.

Definition 2.1. A hypergraph $\mathcal{H}$ is a pair $\mathcal{H}(V, E)$ where V is a finite nonempty set and E is a collection of subsets of V. The elements of V are called vertices and the elements of E are called edges or hyperedges. And $\cup_{e_i \in E} e_i = V$ and $e_i \neq \phi$ are required, for all $e_i \in E$. The number of vertices in $\mathcal{H}$ is called the order of the hypergraph and is denoted by $|V|$. The number of edges in $\mathcal{H}$ is called the size of $\mathcal{H}$ and is denoted by $|E|$. A hypergraph of order n and size m is called a (n, m) hypergraph. The number $|e_i|$ is called the degree (cardinality) of the edges e_i. The rank of a hypergraph $\mathcal{H}$ is $r(\mathcal{H}) = max_{e_i \in E}|e_i|$.

Definition 2.2. For any vertex v in a hypergraph $\mathcal{H}(V, E)$, the set $N[v] = \{u \in V : u \text{ is adjacent to } v\} \cup \{v\}$ is called the closed neighborhood of v in $\mathcal{H}$ and each vertex in the set $N[v] - \{v\}$ is called neighbor of v. The open neighborhood of the vertex v is the set $N[v] \setminus \{v\}$. If $S \subseteq V$ then $N(S) = \cup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$.

Definition 2.3. A simple hypergraph (or Sperner family) is a hypergraph $\mathcal{H}(V, E)$ where $E = \{e_1, e_2, \cdots, e_m\}$ such that $e_i \subset e_j$ implies $i = j$.

Definition 2.4. For any hypergraph $\mathcal{H}(V, E)$, two vertices v and u are said to be adjacent if there exists an edge $e \in E$ that contains both v and u and non-adjacent otherwise.

Definition 2.5. For any hypergraph $\mathcal{H}(V, E)$, two edges are said to be adjacent if their intersection is nonempty. If a vertex $v_i \in V$ belongs to an edge $e_j \in E$ then we say that they are incident to each other.

Definition 2.6. An edge in a hypergraph $\mathcal{H}$ is called a pure hyperedge if it contains at least three vertices; otherwise it is called ordinary, and $\mathcal{H}$ is called a pure hypergraph if each edge of $\mathcal{H}$ is a pure hyperedge.

Definition 2.7. The vertex degree of a vertex v is the number of vertices adjacent to the vertex v in $\mathcal{H}$. It is denoted by $d(v)$. The maximum (minimum) vertex degree of a hypergraph is denoted by $\Delta(\mathcal{H})(\delta(\mathcal{H}))$.

Definition 2.8. The edge degree of a vertex v is the number of edges containing the vertex v. It is denoted by $d_E(v)$. The maximum (minimum) edge degree of a hypergraph is denoted by $\Delta_E(\mathcal{H})(\delta_E(\mathcal{H}))$. A vertex of a hypergraph which is incident to no edge is called an isolated vertex. The edge degree (or vertex degree) of an isolated vertex is trivially 0. An edge of cardinality one is called a singleton (loop), a vertex of edge degree one is called a pendant vertex.

Definition 2.9. The hypergraph $\mathcal{H}(V, E)$ is called connected if for any pair of its vertices, there is a path connecting them. If $\mathcal{H}$ is not connected then it consists of two or more connected components, each of which is a connected hypergraph.

Definition 2.10. A hypergraph is said to be of rank k if each of its edge contains at most k vertices.

Definition 2.11. The complement $\bar{\mathcal{H}}$ of a hypergraph $\mathcal{H}(V, E)$ is defined as $\bar{\mathcal{H}}(V, \bar{E})$ where $\bar{E} = \{\bar{e} | e \in E\}$ with $\bar{e} \in \bar{E}, \bar{e} = \{v \notin e | e \in E\}$.

Definition 2.12. For a hypergraph $\mathcal{H}(V, E)$, a set $D \subseteq V$ is called a dominating set of $\mathcal{H}$ if for every $v \in V \setminus D$ there exists $u \in D$ such that u and v are adjacent in $\mathcal{H}$, that is there exists $e \in E$ such that $u, v \in e$.

Definition 2.13. A dominating set D of a hypergraph $\mathcal{H}$ is called a minimal dominating set, if no proper subset of D is a dominating set of $\mathcal{H}$. The minimum cardinality of a minimal dominating set in a hypergraph $\mathcal{H}$ is called the domination number of $\mathcal{H}$ and is denoted by $\gamma(\mathcal{H})$.

Definition 2.14. Let $D \in D^0(\mathcal{H})$, the set of all minimum dominating sets (of cardinality $\gamma(\mathcal{H})$). An inverse dominating set with respect to D is any dominating set D' of $\mathcal{H}$ such that $D' \subseteq V \setminus D$. The inverse domination number of $\mathcal{H}$ is defined as

$$\gamma^{-1}(\mathcal{H}) = min\{|D'| \ \mid \ D \in D^0(\mathcal{H}), D' \text{ is an inverse dominating set with respect to D}\}$$

Definition 2.15. Let $\mathcal{H}(V, E)$ be a simple and connected hypergraph. Let $V(\mathcal{H})$ be the vertex set of $\mathcal{H}$ and $E(\mathcal{H})$ be the edge set of $\mathcal{H}$. Let P be a set of positive integers such that $|E| = |P|$. Then any bijection $f : E \to P$ is called an edge function of the hypergraph $\mathcal{H}$.

Definition 2.16. The function

$$F(v) = \prod\{f(e)| \text{ edge } e \text{ is incident to the vertex } v\}$$

on $V(\mathcal{H})$ is called an edge product function of the edge function f.

Definition 2.17. The hypergraph $\mathcal{H}(V, E)$ is said to be an edge product hypergraph if there exists an edge function $f : E \to P$ such that the edge function f and the corresponding edge product function F of f on $V(\mathcal{H})$ have the following two conditions:

(1) $F(v) \in P$, for every $v \in V$.
(2) If $f(e_1) \times f(e_2) \times \ldots \times f(e_p) \in P$, for some edges $e_1, e_2, \ldots, e_p \in E$ then the edges $e_1, e_2, \ldots, e_p$ are all incident to a vertex $v \in V$.

Example 2.18. Let $\mathcal{H}(V, E)$ be a hypergraph, where $V = \{v_1, v_2, \ldots, v_{20}\}$ and $E = \{e_1, e_2, \ldots, e_7\}$. In which the edges of $\mathcal{H}$ are defined as follows:

$e_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$, $e_2 = \{v_1, v_2, v_3, v_{10}\}$,
$e_3 = \{v_1, v_2, v_3, v_{11}, v_{12}\}$, $e_4 = \{v_4, v_5, v_{13}\}$,
$e_5 = \{v_4, v_5, v_{14}, v_{15}, v_{16}\}$, $e_6 = \{v_4, v_5, v_{17}, v_{18}\}$, and
$e_7 = \{v_{19}, v_{20}\}$.

Now define the edge function $f : E \to P$ by

$$f(e_1) = 11, \quad f(e_2) = 30, \quad f(e_3) = 4, \quad f(e_4) = 3,$$
$$f(e_5) = 20, \quad f(e_6) = 2, \quad f(e_7) = 1320.$$

The edge product function F of f is defined by,

$$F(v_1) = 1320, \quad F(v_2) = 1320, \quad F(v_3) = 1320, \quad F(v_4) = 1320, \quad F(v_5) = 1320,$$
$$F(v_6) = 11, \quad F(v_7) = 11, \quad F(v_8) = 11, \quad F(v_9) = 11, \quad F(v_{10}) = 30,$$
$$F(v_{11}) = 4, \quad F(v_{12}) = 4, \quad F(v_{13}) = 3, \quad F(v_{14}) = 20, \quad F(v_{15}) = 20,$$
$$F(v_{16}) = 20, \quad F(v_{17}) = 2, \quad F(v_{18}) = 2, \quad F(v_{19}) = 1320, \quad F(v_{20}) = 1320.$$

Hence the given hypergraph is an edge product hypergraph.

Definition 2.19. For an edge product hypergraph $\mathcal{H}(V, E)$ there exists an edge function $f : E \to P$ such that an element $1 \in P$ then the hypergraph $\mathcal{H}$ is called a unit edge product hypergraph.

Theorem 2.20. *Let $\mathcal{H}$ be a unit edge product hypergraph with an edge $e^* \in E$ and $f(e^*) = 1$. Then e^* must adjacent to all the edges of $\mathcal{H}$.*

Theorem 2.21. *Let $\mathcal{H}(V, E)$ be a unit edge product hypergraph and $e^* \in E$ such that $f(e^*) = 1$. Then $\mathcal{H}$ contains at least one edge which is adjacent to only e^*.*

Lemma 2.22. *Let $\mathcal{H}(V, E)$ be an edge product hypergraph. Then $\gamma(\bar{\mathcal{H}}) = \gamma^{-1}(\bar{H}) = 1$.*

Here the edge in a unit edge product hypergraph whose label is one is called a unit edge of that hypergraph and the edge which is not a unit edge is called a non-unit edge of that hypergraph.

3. Domination in Unit Edge Product Hypergraph

In this section, we obtained the dominating sets for some edge product hypergraphs and unit edge product hypergraphs. Also the properties of a unit edge product hypergraph are studied when the domination number of that hypergraph is known.

Theorem 3.1. *If $\mathcal{H}$ is a unit edge product hypergraph, then the unit edge of $\mathcal{H}$ is a dominating set of $\mathcal{H}$.*

Proof. Let $\mathcal{H}(V, E)$ be a unit edge product hypergraph and e^* be the unit edge of $\mathcal{H}$. Since it is a unit edge product hypergraph, the intersection of any edge and the unit edge is non-empty. Hence for every $u \in V \setminus e^*$ we have, a vertex $v \in e^*$ such that v is adjacent to u. Hence the unit edge e^* forms a dominating set of $\mathcal{H}$. □

Theorem 3.2. *Let $\mathcal{H}$ be a unit edge product hypergraph. Then any dominating set contains a vertex from unit edge or a pendant vertex.*

Proof. Let $\mathcal{H}(V, E)$ be a unit edge product hypergraph with an unit edge e^*. Let D be a dominating set of $\mathcal{H}$. Now in a unit edge product hypergraph there exists at least one edge which is adjacent to e^* only. Let e be the edge in $\mathcal{H}$ which is adjacent to e^* only. Then for any $v \in e$, we have a vertex $u \in D$ such that the vertex v is adjacent to u. Therefore the vertex u can be either belongs to the intersection of e^* and e or the pendant vertex in e^*. Hence the proof. □

Theorem 3.3. *If $\mathcal{H}$ is a unit edge product hypergraph, then the vertex with maximum edge degree $\Delta_E(\mathcal{H})$ must belongs to the unit edge of $\mathcal{H}$.*

Proof. Let $\mathcal{H}$ be a unit edge product hypergraph with an unit edge e^*. The mapping $f : E \to P$ is an edge function and F is an edge product function of f in $\mathcal{H}$. Let u be the vertex in $\mathcal{H}$ with maximum edge degree $\Delta_E(\mathcal{H}) = k$, where k is a positive integer. Suppose the vertex u is not in the unit edge e^*. Now let $e_1, e_2, \ldots, e_k$ be the edges incident to the vertex u. Then $F(u) = f(e_1) \times f(e_2) \times \ldots \times f(e_k) \in P$. But $F(u) = F(u) \cdot 1 \in P \Rightarrow F(u) = f(e_1) \times f(e_2) \times \ldots \times f(e_k) \times f(e^*) \in P$. Hence the edges $e_1, e_2, \ldots, e_k, e^*$ are incident to a vertex $v \in V$ with the edge degree $k + 1 > \Delta_E(\mathcal{H})$ which is a contradiction. Hence the vertex u must be in e^*. □

Theorem 3.4. *Let $\mathcal{H}$ be a unit edge product hypergraph with $\gamma(\mathcal{H}) + \Delta(\mathcal{H}) = |V|$ and let v be a vertex of degree $\Delta(\mathcal{H})$. Then the unit edge of $\mathcal{H}$ contains at least $|\mathcal{H} \setminus N[v]|$ number of vertices.*

Proof. Let $\mathcal{H}$ be a hypergraph following the given conditions of the theorem. Let e^* be the unit edge of $\mathcal{H}$. Since it is a unit edge product hypergraph, it follows the edge containing the vertex $u \in \mathcal{H} \setminus N[v]$ is adjacent to the unit edge in $\mathcal{H}$.(the edge containing the vertex $u \in \mathcal{H} \setminus N[v]$ cannot be the unit edge otherwise it would not be an edge product hypergraph) Hence for every $u \in \mathcal{H} \setminus N[v]$ we have, a vertex $y \in e^*$, adjacent to u and belongs to the intersection of the unit edge and the edge containing the vertex u. Now we assume contrary that the cardinality of the unit edge is less than $|\mathcal{H} \setminus N[v]|$. This implies that there does not exist a distinct vertex $y \in e^*$ for each $u \in \mathcal{H} \setminus N[v]$.

Let for $u_1, u_2 \in \mathcal{H} \setminus N[v]$, we have a single vertex $y \in e^*$ then

$$D = \{\{v, y\} \cup (V \setminus (N[v] \cup (N(y) \cap (V \setminus N[v]))))\}$$

is a dominating set of $\mathcal{H}$ with cardinality $|D| = 2 + |V| - \Delta(\mathcal{H}) - 1 - 2$ $= |V| - \Delta(\mathcal{H}) - 1$ which is a contradiction. Hence the unit edge contains at least $|\mathcal{H} \setminus N[v]|$ number of vertices. □

Theorem 3.5. *Let $\mathcal{H}$ be a unit edge product hypergraph with $i(\mathcal{H}) + \Delta(\mathcal{H}) = |V|$ and let u be a vertex of degree $\Delta(\mathcal{H})$. Then there exists at least $|V \setminus N[u]|$ non-unit edges in $\mathcal{H}$.*

Proof. Let $\mathcal{H}$ be a hypergraph with the given conditions of the theorem. Since it is a unit edge product hypergraph, the edge containing the vertex $w \in V \setminus N[u]$ is adjacent to the unit edge in $\mathcal{H}$. Now suppose there are p non-unit edges in $\mathcal{H}$ and $p < |V \setminus N[u]|$. Then there exist two vertices $x, y \in V \setminus N[u]$ such that x and y are adjacent. Hence the cardinality of any strongly independent dominating set of $V \setminus N[u]$ is at most $|V \setminus N[u]| - 1$. Hence $i(\mathcal{H}) \leq |V \setminus N[u]| - 1 + |u| = |V| - \Delta(\mathcal{H}) - 1$, which is a contradiction. Hence there exist at least $|V \setminus N[u]|$ non-unit edges in $\mathcal{H}$. □

Theorem 3.6. *Let $\mathcal{H}$ be a unit edge product hypergraph with $\gamma(\mathcal{H}) = m - 1$. Then any two non-unit edges are non-adjacent.*

Proof. Let e^* be the unit edge of the given hypergraph. Let e and e be the two non-unit edges in $\mathcal{H}$. Suppose the edge e is adjacent to the edge e. Then the intersection of the edges e and e is non-empty. Let $v \in e \cap e$. Now since it is a unit edge product hypergraph, for every $e_i \in E$, we have a vertex $v_i \in e^* \cap e_i$, for $1 \leq i \leq m - 1$. Thus the set containing the vertices $v_1, v_2, \ldots, v_{m-3}$ and v forms a dominating set of $\mathcal{H}$ with cardinality $|D| = m - 3 + 1 < m - 1$ a contradiction. Hence no two non-unit edges are adjacent in $\mathcal{H}$. □

Theorem 3.7. *If $\mathcal{H}$ is an unit edge product hypergraph with $\gamma(\mathcal{H}) = m - 1$ and unit edge contains at least one pendant vertex then any dominating set contains at least one element from unit edge.*

Proof. Let e^* be the unit edge of $\mathcal{H}$ and $v \in V \setminus e^*$. Then the vertex v is a pendant vertex in $\mathcal{H}$, by Theorem 3.6. Hence any subset of $V \setminus e^*$ cannot be a dominating set of $\mathcal{H}$ unless it contains a vertex from an unit edge. Hence the proof. □

4. Nordhaus-Gaddum Theorem

In this section, we obtained some results similar to Nordhaus-Gaddum theorem, relevant to the sums and products of domination parameters in an edge product hypergraph and their compliments. Also in order to avoid the trivial anomalies, whenever we talk about $\bar{\mathcal{H}}$, we restrict ourselves to those hypergraphs which satisfies the condition that, every vertex v of $\mathcal{H}$ is incident with some edge e of cardinality, $2 \leq |e| \leq |v|-2$, and avoiding v and $d_E(v) < |E|$ and $|V| \geq 4$.

Theorem 4.1. *For a unit edge product hypergraph of size $m > 2$,*

(1) $2 \leq \gamma(\mathcal{H}) + \gamma(\bar{\mathcal{H}}) \leq |e^*| + 1$.
(2) $1 \leq \gamma(\mathcal{H}) \cdot \gamma(\bar{\mathcal{H}}) \leq |e^*|$ *and the bounds are sharp. Where e^* is the unit edge of $\mathcal{H}$.*

Proof. We know for any hypergraph $\mathcal{H}$, we have $\gamma(\mathcal{H}) \geq 1$. Hence the lower bounds in 1) and 2) are obvious. Now we proceed to prove for upper bounds. The unit edge e^* of $\mathcal{H}$ forms a dominating set of $\mathcal{H}$, by Theorem 3.1. Hence we have, $\gamma(\mathcal{H}) \leq |e^*|$. Now since it is a unit edge product hypergraph, there exist an edge which is adjacent to e^* only. Let e_j be the edge in $\mathcal{H}$ which is adjacent to e^* only. Let e_k be any edge in $\mathcal{H}$. Then the edges e_j and e_k are two independent edges in $\mathcal{H}$. Now if we take $w \in V|_{e_j \cup e_k}$ then for any $v \in V$, we have a vertex w adjacent to v in $\bar{\mathcal{H}}$. Therefore $\gamma(\bar{\mathcal{H}}) = 1$. Hence the upper bound follows. □

Remark 4.2. The bounds given in the Theorem 4.1 are sharp:
For, consider the hypergraph with the vertex set

$$V = \{v_1, v_2, v_3, u_1, u_2, u_3, w_1, w_2, w_3, x_1, x_2, x_3\}$$

and the edge set

$$E = \{e_1, e_2, e_3, e_4, e_5\}.$$

In which the edges of $\mathcal{H}$ are defined as follows:

$$e_1 = \{v_1, u_1, w_1, x_1\}, \qquad e_2 = \{v_1, v_2, v_3\},$$
$$e_3 = \{u_1, u_2, u_3\}, \qquad e_4 = \{w_1, w_2, w_3\}, \text{ and}$$
$$e_5 = \{x_1, x_2, x_3\}.$$

We define the edge function $f : E \to P$ by

$$f(e_1) = 1, \qquad f(e_2) = 2, \qquad f(e_3) = 3,$$
$$f(e_4) = 5, \qquad f(e_5) = 7,$$

where $P = \{1, 2, 3, 5, 7\}$. Then edge product function F of f will be,

$$F(v_1) = F(v_2) = F(v_3) = 2, \qquad F(u_1) = F(u_2) = F(u_3) = 3,$$
$$F(w_1) = F(w_2) = F(w_3) = 5, \qquad F(x_1) = F(x_2) = F(x_3) = 7.$$

Thus, $\mathcal{H}(V, E)$ is a unit edge product hypergraph. Here the set $\{v_1, u_1, w_1, x_1\}$ forms a minimum dominating set of $\mathcal{H}$ whereas any vertex of $\mathcal{H}$ forms a dominating set of $\bar{\mathcal{H}}$. Hence in this case, $\gamma(\mathcal{H}) + \gamma(\bar{\mathcal{H}}) = |e^*| + 1$ and $\gamma(\mathcal{H}) \cdot \gamma(\bar{\mathcal{H}}) = |e^*|$.

Observation: If $\mathcal{H}$ is an edge product hypergraph with edge product number $\mathcal{EP}_n(\mathcal{H}) = r$ then any dominating set of $\mathcal{H}$ contains at least r pendant vertices.

Theorem 4.3. *For an edge product hypergraph with edge product number $\mathcal{EP}_n(\mathcal{H}) = r$,*

(1) $2 + r \leq \gamma(\mathcal{H}) + \gamma(\bar{\mathcal{H}}) \leq \lfloor \frac{n}{2} \rfloor + r + 1$.

(2) $1 + r \leq \gamma(\mathcal{H}) \cdot \gamma(\bar{\mathcal{H}}) \leq \lfloor \frac{n}{2} \rfloor + r$ *and the bounds are sharp.*

Proof. The lower bounds in 1) and 2) are obvious. Now consider the upper bounds of 1) and 2). Since $\mathcal{H}$ is an edge product hypergraph with edge product number r, it follows $\gamma(\mathcal{H}) \leq \lfloor \frac{n}{2} \rfloor + r$ and $\gamma(\bar{\mathcal{H}}) = 1$, by Lemma 2.22. Hence $\gamma(\mathcal{H}) + \gamma(\bar{\mathcal{H}}) \leq \lfloor \frac{n}{2} \rfloor + r + 1$ and $\gamma(\mathcal{H}) \cdot \gamma(\bar{\mathcal{H}}) \leq \lfloor \frac{n}{2} \rfloor + r$. □

Remark 4.4. Here the bounds are sharp:
Now consider the hypergraph $\mathcal{H} \cup K_2$ with vertex set

$$V = \{v_1, v_2, v_3, v_4\} \cup \{w_1, w_2\}.$$

$E = \{e_1, e_2, e_3\} \cup \{e_4\}$ where

$$e_1 = \{v_1, v_2\}, \qquad e_2 = \{v_2, v_3\},$$
$$e_3 = \{v_3, v_4\} \qquad e_4 = \{w_1, w_2\}.$$

Define the edge function $f : E \to P$ by

$$f(e_1) = 5, \qquad f(e_2) = 2,$$
$$f(e_3) = 10, \qquad f(e_4) = 20.$$

Then edge product function F of f will be,

$$F(v_1) = 5, \qquad F(v_2) = 10, \qquad F(v_3) = 20,$$
$$F(v_4) = 10, \qquad F(w_1) = F(w_2) = 20.$$

Hence $\mathcal{H}$ is an edge product hypergraph and $\mathcal{EP}_n(\mathcal{H}) = 1$. And here in this case, $\gamma(\mathcal{H}) + \gamma(\bar{\mathcal{H}}) = \lfloor \frac{n}{2} \rfloor + r + 1 = 4$ and $\gamma(\mathcal{H}) \cdot \gamma(\bar{\mathcal{H}}) = \lfloor \frac{n}{2} \rfloor + r = 3$.

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Kishor F. Pawar
Department of Mathematics, School of Mathematical Sciences, Kavayitri Bahinabai Chaudhari North Maharashtra University Jalgaon-425 001, India
Email: kfpawar@nmu.ac.in

Megha M. Jadhav
Department of Mathematics, School of Mathematical Sciences, Kavayitri Bahinabai Chaudhari North Maharashtra University Jalgaon-425 001, India
Email: meghachalisgaon@gmail.com