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PICTURE FUZZY HYPERSOFT TOPSIS METHOD BASED ON CORRELATION COEFFICIENT

V. CHINNADURAI AND A. BOBIN

ABSTRACT. Considering today's complexity, we may have to deal with numbers that are dependent, like those of positive, neutral and negative values and require multi- attributes function. Also, the most significant factor is to combine these numbers to generate a single real number. When decision-makers come across such an environment, the decisions are harder to make and decision makers cannot use the soft set theory, a single attribute function. To overcome this hindrance, we introduce the notion of picture fuzzy hypersoft set with technique of order of preference by similarity to ideal solution (TOPSIS) method. This eventually helps the decision-maker to collect the data without any misconceptions. We present some properties of the correlation coefficient and aggregation operators on it. Also, we propose an algorithm for the TOPSIS method based on correlation coefficients to identify a suitable leader, who can bring changes to society in the socio-political context. Finally, we present a comparative study with existing studies to show the effectiveness of the proposed method.

Key Words: Picture fuzzy set, Hypersoft set, TOPSIS, Correlation coefficient.
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1. Introduction

Zadeh [27] defined the concept of fuzzy set (FS). The membership value of each element in FS is specified by a real number from the closed interval [0,1]. Atanassov [4] proposed the notion of intuitionistic fuzzy

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∗Address correspondence to C. Veerappan; E-mail: chinnaduraiau@gmail.com.

set (IFS), an extension of FS. In IFS, the elements possess both membership and non-membership values such that their sum does not exceed unity. Smarandache [9] presented the idea of neutrosophic set (NS), characterized by the values of truth, indeterminacy, and falsity grades for each element of the set. Later, Wang et al. [25] proposed the notion of single-valued NS (SVNS) with a restricted condition for the membership values to overcome the constraints faced in NS. Bui Cong Cuong defined the concept of picture fuzzy sets (PFS), an extension of FS and IFS. Molodtsov [16] introduced the concept of soft set (SS) to deal with uncertainties. Smarandache [10] presented the idea of hypersoft set (HSS) to overcome the restriction faced in soft set.

Khalil et al. [15] presented the notion of interval-valued PFS and studied some of its properties. Wei [26] presented the concept to measure the similarity between PFS. Ganie et al. [11] introduced the idea of correlation coefficient (CC) in PFS. Mohamed Abdel-Basset et al. [1] presented the concept of type-2 neutrosophic numbers and presented a real case study using the technique of order of preference by similarity to ideal solution (TOPSIS). Mohamed Abdel-Basset et al. [2] combined the neutrosophic analytical network process (ANP) method and the ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method for solving supplier chain management problems. Arora and Harish [3] studied the properties of aggregation operators on IFS. Endalkachew Teshome Ayele et al. [24] proposed a method for traffic signal control using interval-valued neutrosophic soft sets. Ejegwa et al. [6] used intuitionistic fuzzy correlation measure and programming language in the medical diagnosis field. Harish and Rishu [12] proposed TOPSIS method based on correlation measures on IFS to solve multi-criteria decision making (MCDM) problems. Jana and Pal [14] used the concept of aggregation operators on SVNS for solving MCDM problems. Rahman et al. [18] generalized the concept of complex fuzzy soft structures to hypersoft structure to handle MCDM problems. Rahman et al. [19] established the properties of convex and concave HSS. Rahman et al. [20] discussed the significance of neutrosophic parameterized HSS (NPHSS) with decision making problems. Saeed et al. [23] studied the fundamental concepts of HSS theory. Rahman et al. [21] introduced the notion of bijective HSS and studied some of its properties. Rahman et al. [22] conceptualized the properties of NPHSS with FS and IFS. Ihsan et al. [13] generalized the concept of soft expert set to hypersoft expert set to solve MCDM problems. Rana Muhammad Zulqarnain et al. [28]

introduced the concept of intuitionistic fuzzy HSS and used the TOPSIS method based on CC. Rana Muhammad Zulqarnain et al. [29] studied the fundamental operations of interval-valued neutrosophic HSS. Saqlain Muhammad et al. [17] defined aggregation operators on neutrosophic HSS and studied some properties.

A single attribute function like SS does not handle today's real-life applications. HSS, an extension of SS, can overcome this limitation by using the multi-attributes function. Also, HSS can be applied to any multi-criteria decision-making (MCDM) problems with no limitations to the attributes by the decision-makers (DMs). By combining HSS with other hybrid fuzzy structures, DMs can collect the data without loss of information. In PFS, the values of positive, neutral, and negative depend on each other, and the sum of these grades cannot be greater than one. Therefore, this study aims to develop a new theory, picture fuzzy HSS (PFHSS) by combining structures of PFS with HSS. Also, helps to rank the alternatives using aggregation operators and the TOPSIS method based on CC in PFHSS. To the best of our knowledge, research on PFHSS is confined to its theory and related development and applications. Here, we examine and provide a suitable solution to the decision-making problem by ranking the alternatives. We discuss an MCDM problem based on the TOPSIS method. We show the effectiveness of this method through the selection of a leader who can influence society in a socio-political context. To prove the efficiency of the proposed method, we illustrate a comparative analysis between the proposed and existing method with examples. We show the PFHSS as a robust tool to decide under uncertainties.

The manuscript consists of the following sections. Section 2 briefs on existing definitions. Section 3 introduces the concept of PFHSS and discusses some properties of CC and weighted CC of PFHSS. Section 4 deals with picture fuzzy hypersoft weighted average operator(PFHSWAO) and picture fuzzy hypersoft weighted geometric operator (PFHSWGO). Section 5 highlights the combination of CC with the TOPSIS method. The paper ends with a conclusion in section 6.

2. Preliminaries

We present some of the basic definitions required for this study. Let us consider the following notations throughout this study unless otherwise specified. Let $\mathcal{V}$ be the universe and $v \in \mathcal{V}$, $P(\mathcal{V})$ the power set of $\mathcal{V}$, $\mathbb{N}$

represents natural numbers, and $\mathcal{P}^U$ the collection of Picture fuzzy sets (PFS) over $\mathcal{V}$.

Definition 2.1. [27] A fuzzy set (FS) is a set of the form $\mathcal{F} = \{(v, \mathcal{P}_{\mathcal{F}}(v)) : v \in \mathcal{V}\}$, where $\mathcal{P}_{\mathcal{F}}(v) : \mathcal{V} \to [0,1]$ defines the degree of membership of the element $v \in \mathcal{V}$.

Definition 2.2. [4] An intuitionistic FS (IFS) is an object of the form $\mathcal{C} = \{(v, \mathcal{P}_{\mathcal{C}}(v), \mathcal{N}_{\mathcal{C}}(v)) : v \in \mathcal{V}\}$, where $\mathcal{P}_{\mathcal{C}}(v) : \mathcal{V} \to [0,1]$ and $\mathcal{N}_{\mathcal{C}}(v) : \mathcal{V} \to [0,1]$ define the degree of membership and degree of non-membership of the element $v \in \mathcal{V}$, respectively and for every $v \in \mathcal{V}$, $0 \le \mathcal{P}_{\mathcal{C}}(v) + \mathcal{N}_{\mathcal{C}}(v) \le 1$, where $\pi_{\mathcal{C}}(v) = 1 - \mathcal{P}_{\mathcal{C}}(v) - \mathcal{N}_{\mathcal{C}}(v)$.

Definition 2.3. [5] A PFS in $\mathcal{V}$ is an object of the form $\Omega = \{\langle v, \mathcal{P}_{\Omega}(v), \mathcal{E}_{\Omega}(v), \mathcal{N}_{\Omega}(v)\rangle\}$, where $\mathcal{P}_{\Omega}(v), \mathcal{E}_{\Omega}(v), \mathcal{N}_{\Omega}(v) : \mathcal{V} \to [0,1]$, are the membership values of positive, neutral and negative of the element $v \in \mathcal{V}$ respectively, such that $0 \le \mathcal{P}_{\Omega}(v) + \mathcal{E}_{\Omega}(v) + \mathcal{N}_{\Omega}(v) \le 1$ and the degree of refusal membership is $1 - (\mathcal{P}_{\Omega}(v) + \mathcal{E}_{\Omega}(v) + \mathcal{N}_{\Omega}(v))\ \forall\ v \in \mathcal{V}$.

Definition 2.4. [16] A pair $(\mathcal{O}, \mathcal{E})$ is called a soft set (SS) over $\mathcal{V}$, if $\mathcal{O} : \mathcal{E} \to \mathcal{P}(\mathcal{V})$. Then for any $p \in \mathcal{E}$, $\mathcal{O}(p) = 1$ is equivalent to $v \in \mathcal{O}(p)$ and $\mathcal{O}(p) = 0$ is equivalent to $v \notin \mathcal{O}(p)$. Thus a SS is not a set, but a parameterized family of subsets of $\mathcal{V}$.

Definition 2.5. [10] Let $\Delta_1, \Delta_2, ..., \Delta_k$, be distinct attribute sets, whose corresponding sub-attributes are $\Delta_1 = \{\lambda_{11}, \lambda_{12}, ..., \lambda_{1f}\}$, $\Delta_2 = \{\lambda_{21}, \lambda_{22}, ..., \lambda_{2g}\}, ..., \Delta_k = \{\lambda_{k1}, \lambda_{k2}, ..., \lambda_{kh}\}$, where $1 \le f \le p,\ 1 \le g \le q,\ 1 \le h \le r$ and $p,\ q,\ r \in \mathbb{N}$, such that $\Delta_i \cap \Delta_j = \emptyset$, for each $i, j \in \{1, 2, ..., k\}$ and $i \ne j$. Then the Cartesian product of the distinct attribute sets $\Delta_1 \times \Delta_2 \times ... \times \Delta_k = \tilde{\Delta} = \{\lambda_{1f} \times \lambda_{2g} \times ... \times \lambda_{kh}\}$, represent a collection of multi- attributes. A pair $(\Omega, \tilde{\Delta})$ is called a hypersoft set (HSS) over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to P(\mathcal{V})$. HSS can be represented as $(\Omega, \tilde{\Delta}) = \left\{(\tilde{\lambda}, \Omega(\tilde{\lambda})) | \tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in P(\mathcal{V})\right\}$.

3. Picture fuzzy hypersoft set

We now present the notion of picture fuzzy hypersoft set (PFHSS). Also, we discuss some basic properties of correlation coefficient (CC) and weighted CC (WCC) on PFHSS.

Definition 3.1. A pair $(\Omega, \tilde{\Delta})$ is called a PFHSS over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to \mathcal{P}^U$. PFHSS can be represented as $(\Omega, \tilde{\Delta}) = \{(\tilde{\lambda}, \Omega(\tilde{\lambda})) | \tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in$

$\mathcal{P}^U \in [0,1]\big\}$, where $\Omega(\tilde{\lambda}) = \Big\{\Big\langle v, \mathcal{P}_{\Omega(\tilde{\lambda})}(v), \mathcal{E}_{\Omega(\tilde{\lambda})}(v), \mathcal{N}_{\Omega(\tilde{\lambda})}(v)\Big\rangle \,|v \in \mathcal{V}\Big\}$, $\mathcal{P}_{\Omega(\tilde{\lambda})}(v)$, $\mathcal{E}_{\Omega(\tilde{\lambda})}(v)$ and $\mathcal{N}_{\Omega(\tilde{\lambda})}(v)$ represent the membership values of positive, neutral and negative, such that $0 \leq \mathcal{P}_{\Omega(\tilde{\lambda})}(v)+\mathcal{E}_{\Omega(\tilde{\lambda})}(v)+\mathcal{N}_{\Omega(\tilde{\lambda})}(v) \leq 1$ and degree of refusal membership is $1-(\mathcal{P}_{\Omega(\tilde{\lambda})}(v)+\mathcal{E}_{\Omega(\tilde{\lambda})}(v)+\mathcal{N}_{\Omega(\tilde{\lambda})}(v))$.

Example 3.2. Let $\mathcal{V} = \{v_1, v_2, v_3\}$ be a set of sociologists responsible to evaluate a leader, the role of the leader is to bring socio-political changes to society. Let Δ_1, Δ_2 and Δ_3 be distinct attribute sets whose corresponding sub-attributes are represented as Δ_1 = leader attributes = $\big\{\lambda_{11}$ = personality variables, λ_{12} = cognitive ability and skills, λ_{13} = sense making$\big\}$, Δ_2 = leader behavior = $\big\{\lambda_{21}$ = setting sub culture, λ_{22} = conflict management$\big\}$, Δ_3 = group behaviors = $\big\{\lambda_{31}$ = living the sub culture$\big\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3$ is the distinct attribute set given by

$$\begin{aligned}\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 &= \{\lambda_{11}, \lambda_{12}, \lambda_{13}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}\}.\\ &=\big\{(\lambda_{11}, \lambda_{21}, \lambda_{31}), (\lambda_{11}, \lambda_{22}, \lambda_{31}), (\lambda_{12}, \lambda_{21}, \lambda_{31}), (\lambda_{12}, \lambda_{22}, \lambda_{31}),\\ &\qquad (\lambda_{13}, \lambda_{21}, \lambda_{31}), (\lambda_{13}, \lambda_{22}, \lambda_{31})\big\}.\\ &=\big\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}, \tilde{\lambda_5}, \tilde{\lambda_6}\big\}.\end{aligned}$$

A PFHSS $(\Omega, \tilde{\Delta})$ is a collection of subsets of $\mathcal{V}$ described by the sociologists for a leader and presented in tabular form below.

TABLE 1. Leadership skills of a leader in PFHSS $(\Omega, \tilde{\Delta})$ form.

$\mathcal{V}$	$\tilde{\lambda_1}$	$\tilde{\lambda_2}$	$\tilde{\lambda_3}$	$\tilde{\lambda_4}$	$\tilde{\lambda_5}$	$\tilde{\lambda_6}$
v_1	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.2, 0.5, 0.1\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.1, 0.4, 0.3\rangle$	$\langle 0.7, 0.1, 0.1\rangle$
v_2	$\langle 0.2, 0.2, 0.3\rangle$	$\langle 0.4, 0.1, 0.4\rangle$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.4, 0.2, 0.4\rangle$	$\langle 0.3, 0.4, 0.2\rangle$	$\langle 0.5, 0.2, 0.1\rangle$
v_3	$\langle 0.4, 0.3, 0.3\rangle$	$\langle 0.3, 0.2, 0.4\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.3, 0.2, 0.5\rangle$	$\langle 0.2, 0.4, 0.3\rangle$	$\langle 0.3, 0.3, 0.2\rangle$

3.1. **Correlation coefficient for PFHSS.** Let $(\Omega_1, \tilde{\Delta}_1) = \big\{(v_i, \mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))|v_i \in \mathcal{V}\big\}$ and $(\Omega_2, \tilde{\Delta}_2) = \big\{(v_i, \mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))|v_i \in \mathcal{V}\big\}$ be two PFHSS over $\mathcal{V}$.

Definition 3.3. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two PFHSS. Then the picture fuzzy informational energies of $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ are represented as

$$\Phi(\Omega_1, \tilde{\Delta}_1) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\right], \tag{3.1}$$

$$\Phi(\Omega_2, \tilde{\Delta}_2) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\right]. \tag{3.2}$$

Definition 3.4. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two PFHSS. Then the correlation measure between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ is defined as

$$\begin{aligned}\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = &\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) \\ &+ (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].\end{aligned} \tag{3.3}$$

Proposition 3.5. *Let* $(\Omega_1, \tilde{\Delta}_1)$ *and* $(\Omega_2, \tilde{\Delta}_2)$ *be two PFHSS. Then,*
(i) $\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_1, \tilde{\Delta}_1)) = \Phi(\Omega_1, \tilde{\Delta}_1)$
(ii) $\mathcal{C}_{\mathcal{M}}((\Omega_2, \tilde{\Delta}_2), (\Omega_2, \tilde{\Delta}_2)) = \Phi(\Omega_2, \tilde{\Delta}_2)$.

Proof. Straight forward □

Definition 3.6. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two PFHSS. Then, the CC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ is given by

$$\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}} \tag{3.4}$$

Proposition 3.7. *Let* $(\Omega_1, \tilde{\Delta}_1)$ *and* $(\Omega_2, \tilde{\Delta}_2)$ *be two PFHSS. Then, the following CC properties hold:*
(i) $0 \le \mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \le 1$*;*
(ii) $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \mathcal{C}_C((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$*;*
(iii) If $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$*, then* $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof. (i) Obviously, $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \ge 0$. Now, we present the proof of $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \le 1$.

$$\begin{aligned}\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = &\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * \\ &(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].\end{aligned}$$

$$= \sum_{k=1}^{m}\Big[\Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_1))$$
$$+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_1))\Big) + \Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_2))$$
$$+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_2))\Big) + ...$$
$$+\Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_n))$$
$$+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_n))\Big)\Big].$$

By applying Cauchy-Schwarz inequality, we get

$$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2 \leq \sum_{k=1}^{m}\Big[\Big\{(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+\Big\{(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+\Big\{(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big] \times$$
$$\sum_{k=1}^{m}\Big[\Big\{(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+\Big\{(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+\Big\{(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big].$$

$$\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2 \leq \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]$$
$$\times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big].$$

$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))^2 \leq \Phi(\Omega_1,\tilde{\Delta}_1) \times \Phi(\Omega_2,\tilde{\Delta}_2).$
$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq \sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}.$
$\Rightarrow \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \leq 1.$
By using Definition 3.5, we get $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$.
Hence, $0 \leq \mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$. □

Proof. (ii) Straight forward. □

Proof. (iii) $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}}$.
Since, $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$.

$\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]} \times \sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]}}$$

$\Rightarrow \mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1.$

□

Definition 3.8. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two PFHSS. Then, the CC between $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2\tilde{\Delta}_2)$ is defined as

$$\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\max\Big\{\Phi(\Omega_1, \tilde{\Delta}_1), \Phi(\Omega_2, \tilde{\Delta}_2)\Big\}}. \tag{3.5}$$

$\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]}{\max\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big], \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big\}}.$$

Proposition 3.9. *Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two PFHSS. Then, the following CC properties hold:*
(i) $0 \leq \tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$;
(ii) $\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \tilde{\mathcal{C}_C}((\Omega_2, \tilde{\Delta}_2), (\Omega_1, \tilde{\Delta}_1))$;
(iii) If $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$, *then* $\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = 1$.

Proof. (i) Obviously, $\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \geq 0$. Now, we present the proof of $\tilde{\mathcal{C}_C}((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$.

$$\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$$

$$= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].$$

$$= \sum_{k=1}^{m}\Big[\Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_1))\Big) + \Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_2))\Big) + ... + \Big((\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_n))\Big)\Big].$$

By applying Cauchy-Schwarz inequality, we get
$\mathcal{C}_{M}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$\leq \Bigg\{\sum_{k=1}^{m}\Big[\Big\{(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big] \times \sum_{k=1}^{m}\Big[\Big\{(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big]\Bigg\}^{\frac{1}{2}}.$$

$$\begin{aligned}
&\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\\
&\quad\leq\Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\times\\
&\qquad\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}^{\frac{1}{2}}.\\
&\quad\leq\Bigg\{\Bigg(\max\Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\\
&\quad\times\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}\Bigg)^2\Bigg\}^{\frac{1}{2}}.\\
&\quad=\max\Bigg\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\times\\
&\quad\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}.\\
&\Rightarrow\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leq\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}.\\
&\Rightarrow\frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}}\leq 1.
\end{aligned}$$

By using Definition 3.8, we get $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leq 1$.
Hence, $0\leq\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\leq 1$.
Proofs of (ii) and (iii) are same as in Proposition 3.6. □

3.2. **Weighted correlation coefficient for PFHSS.** We next present the concept of weighted correlation coefficient (WCC) for PFHSS. WCC facilitates decision-makers (DMs) to provide different weights for each of the alternatives. Consider $\mathcal{D}=\{\mathcal{D}_1,\mathcal{D}_2,...,\mathcal{D}_m\}$ and $\mathcal{W}=\{\mathcal{W}_1,\mathcal{W}_2,...,\mathcal{W}_n\}$ as weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k,\mathcal{W}_i>0$ and $\sum_{k=1}^{m}\mathcal{D}_k=1,\sum_{i=1}^{n}\mathcal{W}_i=1$.

Definition 3.10. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two PFHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \tag{3.6}$$

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i)+\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i)+\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i)\Big]\Big)}{\sqrt{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)}\times\sqrt{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)}}.$$

If $\mathcal{D}=\left\{\frac{1}{m},\frac{1}{m},...,\frac{1}{m},\right\}$ and $\mathcal{W}=\left\{\frac{1}{n},\frac{1}{n},...,\frac{1}{n},\right\}$, then WCC given in Eq.(6) reduces to CC as in Eq.(4).

Proposition 3.11. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two PFHSS. Then, the following WCC properties hold:*
(i) $0\le\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\le 1$;
(ii) $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\mathcal{C}_{C_{\mathcal{W}}}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1)=(\Omega_2,\tilde{\Delta}_2)$, then $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=1$.

Proof. Similar to Proposition 3.6. □

Definition 3.12. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two PFHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\left\{\Phi(\Omega_1,\tilde{\Delta}_1),\Phi(\Omega_2,\tilde{\Delta}_2)\right\}}. \tag{3.7}$$

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]\Big)}{\max\Big\{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{P}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\Big),\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\mathcal{P}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{E}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\mathcal{N}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)\Big\}}.$$

If $\mathcal{D}=\left\{\frac{1}{m},\frac{1}{m},...,\frac{1}{m},\right\}$ and $\mathcal{W}=\left\{\frac{1}{n},\frac{1}{n},...,\frac{1}{n},\right\}$, then WCC given in Eq.(7) reduces to CC as in Eq.(5).

Proposition 3.13. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two PFHSS. Then, the following WCC properties hold:*
(i) $0\le\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\le 1$;

(ii) $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. Similiar to Proposition 3.6. □

4. Aggregation operators for PFHSS

We now present the concept of picture fuzzy hypersoft weighted average operator(PFHSWAO) and picture fuzzy hypersoft weighted geometric operator (PFHSWGO) by using operational laws. Let κ represent the collection of picture fuzzy hypersoft numbers (PFHSNs).

4.1. Operational laws for PFHSS.

Definition 4.1. Let $\Omega_{e_{11}} = (\mathcal{P}_{11},\mathcal{E}_{11},\mathcal{N}_{11})$ and $\Omega_{e_{12}} = (\mathcal{P}_{12},\mathcal{E}_{12},\mathcal{N}_{12})$ be two PFHSS and β a positive integer. Then,
(i) $\Omega_{e_{11}} \oplus \Omega_{e_{12}} = \langle \mathcal{P}_{11}+\mathcal{P}_{12}-\mathcal{P}_{11}\mathcal{P}_{12}, \mathcal{E}_{11}+\mathcal{E}_{12}-\mathcal{E}_{11}\mathcal{E}_{12}, \mathcal{N}_{11}\mathcal{N}_{12}\rangle$;
(ii) $\Omega_{e_{11}} \otimes \Omega_{e_{12}} = \langle \mathcal{P}_{11}\mathcal{P}_{12}, \mathcal{E}_{11}\mathcal{E}_{12}, \mathcal{N}_{11}+\mathcal{N}_{12}-\mathcal{N}_{11}\mathcal{N}_{12}\rangle$;
(iii) $\beta\Omega_{e_{11}} = \big\langle \big[(1-(1-\mathcal{P}_{11})^{\beta}, (1-(1-\mathcal{E}_{11})^{\beta}, (\mathcal{N}_{11})^{\beta}\big]\big\rangle$;
(iv) $(\Omega_{e_{11}})^{\beta} = \big\langle \big[(\mathcal{P}_{11})^{\beta}, (\mathcal{E}_{11})^{\beta}, (1-(1-\mathcal{N}_{11})^{\beta}\big]\big\rangle$.

4.2. Picture fuzzy hypersoft weighted average operator.

Definition 4.2. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m}\mathcal{D}_k = 1, \sum_{i=1}^{n}\mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\mathcal{P}_{ik},\mathcal{E}_{ik},\mathcal{N}_{ik})$ be a PFHSN, where $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. Then the PFHSWAO $\mathcal{A}:\kappa^n \to \kappa$ is represented as

$$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{nm}}) = \bigoplus_{k=1}^{m}\mathcal{D}_k(\bigoplus_{i=1}^{n}\mathcal{W}_i\Omega_{e_{ik}}).$$

Theorem 4.3. *Let* $\Omega_{e_{ik}} = (\mathcal{P}_{ik},\mathcal{E}_{ik},\mathcal{N}_{ik})$ *be a PFHSN, where* $i = \{1,2,...n\}$, $k = \{1,2,...m\}$. *Then, the aggregated value of PFHSWAO is also a PFHSN is given by*
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{nm}})$

$$= \left\langle 1-\prod_{k=1}^{m}(\prod_{i=1}^{n}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{m}(\prod_{i=1}^{n}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{m}(\prod_{i=1}^{n}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

Proof. If $n = 1$, then $\mathcal{W}_1 = 1$, using Definition 4.1, we get
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{1m}}) = \bigoplus_{k=1}^{m}\mathcal{D}_k\Omega_{e_{1k}}$.

$$= \left\langle 1-\prod_{k=1}^{m}(\prod_{i=1}^{1}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{m}(\prod_{i=1}^{1}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{m}(\prod_{i=1}^{1}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

If $m=1$, then $\mathcal{D}_1=1$, using Definition 4.2, we get
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{21}},...,\Omega_{e_{n1}})=\bigoplus_{i=1}^{n}\mathcal{W}_i\Omega_{e_{i1}}$.

$$=\left\langle 1-\prod_{k=1}^{1}(\prod_{i=1}^{n}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{1}(\prod_{i=1}^{n}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{1}(\prod_{i=1}^{n}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

Hence, the results hold for $n=1$ and $m=1$.
Now, if $m=l_1+1$ and $n=l_2$, then,
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{l_2(l_1+1)}})=\bigoplus_{k=1}^{l_1+1}\mathcal{D}_k\big(\bigoplus_{i=1}^{l_2}\mathcal{W}_i\Omega_{e_{ik}}\big)$.

$$=\left\langle 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

Similarly, if $m=l_1$, $n=l_2+1$, then,
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{(l_2+1)l_1}})=\bigoplus_{k=1}^{l_1}\mathcal{D}_k\big(\bigoplus_{i=1}^{l_2+1}\mathcal{W}_i\Omega_{e_{ik}}\big)$.

$$=\left\langle 1-\prod_{k=1}^{l_1}(\prod_{i=1}^{l_2+1}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1}(\prod_{i=1}^{l_2+1}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{l_1}(\prod_{i=1}^{l_2+1}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

Now, if $m=l_1+1$, $n=l_2+1$, then,
$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{(l_2+1)(l_1+1)}})$

$$=\bigoplus_{k=1}^{l_1+1}\mathcal{D}_k(\bigoplus_{i=1}^{l_2+1}\mathcal{W}_i\Omega_{e_{ik}}).$$
$$=\bigoplus_{k=1}^{l_1+1}\mathcal{D}_k(\bigoplus_{i=1}^{l_2}\mathcal{W}_i\Omega_{e_{ik}})\bigoplus_{k=1}^{l_1+1}\mathcal{D}_k(\mathcal{W}_{l_2+1}\Omega_{e_{(l_2+1)k}}).$$

$\mathcal{A}(\Omega_{e_{11}},\Omega_{e_{12}},...,\Omega_{e_{(l_2+1)(l_1+1)}})$

$$\begin{aligned}=\Big\langle & 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\oplus 1-\prod_{k=1}^{l_1+1}((1-\mathcal{P}_{(l_2+1)k})^{\mathcal{W}_{(l_2+1)}})^{\mathcal{D}_k},\\ & 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\oplus 1-\prod_{k=1}^{l_1+1}((1-\mathcal{E}_{(l_2+1)k})^{\mathcal{W}_{(l_2+1)}})^{\mathcal{D}_k},\\ & \prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\oplus\prod_{k=1}^{l_1+1}((\mathcal{N}_{(l_2+1)k})^{\mathcal{W}_{(l_2+1)}})^{\mathcal{D}_k}\Big\rangle.\end{aligned}$$
$$=\left\langle 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2+1}(1-\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1-\prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2+1}(1-\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1}(\prod_{i=1}^{l_2+1}(\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}\right\rangle.$$

Hence, the results hold for $n=l_2+1$ and $m=l_1+1$.
Therefore, by induction method, the result is true $\forall\ m,n\geq 1$.

Since,

$$0 \leq \mathcal{P}_{ik} + \mathcal{E}_{ik} + \mathcal{N}_{ik} \leq 1.$$
$$\Leftrightarrow 1 - \prod_{k=1}^{m} (\prod_{i=1}^{n} (1 - \mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m} (\prod_{i=1}^{n} (1 - \mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} + ((\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} \leq 1.$$

Therefore, the aggregated value given by PFHSWAO is also a PFHSN. □

Example 4.4. Let us consider the same values as in Example 3.2. Also, let $\mathcal{W}_i = \{0.50, 0.30, 0.20\}$ and $\mathcal{D}_k = \{0.14, 0.13, 0.23, 0.20, 0.18, 0.12\}$ be the weight of sociologists and attributes, respectively. Then,
$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{36}})$

$$= \left\langle 1 - \prod_{k=1}^{6} (\prod_{i=1}^{3} (1 - \mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1 - \prod_{k=1}^{6} (\prod_{i=1}^{3} (1 - \mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{6} (\prod_{i=1}^{3} (\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} \right\rangle.$$
$$= \langle 0.48, 0.24, 0.22 \rangle.$$

4.3. **Picture fuzzy hypersoft weighted geometric operator.**

Definition 4.5. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1, \sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\mathcal{P}_{ik}, \mathcal{E}_{ik}, \mathcal{N}_{ik})$ be a PFHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then the PFHSWGO $\mathcal{G} : \kappa^n \to \kappa$ is defined as

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \bigotimes_{k=1}^{m} \left(\bigotimes_{i=1}^{n} \left(\Omega_{e_{ik}} \right)^{\mathcal{W}_i} \right)^{\mathcal{D}_k}.$$

Theorem 4.6. *Let $\Omega_{e_{ik}} = (\mathcal{P}_{ik}, \mathcal{E}_{ik}, \mathcal{N}_{ik})$ be a PFHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, the aggregated value of PFHSWGO is also a PFHSN is given by*
$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}})$

$$= \left\langle \prod_{k=1}^{m} (\prod_{i=1}^{n} (\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{m} (\prod_{i=1}^{n} (\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} (\prod_{i=1}^{n} (1 - \mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} \right\rangle.$$

Proof. Similar to Theorem 4.3. □

Example 4.7. Let us consider the same values given in Example 3.2 and the weight of sociologists and attributes be as in Example 4.4. Then,
$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{36}})$

$$= \left\langle \prod_{k=1}^{6} (\prod_{i=1}^{3} (\mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{6} (\prod_{i=1}^{3} (\mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1 - \prod_{k=1}^{6} (\prod_{i=1}^{3} (1 - \mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} \right\rangle.$$
$$= \langle 0.36, 0.19, 0.30 \rangle.$$

5. MCDM problems based on TOPSIS and CC method

TOPSIS method helps to find the best alternative based on minimum and maximum distance from the picture fuzzy positive ideal solution (PFPIS) and picture fuzzy negative ideal solution (PFNIS). Also, when TOPSIS method is combined with CC instead of similarity measures, it provides reliable results for evaluating the closeness coefficients. We present an algorithm and a case study to illustrate the PFHSS TOPSIS method based on CC.

5.1. **Algorithm to solve MCDM problems with PFHSS data based on TOPSIS and CC method.** Let $\mathcal{A} = \left\{\mathcal{A}^1, \mathcal{A}^2, ..., \mathcal{A}^x\right\}$ be a set of selected leaders aspiring to bring in socio-political changes to society and $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ be a set of sociologists responsible to evaluate the leaders with weights $\mathcal{W}_i = (\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n)$, such that $\mathcal{W}_i > 0$ and $\sum_{i=1}^{n} \mathcal{W}_i = 1$. Let $\tilde{\Delta} = \left\{\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_m\right\}$ be a set of multi-valued sub-attributes with weights $\mathcal{D}_k = (\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_m)$, such that $\mathcal{D}_k > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$. The evaluation of leaders $\mathcal{A}^t$, $(t = 1, 2, ..., x)$ is carried out by the sociologists v_i, $(i = 1, 2, ..., n)$ based on the multi-valued sub-attributes $\tilde{\lambda}_k$, $(k = 1, 2, ..., m)$ given in PFHSS form and represented as $\Omega^t_{ik} = \left\langle \mathcal{P}^t_{ik}, \mathcal{E}^t_{ik}, \mathcal{N}^t_{ik} \right\rangle$, subject to the conditions $0 \leq \mathcal{P}^t_{ik} + \mathcal{E}^t_{ik} + \mathcal{N}^t_{ik} \leq 1$ $\forall$ i, k.

Step 1. Construct the matrix for each multi-valued sub-attributes in PFHSS form as below:

$[\mathcal{A}^t, \tilde{\Delta}]_{n\times m} = [\mathcal{A}^t]_{n\times m}$

$$
= \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{array}
\begin{array}{c}
\begin{array}{cccc} \quad\tilde{\lambda}_1 \quad & \quad\tilde{\lambda}_2\quad & \cdots & \quad\tilde{\lambda}_m\quad \end{array} \\
\begin{bmatrix}
\langle \mathcal{P}^t_{11}, \mathcal{E}^t_{11}, \mathcal{N}^t_{11} \rangle & \langle \mathcal{P}^t_{12}, \mathcal{E}^t_{12}, \mathcal{N}^t_{12} \rangle & \cdots & \langle \mathcal{P}^t_{1m}, \mathcal{E}^t_{1m}, \mathcal{N}^t_{1m} \rangle \rangle \\
\langle \mathcal{P}^t_{21}, \mathcal{E}^t_{21}, \mathcal{N}^t_{21} \rangle & \langle \mathcal{P}^t_{22}, \mathcal{E}^t_{22}, \mathcal{N}^t_{22} \rangle & \cdots & \langle \mathcal{P}^t_{2m}, \mathcal{E}^t_{2m}, \mathcal{N}^t_{2m} \rangle \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle \mathcal{P}^t_{n1}, \mathcal{E}^t_{n1}, \mathcal{N}^t_{n1} \rangle & \langle \mathcal{P}^t_{n2}, \mathcal{E}^t_{n2}, \mathcal{N}^t_{n2} \rangle & \cdots & \langle \mathcal{P}^t_{nm}, \mathcal{E}^t_{nm}, \mathcal{N}^t_{nm} \rangle \rangle
\end{bmatrix}
\end{array}
$$

Step 2. Obtain the weighted decision matrix for each multi-valued sub-attributes,

$[\tilde{A}^t_{ik}]_{n\times m}$

$$
= \left\langle 1 - \prod_{k=1}^{m} (\prod_{i=1}^{n} (1 - \mathcal{P}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} (\prod_{i=1}^{n} (1 - \mathcal{E}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k}, \prod_{k=1}^{m} (\prod_{i=1}^{n} (\mathcal{N}_{ik})^{\mathcal{W}_i})^{\mathcal{D}_k} \right\rangle
$$

$$
= \left\langle \tilde{\mathcal{P}}_{ik}, \tilde{\mathcal{E}}_{ik}, \tilde{\mathcal{N}}_{ik} \right\rangle .
$$

Step 3. Determine the PFPIS and PFNIS for weighted PFHSS as below:

$$\tilde{\mathcal{A}}^{+} = \left\langle \tilde{\mathcal{P}}^{+}, \tilde{\mathcal{E}}^{+}, \tilde{\mathcal{N}}^{+} \right\rangle_{n\times m} = \left\langle \tilde{\mathcal{P}}^{(\vee_{ij})}, \tilde{\mathcal{E}}^{(\wedge_{ij})}, \tilde{\mathcal{N}}^{(\wedge_{ij})} \right\rangle \text{ and}$$

$$\tilde{\mathcal{A}}^{-} = \left\langle \tilde{\mathcal{P}}^{-}, \tilde{\mathcal{E}}^{-}, \tilde{\mathcal{N}}^{-} \right\rangle_{n\times m} = \left\langle \tilde{\mathcal{P}}^{(\wedge_{ij})}, \tilde{\mathcal{E}}^{(\wedge_{ij})}, \tilde{\mathcal{N}}^{(\vee_{ij})} \right\rangle,$$

where $\vee_{ij} = \arg\max_t \left\{ \varphi_{ij}^t \right\}$ and $\wedge_{ij} = \arg\min_t \left\{ \varphi_{ij}^t \right\}$.

Step 4. Determine the CC for each alternative from PFPIS and PFNIS.

$$\chi^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^+) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^+)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{(\Phi\tilde{A}^+)}} \text{ and}$$

$$\lambda^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^-) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^-)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^-)}}$$

Step 5. Compute the closeness coefficient of picture fuzzy ideal solution as:

$$\epsilon^t = \frac{1 - \lambda^t}{2 - \chi^t - \lambda^t}$$

Step 6. Arrange the ϵ^t values in descending order and determine the rank of the alternatives $\mathcal{A}^t$, $(t = 1, 2, ..., x)$. The one with the maximum value is the best alternative.

5.2. **Application based on TOPSIS and CC method.** Let $\mathcal{A} = \left\{ \mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3, \mathcal{A}^4 \right\}$ be a set of leaders aspiring to bring in socio-political changes with their leadership skills and Δ_1 and Δ_2 be distinct attribute sets whose corresponding sub-attributes are represented as

$$\begin{aligned} \Delta_1 &= \text{leader attributes} = \big\{ \lambda_{11} = \text{personality variables}, \\ &\lambda_{12} = \text{cognitive ability and skills} \big\}, \Delta_2 = \text{leader behaviors} \\ &= \{ \lambda_{21} = \text{setting sub culture}, \lambda_{22} = \text{conflict management} \}. \end{aligned}$$

Then $\tilde{\Delta} = \Delta_1 \times \Delta_2$ is the distinct attribute set is given by

$$\begin{aligned} \tilde{\Delta} &= \Delta_1 \times \Delta_2 = \{ \lambda_{11}, \lambda_{12} \} \times \{ \lambda_{21}, \lambda_{22} \}. \\ &= \left\{ (\lambda_{11}, \lambda_{21}), (\lambda_{11}, \lambda_{22}), (\lambda_{12}, \lambda_{21}), (\lambda_{12}, \lambda_{21}) \right\}. \\ &= \left\{ \tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4} \right\} \text{ with weights } \mathcal{D}_k = (0.20, 0.25, 0.30, 0.25). \end{aligned}$$

Let $\mathcal{V} = \{ v_1, v_2, v_3, v_4 \}$ be a set of sociologists responsible to evaluate the leaders with weights $\mathcal{W}_i = (0.35, 0.15, 0.30, 0.20)$. The aim is to find a leader who can bring major socio-political changes in a larger way to society.

Step 1. Construct $\mathcal{A}^1$, $\mathcal{A}^2$, $\mathcal{A}^3$ and $\mathcal{A}^4$ matrices for each multi-valued sub-attributes in PFHSS form.

TABLE 2. Representation of values in PFHSS form for $\mathcal{A}^1$.

$\mathcal{A}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.42, 0.34, 0.21\rangle$	$\langle 0.23, 0.48, 0.25\rangle$	$\langle 0.17, 0.70, 0.10\rangle$	$\langle 0.23, 0.24, 0.35\rangle$
v_2	$\langle 0.23, 0.31, 0.24\rangle$	$\langle 0.43, 0.30, 0.25\rangle$	$\langle 0.42, 0.29, 0.12\rangle$	$\langle 0.23, 0.13, 0.45\rangle$
v_3	$\langle 0.13, 0.25, 0.35\rangle$	$\langle 0.45, 0.20, 0.35\rangle$	$\langle 0.67, 0.12, 0.19\rangle$	$\langle 0.41, 0.15, 0.34\rangle$
v_4	$\langle 0.53, 0.11, 0.31\rangle$	$\langle 0.52, 0.23, 0.19\rangle$	$\langle 0.49, 0.34, 0.12\rangle$	$\langle 0.18, 0.23, 0.53\rangle$

TABLE 3. Representation of values in PFHSS form for $\mathcal{A}^2$.

$\mathcal{A}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.48, 0.27, 0.24\rangle$	$\langle 0.23, 0.43, 0.25\rangle$	$\langle 0.54, 0.22, 0.12\rangle$	$\langle 0.32, 0.41, 0.24\rangle$
v_2	$\langle 0.44, 0.23, 0.21\rangle$	$\langle 0.25, 0.56, 0.15\rangle$	$\langle 0.34, 0.33, 0.31\rangle$	$\langle 0.42, 0.23, 0.15\rangle$
v_3	$\langle 0.35, 0.12, 0.45\rangle$	$\langle 0.37, 0.32, 0.25\rangle$	$\langle 0.45, 0.44, 0.11\rangle$	$\langle 0.45, 0.11, 0.43\rangle$
v_4	$\langle 0.45, 0.13, 0.35\rangle$	$\langle 0.45, 0.23, 0.16\rangle$	$\langle 0.12, 0.55, 0.24\rangle$	$\langle 0.34, 0.12, 0.51\rangle$

TABLE 4. Representation of values in PFHSS form for $\mathcal{A}^3$.

$\mathcal{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.32, 0.12, 0.34\rangle$	$\langle 0.51, 0.21, 0.21\rangle$	$\langle 0.21, 0.45, 0.23\rangle$	$\langle 0.63, 0.13, 0.19\rangle$
v_2	$\langle 0.34, 0.42, 0.21\rangle$	$\langle 0.54, 0.14, 0.19\rangle$	$\langle 0.34, 0.41, 0.21\rangle$	$\langle 0.53, 0.12, 0.25\rangle$
v_3	$\langle 0.29, 0.23, 0.35\rangle$	$\langle 0.49, 0.23, 0.24\rangle$	$\langle 0.21, 0.31, 0.45\rangle$	$\langle 0.23, 0.21, 0.34\rangle$
v_4	$\langle 0.35, 0.40, 0.24\rangle$	$\langle 0.34, 0.41, 0.21\rangle$	$\langle 0.45, 0.25, 0.19\rangle$	$\langle 0.45, 0.31, 0.17\rangle$

TABLE 5. Representation of values in PFHSS form for $\mathcal{A}^4$.

$\mathcal{A}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.18, 0.46, 0.12\rangle$	$\langle 0.35, 0.40, 0.24\rangle$	$\langle 0.23, 0.32, 0.42\rangle$	$\langle 0.21, 0.34, 0.45\rangle$
v_2	$\langle 0.47, 0.14, 0.14\rangle$	$\langle 0.23, 0.40, 0.34\rangle$	$\langle 0.13, 0.34, 0.52\rangle$	$\langle 0.41, 0.23, 0.31\rangle$
v_3	$\langle 0.54, 0.12, 0.23\rangle$	$\langle 0.36, 0.34, 0.26\rangle$	$\langle 0.12, 0.23, 0.43\rangle$	$\langle 0.23, 0.34, 0.41\rangle$
v_4	$\langle 0.42, 0.40, 0.16\rangle$	$\langle 0.46, 0.23, 0.31\rangle$	$\langle 0.18, 0.32, 0.45\rangle$	$\langle 0.32, 0.50, 0.12\rangle$

Step 2. Obtain $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$, the weighted matrices for each multi-valued sub-attributes.

TABLE 6. Shows weighted values in PFHSS form for $\tilde{\mathcal{A}}^1$.

$\tilde{\mathcal{A}}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.0375, 0.0287, 0.8966\rangle$	$\langle 0.0227, 0.0557, 0.8858\rangle$	$\langle 0.0194, 0.1188, 0.7853\rangle$	$\langle 0.0227, 0.0238, 0.9123\rangle$
v_2	$\langle 0.0079, 0.0111, 0.9581\rangle$	$\langle 0.0209, 0.0133, 0.9494\rangle$	$\langle 0.0243, 0.0153, 0.9090\rangle$	$\langle 0.0098, 0.0053, 0.9705\rangle$
v_3	$\langle 0.0084, 0.0172, 0.9390\rangle$	$\langle 0.0439, 0.0166, 0.9243\rangle$	$\langle 0.0950, 0.0115, 0.8612\rangle$	$\langle 0.0388, 0.0122, 0.9223\rangle$
v_4	$\langle 0.0298, 0.0047, 0.9543\rangle$	$\langle 0.0361, 0.0130, 0.9204\rangle$	$\langle 0.0396, 0.0247, 0.8806\rangle$	$\langle 0.0099, 0.0130, 0.9688\rangle$

TABLE 7. Shows weighted values in PFHSS form for $\tilde{\mathcal{A}}^2$.

$\tilde{\mathcal{A}}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_2$	$\tilde{\lambda}_4$
v_1	$\langle 0.0448, 0.0218, 0.9050\rangle$	$\langle 0.0227, 0.0480, 0.8858\rangle$	$\langle 0.0784, 0.0258, 0.8005\rangle$	$\langle 0.0332, 0.0452, 0.8827\rangle$
v_2	$\langle 0.0173, 0.0079, 0.9543\rangle$	$\langle 0.0108, 0.0304, 0.9314\rangle$	$\langle 0.0186, 0.0179, 0.9487\rangle$	$\langle 0.0203, 0.0098, 0.9314\rangle$
v_3	$\langle 0.0256, 0.0077, 0.9533\rangle$	$\langle 0.0341, 0.0286, 0.9013\rangle$	$\langle 0.0524, 0.0509, 0.8199\rangle$	$\langle 0.0439, 0.0088, 0.9387\rangle$
v_4	$\langle 0.0237, 0.0056, 0.9589\rangle$	$\langle 0.0295, 0.0130, 0.9125\rangle$	$\langle 0.0077, 0.0468, 0.9180\rangle$	$\langle 0.0206, 0.0064, 0.9669\rangle$

TABLE 8. Shows weighted values in PFHSS form for $\tilde{\mathcal{A}}^3$.

$\tilde{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.0267, 0.0090, 0.9273\rangle$	$\langle 0.0606, 0.0205, 0.8724\rangle$	$\langle 0.0245, 0.0609, 0.8571\rangle$	$\langle 0.0834, 0.0122, 0.8648\rangle$
v_2	$\langle 0.0124, 0.0163, 0.9543\rangle$	$\langle 0.0287, 0.0057, 0.9397\rangle$	$\langle 0.0186, 0.0235, 0.9322\rangle$	$\langle 0.0280, 0.0048, 0.9494\rangle$
v_3	$\langle 0.0204, 0.0156, 0.9390\rangle$	$\langle 0.0493, 0.0195, 0.8985\rangle$	$\langle 0.0210, 0.0329, 0.9307\rangle$	$\langle 0.0195, 0.0176, 0.9223\rangle$
v_4	$\langle 0.0171, 0.0203, 0.9446\rangle$	$\langle 0.0206, 0.0261, 0.9250\rangle$	$\langle 0.0353, 0.0172, 0.9052\rangle$	$\langle 0.0295, 0.0184, 0.9153\rangle$

TABLE 9. Shows weighted values in PFHSS form for $\tilde{\mathcal{A}}^4$.

$\tilde{\mathcal{A}}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle 0.0138, 0.0423, 0.8621\rangle$	$\langle 0.0370, 0.0438, 0.8827\rangle$	$\langle 0.0271, 0.0397, 0.9130\rangle$	$\langle 0.0205, 0.0358, 0.9326\rangle$
v_2	$\langle 0.0189, 0.0046, 0.9428\rangle$	$\langle 0.0098, 0.0190, 0.9604\rangle$	$\langle 0.0063, 0.0186, 0.9711\rangle$	$\langle 0.0196, 0.0098, 0.9571\rangle$
v_3	$\langle 0.0456, 0.0077, 0.9156\rangle$	$\langle 0.0330, 0.0307, 0.9040\rangle$	$\langle 0.0115, 0.0233, 0.9269\rangle$	$\langle 0.0195, 0.0307, 0.9354\rangle$
v_4	$\langle 0.0216, 0.0203, 0.9294\rangle$	$\langle 0.0304, 0.0130, 0.9432\rangle$	$\langle 0.0119, 0.0229, 0.9533\rangle$	$\langle 0.0191, 0.0341, 0.8995\rangle$

Step 3. Determine the PFPIS and PFNIS from the weighted matrices, $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$.

$$\tilde{\mathcal{A}}^+ = \begin{bmatrix} \langle 0.0448, 0.0090, 0.8621\rangle & \langle 0.0606, 0.0205, 0.8724\rangle & \langle 0.0784, 0.0258, 0.7853\rangle & \langle 0.0834, 0.0122, 0.8648\rangle \\ \langle 0.0189, 0.0046, 0.9428\rangle & \langle 0.0287, 0.0057, 0.9314\rangle & \langle 0.0243, 0.0153, 0.9090\rangle & \langle 0.0280, 0.0048, 0.9314\rangle \\ \langle 0.0456, 0.0077, 0.9156\rangle & \langle 0.0493, 0.0166, 0.8985\rangle & \langle 0.0950, 0.0115, 0.8199\rangle & \langle 0.0439, 0.0088, 0.9223\rangle \\ \langle 0.0298, 0.0047, 0.9294\rangle & \langle 0.0361, 0.0130, 0.9125\rangle & \langle 0.0396, 0.0172, 0.8806\rangle & \langle 0.0295, 0.0064, 0.8995\rangle \end{bmatrix}$$

$$\tilde{\mathcal{A}}^- = \begin{bmatrix} \langle 0.0138, 0.0090, 0.9273\rangle & \langle 0.0227, 0.0205, 0.8858\rangle & \langle 0.0194, 0.0258, 0.9130\rangle & \langle 0.0205, 0.0122, 0.9326\rangle \\ \langle 0.0079, 0.0046, 0.9581\rangle & \langle 0.0098, 0.0057, 0.9604\rangle & \langle 0.0063, 0.0153, 0.9711\rangle & \langle 0.0098, 0.0048, 0.9705\rangle \\ \langle 0.0084, 0.0046, 0.9533\rangle & \langle 0.0330, 0.0166, 0.9243\rangle & \langle 0.0115, 0.0115, 0.9307\rangle & \langle 0.0195, 0.0088, 0.9387\rangle \\ \langle 0.0171, 0.0047, 0.9589\rangle & \langle 0.0206, 0.0130, 0.9432\rangle & \langle 0.0077, 0.0172, 0.9533\rangle & \langle 0.0099, 0.0064, 0.9688\rangle \end{bmatrix}$$

Step 4. Determine the CC for the alternatives by using the values of PFPIS and PFNIS.

$$\chi^1 = 0.9989, \chi^2 = 0.9993, \chi^3 = 0.9989 \text{ and } \chi^4 = 0.9982.$$
$$\lambda^1 = 0.9985, \lambda^2 = 0.9988, \lambda^3 = 0.9993 \text{ and } \lambda^4 = 0.9995.$$

Step 5. Compute the closeness coefficients of picture fuzzy ideal solution as below.

$$\epsilon^1 = 0.5769, \epsilon^2 = 0.6316, \epsilon^3 = 0.3889 \text{ and } \epsilon^4 = 0.2174.$$

Step 6. Arrange the values in descending order.

$$\epsilon^2 > \epsilon^1 > \epsilon^3 > \epsilon^4.$$

$$\Rightarrow \mathcal{A}^2 > \mathcal{A}^1 > \mathcal{A}^3 > \mathcal{A}^4.$$

Hence, $\mathcal{A}^2$ is the best leader among the group and can play a significant role in bringing socio-political changes to society.

5.3. **Comparative study.** We compare the proposed method with existing methods.

TABLE 10. Comparing proposed method with existing methods

Authors	Methods	Remarks
Farooq and Saqlain [7]	neutrosophic HSS TOPSIS method	In this method, the values of truth, indeterminacy, and falsity are independent.
Fatma and Cengiz [8]	spherical fuzzy TOPSIS method	Single attribute function is defined to solve MCDM problems.
Proposed method	picture fuzzy HSS TOPSIS method	Dependent grades between positive, neutral and negative and have used multi-attributes function.

6. CONCLUSIONS

In this study, we have introduced the notion of PFHSS and established some of its properties. We have developed the concept of PFHSWAO and PFHSWGO by using operational laws. Also, we have proposed a real-life application using the TOPSIS method based on CC. We have applied CC instead of the usual distance or similarity measures in the TOPSIS method to evaluate the closeness coefficients. Finally, we have discussed the merits of the proposed method with existing methods. Future work may include the study of the proposed method with existing hybrid structures like interval-valued PFHSS, cubic PFHSS, cubic HSS, cubic intuitionistic fuzzy HSS, octahedron HSS, and cubic hesitant fuzzy HSS.

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V.Chinnadurai
Department of Mathematics, Annamalai University, Tamil Nadu, India.
Email: `chinnaduraiau@gmail.com`

A.Bobin
Department of Mathematics, Annamalai University, Tamil Nadu, India.
Email: `bobinalbert@gmail.com`