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A STUDY ON THE GROWTH OF GENERALIST ITERATED ENTIRE FUNCTIONS

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Abstract. In this paper we study growth properties of generalist iterated entire functions.

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1. Introduction, Definitions and Notations

For any two transcendental entire functions $f(z)$ and $g(z)$ defined in the open complex plane C, it is well known [3] that $\lim_{r\to\infty} \frac{\log T(r,f_og)}{T(r,f)} = \infty$ and $\lim_{r\to\infty} \frac{\log T(r,f_og)}{T(r,g)} = 0$. Later on Singh [12] investigated some comparative growth of $logT(r, f_og)$ and $T(r, f)$. Farther in [12] he raised the problem of investing the comparative growth of $logT(r, f_og)$ and $T(r, g)$. However some results on the comparative growth of $logT(r, f_og)$ and $T(r, g)$ are proved in [8].

Recently Banerjee and Dutta [1], and Dutta [4], [5], [6] made close investigation on comparative growth properties of iterated entire functions to generalist some earlier results.

In this paper we consider three entire functions $f(z)$, $g(z)$ and $h(z)$ and following Banerjee and Mandal [2] form the iterations of $f(z)$ with respect to $g(z)$ and $h(z)$ [defined below] and generalist the results of Banerjee and Dutta [1] in this direction.

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$$\begin{aligned}
f_1(z) &= f(z) \\
f_2(z) &= f(g(z)) = f(g_1(z)) \\
f_3(z) &= f(g(h(z))) = f(g(h_1(z))) = f(g_2(z)) \\
f_4(z) &= f(g(h(f(z)))) = f(g(h_2(z))) = f(g_3(z)) \\
&\vdots \\
f_n(z) &= f(g(h(f..(f(z) \text{ or } g(z) \text{ or } h(z) \text{ according as } n = 3m-2 \text{ or } 3m-1 \\
&\quad \text{or } 3m)...))) \\
&= f(g_{n-1}(z)) = f(g(h_{n-2}(z))).
\end{aligned}$$

Similarly,

$$\begin{aligned}
g_1(z) &= g(z) \\
g_2(z) &= g(h(z)) = g(h_1(z)) \\
g_3(z) &= g(h(f(z))) = g(h(f_1(z))) = g(h_2(z)) \\
g_4(z) &= g(h(f(g(z)))) = g(h(f_2(z))) = g(h_3(z)) \\
&\vdots \\
g_n(z) &= g(h(f(g...(g(z) \text{ or } h(z) \text{ or } f(z) \text{ according as } n = 3m-2 \text{ or } 3m-1 \\
&\quad \text{or } 3m)...))) \\
&= g(h_{n-1}(z)) = g(h(f_{n-2}(z)))
\end{aligned}$$

and

$$\begin{aligned}
h_1(z) &= h(z) \\
h_2(z) &= h(f(z)) = h(f_1(z)) \\
h_3(z) &= h(f(g(z))) = h(f(g_1(z))) = h(f_2(z)) \\
h_4(z) &= h(f(g(h(z)))) = h(f(g_2(z))) = h(f_3(z)) \\
&\vdots \\
h_n(z) &= h(f(g(h...(h(z) \text{ or } f(z) \text{ or } g(z) \text{ according as } n = 3m-2 \text{ or } 3m-1 \\
&\quad \text{or } 3m)...))) \\
&= h(f_{n-1}(z)) = h(f(g_{n-2}(z))).
\end{aligned}$$

Clearly all $f_n(z)$, $g_n(z)$ and $h_n(z)$ are entire functions.

For two non-constant entire functions $f(z)$ and $g(z)$, we have the well known inequality

$$\log M(r, f(g)) \leq \log M(M(r, g), f). \tag{1.1}$$

Definition 1.1. The order ρ_f and lower order λ_f of a meromorphic function $f(z)$ is defined as

$$\rho_f = \lim\sup_{r\to\infty} \frac{\log T(r, f)}{\log r}$$

and

$$\lambda_f = \lim\inf_{r\to\infty} \frac{\log T(r, f)}{\log r}.$$

If $f(z)$ is entire then

$$\rho_f = \lim\sup_{r\to\infty} \frac{\log\log M(r, f)}{\log r}$$

and

$$\lambda_f = \lim\inf_{r\to\infty} \frac{\log\log M(r, f)}{\log r}.$$

Definition 1.2. A function $\lambda_f(r)$ is called a lower proximate order of a meromorphic function $f(z)$ if

(i) $\lambda_f(r)$ is nonnegative and continuous for $r \geq r_0$, say;

(ii) $\lambda_f(r)$ is differentiable for $r \geq r_0$ except possibly at isolated points at which $\lambda'_f(r-0)$ and $\lambda'_f(r+0)$ exist;

(iii) $\lim_{r\to\infty} \lambda_f(r) = \lambda_f < \infty$;

(iv) $\lim_{r\to\infty} r\lambda'_f(r) \log r = 0$; and

(v) $\lim\inf_{r\to\infty} \frac{T(r,f)}{r^{\lambda_f(r)}} = 1$.

Notation 1.3. [11] Let $log^{[0]}x = x$, $exp^{[0]}x = x$ and for positive integer m, $log^{[m]}x = log(log^{[m-1]}x)$, $exp^{[m]}x = exp(exp^{[m-1]}x)$.

Throughout we assume $f(z), g(z), h(z)$ etc. are non constant entire functions having respective orders ρ_f, ρ_g, ρ_h and respective lower orders $\lambda_f, \lambda_g, \lambda_h$. Also we do not explain the standard notations and definitions of the theory of entire and meromorphic functions because those are available in [7].

2. Lemmas

The following lemmas will be needed in the sequel.

Lemma 2.1. [7] *Let $f(z)$ be an entire function. For $0 \le r < R < \infty$, we have*

$$T(r,f) \le \log^+ M(r,f) \le \frac{R+r}{R-r} T(R,f).$$

Lemma 2.2. [10] *Let $f(z)$ and $g(z)$ be two entire functions. Then we have*

$$T(r,f(g)) \ge \frac{1}{3} \log M\left(\frac{1}{8} M\left(\frac{r}{4}, g\right) + O(1), f\right).$$

Lemma 2.3. [9] *Let $f\ (z)$be a meromorphic function. Then for $\delta(>0)$ the function $r^{\lambda_f\ +\delta-\lambda_f(r)}$ is an increasing function of r.*

Lemma 2.4. *Let $f(z)$, $g(z)$ and $h(z)$ be three non-constant entire functions of finite order and nonzero lower order. Then for any ε $(0 < \varepsilon < min\{\lambda_f, \lambda_g, \lambda_h\})$*

$$\log^{[n-1]} T(r,f_n) \le \begin{cases} (\rho_g+\varepsilon)\log M(r,h)+O(1) & when\ n = 3k \\ (\rho_h+\varepsilon)\log M(r,f)+O(1) & when\ n = 3k+1 \\ (\rho_f+\varepsilon)\log M(r,g)+O(1) & when\ n = 3k+2 \end{cases}$$

and

$$\log^{[n-1]} T(r,f_n) \ge \begin{cases} (\lambda_g-\varepsilon)\log M\left(\frac{r}{4^{n-1}},h\right)+O(1) & when\ n = 3k \\ (\lambda_h-\varepsilon)\log M\left(\frac{r}{4^{n-1}},f\right)+O(1) & when\ n = 3k+1 \\ (\lambda_f-\varepsilon)\log M\left(\frac{r}{4^{n-1}},g\right)+O(1) & when\ n = 3k+2. \end{cases}$$

Proof. For $\varepsilon(>0)$ we get from Lemma 2.1 and (1.1) for all large values of r

$$\begin{aligned} T(r,f_n) &\le \log M(r,f_n) \\ &\le \log M(M(r,g_{n-1}),f) \\ &\le [M(r,g_{n-1})]^{\rho_f+\varepsilon}, \\ \text{that is, } \log\ T(r,f_n) &\le (\rho_f+\varepsilon)\log M(r,g_{n-1}) \\ &\le (\rho_f+\varepsilon)\log M(M(r,h_{n-2}),g) \\ &\le (\rho_f+\varepsilon)[M(r,h_{n-2})]^{\rho_g+\varepsilon}. \\ \text{So, } \log^{[2]}\ T(r,f_n) &\le (\rho_g+\varepsilon)\log M(M(r,f_{n-3}),h)+O(1) \\ &\le (\rho_g+\varepsilon)[M(r,f_{n-3})]^{\rho_h+\varepsilon}+O(1). \\ \text{Therefore, } \log^{[n-1]}\ T(r,f_n) &\le (\rho_g+\varepsilon)\log M(r,h)+O(1) \quad \text{when } n=3k. \end{aligned}$$

Similarly

$$\log^{[n-1]}\ T(r,f_n) \le (\rho_h+\varepsilon)\log M(r,f) + O(1) \quad \text{when } n = 3k+1,$$

and

$$\log^{[n-1]}\ T(r,f_n) \le (\rho_f+\varepsilon)\log M(r,g) + O(1) \quad \text{when } n = 3k+2.$$

Again for ε ($0 < \varepsilon <\min\{\lambda_f,\lambda_g,\lambda_h\}$), we get from Lemma 2.1 and Lemma 2.2, for all large values of r

$$\begin{aligned}
T(r,f_n) &= T(r,f(g_{n-1})) \\
&\ge \frac{1}{3}\log M\left(\frac{1}{8}M\left(\frac{r}{4},g_{n-1}\right)+O(1),f\right) \\
&\ge \frac{1}{3}\left[\frac{1}{8}M\left(\frac{r}{4},g_{n-1}\right)+O(1)\right]^{\lambda_f-\varepsilon} \\
&\ge \frac{1}{3}\left[\frac{1}{9}M\left(\frac{r}{4},g_{n-1}\right)\right]^{\lambda_f-\varepsilon}, \\
\text{that is, } \log T(r,f_n) &\ge (\lambda_f-\varepsilon)\log M\left(\frac{r}{4},g_{n-1}\right)+O(1) \\
&\ge (\lambda_f-\varepsilon)T\left(\frac{r}{4},g_{n-1}\right)+O(1) \\
&\ge (\lambda_f-\varepsilon)\frac{1}{3}\log M\left(\frac{1}{8}M\left(\frac{r}{4^2},h_{n-2}\right)+O(1),g\right)+O(1) \\
&\ge (\lambda_f-\varepsilon)\frac{1}{3}\left[\frac{1}{8}M\left(\frac{r}{4^2},h_{n-2}\right)+O(1)\right]^{\lambda_g-\varepsilon}+O(1) \\
&\ge (\lambda_f-\varepsilon)\frac{1}{3}\left[\frac{1}{9}M\left(\frac{r}{4^2},h_{n-2}\right)\right]^{\lambda_g-\varepsilon}+O(1), \\
\text{that is, } \log^{[2]} T(r,f_n) &\ge (\lambda_g-\varepsilon)\log M\left(\frac{r}{4^2},h_{n-2}\right)+O(1). \\
\text{So, } \log^{[n-1]}\ T(r,f_n) &\ge (\lambda_g-\varepsilon)\log M\left(\frac{r}{4^{n-1}},h\right)+O(1) \quad \text{when } n=3k.
\end{aligned}$$

Similarly

$$\log^{[n-1]}\ T(r,f_n) \ge (\lambda_h-\varepsilon)\log M\left(\frac{r}{4^{n-1}},f\right)+O(1) \quad \text{when } n= 3k+1,$$

and

$$\log^{[n-1]}\ T(r,f_n) \ge (\lambda_f-\varepsilon)\log M\left(\frac{r}{4^{n-1}},g\right)+O(1) \quad \text{when } n= 3k+2.$$

This proves the lemma. □

3. Theorems

Theorem 3.1. *Let $f(z)$, $g(z)$ and $h(z)$ be three non-constant entire functions of finite order and nonzero lower order, then*

$$(i)\quad \liminf_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,h)} \leq 3\rho_g 2^{\lambda_h},$$

$$(ii)\quad \limsup_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,h)} \geq \frac{\lambda_g}{(4^{n-1})^{\lambda_h}}$$

when $n = 3k$
and

$$(iii)\quad \liminf_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,\ f)} \leq 3\rho_h 2^{\lambda_f},$$

$$(iv)\quad \limsup_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,f)} \geq \frac{\lambda_h}{(4^{n-1})^{\lambda_f}}$$

when $n = 3k+1$. Also when $n = 3k+2$,

$$(v)\quad \liminf_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,\ g)} \leq 3\rho_f 2^{\lambda_g},$$

$$(vi)\quad \limsup_{r\to\infty} \frac{\log^{[n-1]} T(r,f_n)}{T(r,g)} \geq \frac{\lambda_f}{(4^{n-1})^{\lambda_g}}.$$

Proof. Since $f(z)$, $g(z)$ and $h(z)$ are three non-constant entire functions of finite order and nonzero lower order so from Lemma 2.4 for arbitrary $\varepsilon > 0$,

$$\log^{[n-1]}\ T(r,f_n) \leq (\rho_g+\varepsilon)\log M(r,h) + O(1) \tag{3.1}$$

when $n = 3k$.

Let $0 < \varepsilon <$min$\{1, \lambda_f, \lambda_g, \lambda_h\}$. Since

$$\liminf_{r\to\infty} \frac{T(r,h)}{r^{\lambda_h(r)}} = 1,$$

there is a sequence of values of r tending to infinity for which

$$T(r,h) < (1+\varepsilon) r^{\lambda_h(r)} \tag{3.2}$$

and for all large value of r

$$T(r,h) > (1-\varepsilon) r^{\lambda_h(r)}. \tag{3.3}$$

Thus for a sequence of values of r tending to infinity we get for any $\delta(>0)$

$$\begin{aligned}\frac{\log M(r,h)}{T(r,h)} &\leq \frac{3T(2r,h)}{T(r,h)} \leq \frac{3(1+\varepsilon)}{1-\varepsilon}\frac{(2r)^{\lambda_h+\delta}}{(2r)^{\lambda_h+\delta-\lambda_h(2r)}}\frac{1}{r^{\lambda_h(r)}} \\ &\leq \frac{3(1+\varepsilon)}{1-\varepsilon}2^{\lambda_h+\delta}\end{aligned}$$

because $r^{\lambda_h+\delta-\lambda_h(r)}$ is an increasing function of r.
Since ε, $\delta > 0$ be arbitrary, we have

$$\liminf_{r\to\infty}\frac{\log M(r,h)}{T(r,h)} \leq 3.2^{\lambda_h}. \tag{3.4}$$

Therefore from (3.1) and (3.4) we get

$$\liminf_{r\to\infty}\frac{\log^{[n-1]} T(r,f_n)}{T(r,h)} \leq 3\rho_g 2^{\lambda_h},$$

when $n = 3k$.
Again for $n = 3k$ we have from Lemma 2.4,

$$\begin{aligned}\log^{[n-1]} T(r,f_n) &\geq (\lambda_g-\varepsilon)\log M\left(\frac{r}{4^{n-1}},h\right)+O(1) \\ &\geq (\lambda_g-\varepsilon)T\left(\frac{r}{4^{n-1}},h\right)+O(1) \\ &\geq (\lambda_g-\varepsilon)(1-\varepsilon)(1+O(1))\frac{\left(\frac{r}{4^{n-1}}\right)^{\lambda_h+\delta}}{\left(\frac{r}{4^{n-1}}\right)^{\lambda_h+\delta-\lambda_h\left(\frac{r}{4^{n-1}}\right)}}, \text{ by (3.3)}.\end{aligned}$$

Since $r^{\lambda_h+\delta-\lambda_h(r)}$ is an increasing function of r, we have

$$\log^{[n-1]} T(r,f_n) \geq (\lambda_g-\varepsilon)(1-\varepsilon)(1+O(1))\frac{r^{\lambda_h(r)}}{(4^{n-1})^{\lambda_h+\delta}}$$

for all large values of r.

So by (3.2) for a sequence of values of r tending to infinity

$$\log^{[n-1]} T(r,f_n) \geq (\lambda_g-\varepsilon)\frac{1-\varepsilon}{1+\varepsilon}(1+O(1))\frac{T(r,h)}{(4^{n-1})^{\lambda_h+\delta}}.$$

Since ε and δ are arbitrary, it follows from the above that

$$\limsup_{r\to\infty}\frac{\log^{[n-1]} T(r,f_n)}{T(r,h)} \geq \frac{\lambda_g}{(4^{n-1})^{\lambda_h}}.$$

Similarly for $n = 3k+1$ and $3k+2$ we get the other results.
This proves the theorem. □

Theorem 3.2. *Let $f(z), g(z)$ and $h(z)$ be three entire functions with nonzero lower order and finitre order, then for $k = 0, 1, 2, 3, ...$*

$$\lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} = 0 \quad \textit{for all natural number } n.$$

Proof. First suppose $n = 3k$ then by Lemma 2.4, for all sufficiently large values of r and $\varepsilon(0 < \varepsilon < min\{\lambda_f, \lambda_g, \lambda_h\})$,

$$\begin{aligned} \log^{[n-1]} \ T(r, f_n) &\leq (\rho_g + \varepsilon) \log M(r, h) + O(1), \\ \log M(r, h) &< r^{\rho_h+\varepsilon} \\ \text{and} \quad T(\exp(r), f^{(k)}) &> e^{r^{(\lambda_f-\varepsilon)}}. \end{aligned}$$

lSo

$$\begin{aligned} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &\leq \frac{(\rho_g + \varepsilon) r^{\rho_h+\varepsilon}}{e^{r^{(\lambda_f-\varepsilon)}}} + o(1). \\ \therefore \lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &= 0. \end{aligned}$$

Similarly for $n = 3k + 1$, we have

$$\begin{aligned} \log^{[n-1]} \ T(r, f_n) &\leq (\rho_h + \varepsilon) \log M(r, f) + O(1), \\ \text{and} \quad \log M(r, f) &< r^{\rho_f+\varepsilon}. \end{aligned}$$

So

$$\begin{aligned} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &\leq \frac{(\rho_h + \varepsilon) r^{\rho_f+\varepsilon}}{e^{r^{(\lambda_f-\varepsilon)}}} + o(1). \\ \therefore \lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &= 0. \end{aligned}$$

Also when $n = 3k + 2$, then,

$$\begin{aligned} \log^{[n-1]} \ T(r, f_n) &\leq (\rho_f + \varepsilon) \log M(r, g) + O(1), \\ \text{and} \quad \log M(r, g) &< r^{\rho_g+\varepsilon}. \end{aligned}$$

So

$$\begin{aligned} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &\leq \frac{(\rho_f + \varepsilon) r^{\rho_g+\varepsilon}}{e^{r^{(\lambda_f-\varepsilon)}}} + o(1). \\ \therefore \lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} &= 0. \end{aligned}$$

This proves the theorem. □

Remark 3.3. The finite order of the functions is necessary for Theorem 3.2, which is shown by the following example.

Example 3.4. Let $f(z) = g(z) = \exp z$ and $h(z) = \exp^{[2]} z$ then $\lambda_f = \rho_f = \lambda_g = \rho_g = 1$ and $\rho_h = \infty$.

Now when $n = 3k$

$$f_n(z) = \exp^{[\frac{4n}{3}]} z.$$

Therefore,

$$\begin{aligned} 3T(2r, f_n) &\geq \log M(r, f_n) = \exp^{[\frac{4n}{3}-1]} r \\ \text{i.e.} \quad T(r, f_n) &\geq \frac{1}{3} \exp^{[\frac{4n}{3}-1]} \frac{r}{2} \\ \therefore \quad \log^{[n-1]} T(r, f_n) &\geq \exp^{[\frac{4n}{3}-1-n+1]} \frac{r}{2} + o(1) \\ &= \exp^{[\frac{n}{3}]} \frac{r}{2} + o(1). \end{aligned}$$

Also when $n = 3k + 1$,

$$f_n(z) = \exp^{[\frac{4n-1}{3}]} z.$$

Therefore

$$\begin{aligned} 3T(2r, f_n) &\geq \log M(r, f_n) = \exp^{[\frac{4n-1}{3}-1]} r \\ \text{i.e.} \quad T(r, f_n) &\geq \frac{1}{3} \exp^{[\frac{4n-1}{3}-1]} \frac{r}{2} \\ \therefore \quad \log^{[n-1]} T(r, f_n) &\geq \exp^{[\frac{4n-1}{3}-1-n+1]} \frac{r}{2} + o(1) \\ &= \exp^{[\frac{n-1}{3}]} \frac{r}{2} + o(1). \end{aligned}$$

If $n = 3k + 1$,

$$f_n(z) = \exp^{[\frac{4n-2}{3}]} z.$$

Therefore

$$\begin{aligned} 3T(2r, f_n) &\geq \log M(r, f_n) = \exp^{[\frac{4n-2}{3}-1]} r \\ \text{i.e.} \quad T(r, f_n) &\geq \frac{1}{3} \exp^{[\frac{4n-2}{3}-1]} \frac{r}{2} \\ \therefore \quad \log^{[n-1]} T(r, f_n) &\geq \exp^{[\frac{4n-2}{3}-1-n+1]} \frac{r}{2} + o(1) \\ &= \exp^{[\frac{n-2}{3}]} \frac{r}{2} + o(1). \end{aligned}$$

Also

$$T(\exp(r), f^{(k)}) = \frac{e^r}{\pi}.$$

Therefore

$$\frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} \geq \frac{\exp^{[\frac{n}{3}]} \frac{r}{2} + o(1)}{e^r/\pi} \nrightarrow 0 \text{ as } r \to \infty \text{ and } n = 3k,$$

$$\frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} \geq \frac{\exp^{[\frac{n-1}{3}]} \frac{r}{2} + o(1)}{e^r/\pi} \nrightarrow 0 \text{ as } r \to \infty \text{ and } n = 3k+1,$$

$$\frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), f^{(k)})} \geq \frac{\exp^{[\frac{n-2}{3}]} \frac{r}{2} + o(1)}{e^r/\pi} \nrightarrow 0 \text{ as } r \to \infty \text{ and } n = 3k+2.$$

Theorem 3.5. *Let $f(z), g(z)$ and $h(z)$ be three entire functions with nonzero lower order and finitre order, then for $k = 0, 1, 2, 3, ...$*

$$\lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), g^{(k)})} = 0 \text{ and } \lim_{r\to\infty} \frac{\log^{[n-1]} T(r, f_n)}{T(\exp(r), h^{(k)})} = 0 \text{ for all natural number } n.$$

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