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PSEDUO-INEQUALITY APPLICATION IN CODING THEORY USING δ-NORM INACCURACY MEASURE

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ABSTRACT. In this paper we prove two pseudo-generalizations of Shannon inequality for the case of norm Inaccuracy Measure and norm entropy. Further, we establish a result on noiseless coding theorem for the proposed mean code length interms of generalized inaccuracy measure.

Key Words: Shannon inequality, δ-norm inaccuracy, δ-norm entropy, Codeword length, Kerridges inaccuracy.
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1. INTRODUCTION

Let

$$L_n = \{A = (a_1, \dots, a_n) : a_k \geqslant 0, \sum_{k=1}^{n} a_k = 1\} \quad , n \geqslant 2$$

be sets of n - complete probability distributions. For

$$(a_1, \dots, a_n) = A \in L_n,$$

Shannons measure of information [15] is defined as

$$I_s(A) = -\sum_{k=1}^{n} a_k \log_2 a_k \tag{1.1}$$

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The measure (1.1) has been generalized by various authors and has found applications in various disciplines such as economics, accounting, crime, physics, etc. For

$$A, B \in L_n,$$

Kerridge [10] introduced a quantity known as inaccuracy defined as

$$I_k(A;B) = -\sum_{k=1}^{n} a_k \log_2 a_k \tag{1.2}$$

There is well- known relation between

$$I_s(A)$$

and

$$I_k(A;B)$$

which is given by

$$I_s(A) \leq I_k(A;B) \tag{1.3}$$

. The relation (1.3) is known as Shannon inequality and its importance is well known in coding theory.Van der Lubbe [17] generalized (1.3) in the another form, which he called a pseudo generalization of (1.3). In fact he proved the following:

$$\begin{aligned} I_1(A;\delta) =& \frac{1}{\delta-1}(1-\sum_{k=1}^{n} a_k^{\delta}) \leq I_{k,1}(A;B,\delta) \\ =& \frac{1}{\delta-1}\left(1-\left(\sum_{k=1}^{n} a_k b_k^{(\frac{\delta-1}{\delta})}\right)\right)^{\delta}, \quad \delta > 0\ (\neq 1) \end{aligned} \tag{1.4}$$

The equality holds in (1.4) if and only if $b_k = \frac{a_k^{\delta}}{\sum_{k=1}^{n} a_k^{\delta}}, k = 1, 2, 3, ..., n$. In fact $I_{k,1}(A;B,\delta)$ is not a measure of inaccuracy in its usual sense [i.e.,$I_{k,1}(A;A,\delta) \neq I_1(A,\delta)$], but as $\delta \to 1, \lim I_{k,1}(A;B,\delta) = I_k(A;B)$. Where $I_1(A;\delta) = \frac{1}{\delta-1}(1-\sum_{k=1}^{n} a_k^{\delta})$ is the Tsallis entropy which is also generalized by Litegebe and Satish [11]. For $A, B \epsilon L_n$, we define a δ-norm inaccuracy measure of type β as

$$I_{k,2}(A;B,\delta,\beta) = \frac{\delta}{\delta-1}\left(1-\left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)\right)^{\frac{1}{\delta}}, \tag{1.5}$$

for $\delta > 0\ (\neq 1), \beta > 0$.

(1) If $b_k = a_k$, (1.5) reduces to a non-additive δ-norm entropy of type β. i.e.

$$(1.6)\qquad I_{k,2}(A;A,\delta,\beta) = \frac{\delta}{\delta-1}\left(1-\left(\frac{\sum_{k=1}^{n} a_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta}}\right)\right)^{\frac{1}{\delta}},$$

for $\delta > 0$ ($\neq 1$), which is studied by Satish and Arun [13].
(2) If $b_k = a_k$ and $\beta = 1$, (1.5) becomes δ-norm entropy [4].
i.e.

$$(1.7)\qquad I(A;\delta) = \frac{\delta}{\delta-1}\left(1-\left(\sum_{k=1}^{n} a_k^{\delta}\right)\right)^{\frac{1}{\delta}}, \delta > 0\ (\neq 1)$$

(3) If $b_k = a_k$,$\beta = 1$ and $\delta \to 1$, (1.5) becomes (1.1).
Further,if $\beta = 1$ and $\delta \to 1$, (1.5) becomes (1.2).

2. PSEUDO - INEQUALITY

For $A, B \in L_n$ define a measure of inaccuracy, denoted by$I_{k,3}(A;B,\delta,\beta)$ as

$$(2.1)\qquad I_{k,3}(A;B,\delta,\beta) = \frac{\delta}{\delta-1}\left[1-\left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right)\right],$$

where $\delta > 0$ ($\neq 1$), $\beta > 0$. Since $I_{k,3}(A;A,\delta,\beta) \neq I_{k,2}(A;A,\delta,\beta)$ and $I_{k,3}(A;B,\delta,\beta) \neq I_{k,2}(A;B,\delta,\beta)$, we will not interpret (2.1) as a measure of inaccuracy. But $I_{k,3}(A;B,\delta,\beta)$ is a pseudo- generalization of the measure of inaccuracy defined in (1.5) and (1.6). In the following theorems, we will determine two relations between (1.5) and (2.1), and (1.6) and (2.1) of the type (1.3).
Theorem 1. Let $A, B \in L_n$ then

$$(2.2)\qquad I_{k,2}(A;B,\delta,\beta) \leqslant I_{k,3}(A;B,\delta,\beta), \delta > 0\ (\neq 1)$$

with equality holds if $b_k = \frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}$, $k = 1, 2, 3, ..., n$, under the condition

$$(2.3)\qquad \sum_{k=1}^{n} a_k^{\beta} b_k^{1-\beta} \leqslant 1$$

Proof. By Hölders inequality, we have

$$(2.4)\qquad \left(\sum_{k=1}^{n} x_k^{p}\right)^{\frac{1}{p}}\left(\sum_{k=1}^{n} y_k^{q}\right)^{\frac{1}{q}} \leqslant \sum_{k=1}^{n} x_k y_k$$

where $\frac{1}{p}+\frac{1}{q}=1; p(\neq 0)<1, q<0$ or $q(\neq 0)<1, p<0; x_k, y_k>0$ for each k. Note that the direction of Hölders inequality is the reverse of the usual one for $p<1$ (see Beckenbach and Bellman [3]). Let $p=\frac{\delta-1}{\delta}, q=\delta-1, x_k=a_k^{\frac{\delta\beta}{\delta-1}}\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta-1}} b_k, y_k=a_k^{\frac{\beta}{1-\delta}}\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{1-\delta}} b_k^{-\beta}$, where $(k=1,2,3,\ldots,n)$. Subsituting all these values into (2.4), we get

$$\left(\sum_{k=1}^n a_k^\beta\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}}\left(\frac{\sum_{k=1}^n a_k^\beta b_k^{\beta(\delta-1)}}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{1-\delta}} \leqslant \sum_{k=1}^n a_k^\beta b_k^{1-\beta} \leqslant 1; \quad \delta>0$$

where we used (2.3) too. This implies

$$\left(\sum_{k=1}^n a_k^\beta\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}} \leqslant\left(\frac{\sum_{k=1}^n a_k^\beta b_k^{\beta(\delta-1)}}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta-1}} \tag{2.5}$$

For $\delta>1$ (2.5) becomes

$$\left(\sum_{k=1}^n a_k^\beta\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right) \leqslant\left(\frac{\sum_{k=1}^n a_k^\beta b_k^{\beta(\delta-1)}}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta}} \tag{2.6}$$

using (2.6) and the fact that $\delta>1$ we get $I_{k,2}(A;B,\delta,\beta) \leqslant I_{k,3}(A;B,\delta,\beta)$. For $0<\delta<1$ the above inequality can be proved in a similar way.
Theorem 2. Let $A, B \in L_n$.Then

$$I_{k,2}(A;A,\delta,\beta) \leqslant I_{k,3}(A;B,\delta,\beta), \delta>0\ (\neq 1) \tag{2.7}$$

with equality holds if $b_k=\frac{a_k^{\delta\beta}}{\sum_{k=1}^n a_k^{\beta\beta}}, k=1,2,3,\ldots,n$.

Proof. Substituting $p=\frac{\delta-1}{\delta}, q=1-\delta, x_k=a_k^{\frac{\delta\beta}{\delta-1}}\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta-1}} b_k, y_k= a_k^{\frac{\beta\delta}{1-\delta}}\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{1-\delta}}$, $(k=1,2,3,\ldots,n)$, in to (2.4), we get
$\left(\sum_{k=1}^n a_k^\beta\left(\frac{1}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}}\left(\frac{\sum_{k=1}^n a_k^{\delta\beta}}{\sum_{k=1}^n a_k^\beta}\right)^{\frac{1}{1-\delta}} \leqslant \sum_{k=1}^n b_k=1.$

This implies

$$(2.8)\qquad \left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right)^{\frac{\delta}{\delta-1}} \leqslant \left(\frac{\sum_{k=1}^{n} a_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta-1}}$$

For $\delta > 1$, (2.8) becomes

$$(2.9)\qquad \left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} b_k^{\frac{\delta-1}{\delta}}\right) \leqslant \left(\frac{\sum_{k=1}^{n} a_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}}$$

using (2.9) and the fact that $\delta > 1$, we get $I_{k,2}(A; A, \delta, \beta) \leqslant I_{k,3}(A; B, \delta, \beta)$ For $0 < \delta < 1$ the above inequality can be proved in a similar way.

Now we discuss an application of inequality (2.2) in coding theory for $L_n = \{A = (a_1, a_2, \cdots, a_n); 0 < a_k \leqslant 1, \sum_{k=1}^{n} a_k = 1\}$. Let a finite set of n input symbols $X = \{x_1, x_2, \cdots, x_n\}$ be encoded using alphabet of D symbols, then it has been shown by Feinstien [7] that there is a uniquely decipherable code with lengths $N_1, N_2, \cdots, N_n$ iff the Kraft inequality holds. That is,

$$(2.10)\qquad \sum_{k=1}^{n} D^{-N_k} \leqslant 1,$$

where D is the size of code alphabet. Furthermore, if

$$(2.11)\qquad L = \sum_{k=1}^{n} N_k a_k$$

is the average codeword length, then for a code satisfying (2.10), the inequality

$$(2.12)\qquad L \geqslant I_s(A)$$

is also fulfilled and equality holds if and only if

$$(2.13)\qquad N_k = -\log_D(a_k), (k = 1, 2, 3, \cdots, n),$$

and that by suitable encoded into words of long sequences, the average length can be made arbitrarily close to $I_s(A)$, (see Feinstein [7]). This is Shannons noiseless coding theorem. Let us introduce another measure of length:

$$(2.14)\qquad L(\delta, \beta) = \frac{\delta}{\delta - 1}\left[1 - \left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{N_k\left(\frac{1-\delta}{\delta}\right)}\right)\right],$$

where $\delta > 0\ (\neq 1)$ and $A = (a_1, a_2, \cdots, a_n) \in L_n$ and $D, N_1, N_2, \cdots, N_n$ are positive integers so that

$$\sum_{k=1}^{n} a_k^{\beta} b_k^{-\beta} D^{-N_k} \leqslant 1 \tag{2.15}$$

Since (2.15) reduces to Kraft inequality when $a_k = b_k, \forall k = 1, 2, 3, \cdots, n$, therefore it is called generalized Kraft inequality and codes obtained under this generalized inequality are called personal codes.
Theorem 3. Let $n \in N, \delta > 0(\neq 1)$ be arbitrarily fixed. Then there exist code length $N_1, N_2, \cdots, N_n$ so that

$$I_{k,2}(A;B,\delta,\beta) \leq L(\delta,\beta) < D^{\frac{1-\delta}{\delta}} I_{k,2}(A;B,\delta,\beta) + \frac{\delta}{\delta-1}\left(1 - D^{\frac{1-\delta}{\delta}}\right) \tag{2.16}$$

holds under the condition (2.15) and equality holds if and only if

$$N_k = -\log_D \left(\frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}} \right); k = 1, 2, 3, \cdots, n. \tag{2.17}$$

Where $I_{k,2}(A;B,\delta,\beta)$ and $L(\delta,\beta)$ are given by (1.5) and (2.14) respectively.
Proof. First of all we shall prove the lower bound of $L(\delta,\beta)$.
Let $p = \frac{\delta-1}{\delta}, q = 1-\delta, x_k = a_k^{\frac{\delta\beta}{\delta-1}} \left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}} \right)^{\frac{1}{\delta-1}} D^{-N_k}, y_k$
$, \quad = a_k^{\frac{\beta}{1-\delta}} \left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}} \right)^{\frac{1}{1-\delta}} b_k^{-\beta}, (k = 1, 2, 3, ..., n)$. Putting these values into (2.4), we get

$$\left(\sum_{k=1}^{n} a_k^{\beta} \left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}} \right)^{\frac{1}{\delta}} D^{-N_k\left(\frac{\delta-1}{\delta}\right)} \right)^{\frac{\delta}{\delta-1}} \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}} \right)^{\frac{1}{1-\delta}}$$
$$\leqslant \sum_{k=1}^{n} a_k^{\beta} b_k^{-\beta} D^{-N_k} \leqslant 1$$

where we used (2.15) too. This implies

$$\left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{-N_k\left(\frac{\delta-1}{\delta}\right)}\right)^{\frac{\delta}{\delta-1}} \leqslant \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta-1}} \tag{2.18}$$

For $\delta > 1$ (2.18) becomes

$$\left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{-N_k\left(\frac{\delta-1}{\delta}\right)}\right) \leqslant \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}}$$

using (2.19) and the fact that $\delta > 1$, we get

$$I_{k,2}(A; B, \delta, \beta) \leq L(\delta, \beta) \tag{2.19}$$

For $0 < \delta < 1$ the inequality (2.20) can be proved in a similar way, by noting that the inequality sign of (2.19) is reversed since $\frac{\delta}{\delta-1} < 0$ for $0 < \delta < 1$. From (2.17) and after simplification, we get

$$D^{-N_k\left(\frac{\delta-1}{\delta}\right)} = \left(\frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}\right)^{\frac{\delta-1}{\delta}}$$

This implies

$$\left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{-N_k\left(\frac{\delta-1}{\delta}\right)}\right) = \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}}$$

which gives $L(\delta, \beta) = I_{k,2}(A; B, \delta, \beta)$. Then equality sign holds in (2.20). Now we will prove the inequality (2.16) for upper bound of $L(\delta, \beta)$. We choose the code word lengths $N_k, k = 1, 2, 3, \cdots, n$ in such a way that

$$-\log_D\left(\frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}\right) \leq N_k$$

$$< -\log_D\left(\frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}\right) + 1 \tag{2.20}$$

is fulfilled for all $k = 1, 2, 3, \cdots, n$. From the left inequality of (2.22), we have

$$D^{-N_k} \leq \frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}} \tag{2.21}$$

multiplying both sides by $a_k^{\beta} b_k^{-\beta}$ and then taking sum over k, we get the generalized inequality (2.15).So there exists a generalized code with code lengths $N_k, k = 1, 2, 3, \cdots, n$.
From right-hand side of (2.22), we have

$$D^{-N_k} > \left(\frac{b_k^{\delta\beta}}{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}\right) D^{-1} \tag{2.22}$$

Since $\delta > 1$, (2.24) leads to

$$\left(\sum_{k=1}^{n} a_k^{\beta}\left(\frac{1}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{-N_k\left(\frac{\delta-1}{\delta}\right)}\right)$$

$$> \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)} D^{(1-\delta)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} \tag{2.23}$$

Finally we find

$$L(\delta, \beta) \leq \frac{\delta}{\delta - 1}\left(1 - \left(\frac{\sum_{k=1}^{n} a_k^{\beta} b_k^{\beta(\delta-1)}}{\sum_{k=1}^{n} a_k^{\beta}}\right)^{\frac{1}{\delta}} D^{\left(\frac{1-\delta}{\delta}\right)}\right) \tag{2.24}$$

where the right- hand side of (2.26) is equivalent to the right- hand side in (2.16). For $0 < \delta < 1$, the proof of the upper bound of $L(\delta, \beta)$ follows along similar lines. Since $D \geq 2$, we have $\frac{\delta}{\delta-1}\left(1 - D^{\frac{1-\delta}{\delta}}\right)$ from which it follows that the upper bound of $L(\delta, \beta)$ in (2.16) is greater than unity.

Particular cases:

(i). For $\beta = 1, a_k = b_k, k = 1, 2, 3, \cdots, n$ and $\delta \to 1$ then (2.16) becomes

$$\frac{I_s(A)}{\log D} \leq L < \frac{I_s(A)}{\log D} + 1$$

which is the well-known result due to Shannon (see Aczel (1975)).

(ii). For $\beta = 1, a_k = b_k, k = 1, 2, 3, \cdots, n$, then (2.16) becomes

$$I(A;\delta) \leq L(\delta) < D^{\frac{1-\delta}{\delta}} I(A;\delta) + \frac{\delta}{\delta - 1}\left(1 - D^{\frac{1-\delta}{\delta}}\right),$$

which is the well known result studied by Boekee and Lubbe [4].

(iii). For $a_k = b_k, k = 1, 2, 3, \cdots, n$, (2.16) becomes

$$I(A; A, \delta, \beta) \leq L(\delta, \beta) < D^{\frac{1-\delta}{\delta}} I(A; A, \delta, \beta) + \frac{\delta}{\delta - 1}\left(1 - D^{\frac{1-\delta}{\delta}}\right)$$

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