Journal of Hyperstructures 8 *(1)* (*2019*), *48-57.*
ISSN: 2251-8436 print/2322-1666 online

A NEW VIEW ON NEUTROSOPHIC MATRIX

BANU PAZAR VAROL, VILDAN ÇETKIN AND HALIS AYGÜN

ABSTRACT. In the present paper, we define a new kind of matrix called by a neutrosophic matrix, whose entries are all single-valued neutrosophic sets. So, we aim to be introduce a convenient tool for the problems, have uncertain inputs. We first give the definition of a neutrosophic matrix with its basic operations. Then we investigate the properties of the given operations and also prove that the family of all neutrosophic matrices is a vector space over a classical field.

Key Words: Neutrosophic set, single valued neutrosophic set, neutrosophic matrix.
2010 Mathematics Subject Classification: Primary: 03E72; Secondary: 15B15.

1. INTRODUCTION

Neutrosopy was introduced by Smarandache to handle the indeterminate information. In the neutrosophic set, a truth-membership, an indeterminacy- membership and a falsity-membership are represented independently. Then Wang et al. [19] specified the definition of neutrosophic set which is called single valued neutrosophic set. The single valued neutrosophic set is a generalization of classical set, fuzzy set, intuitionistic fuzzy set etc. Single valued neutrosophic set is applied to algebraic and topological structures (see [3, 4, 5, 12, 14, 15]). Çetkin and Aygün [7] proposed the definitions of neutrosophic subgroups [5] and neutrosophic subrings [7] of a given classical group and classical ring, respectively. Also, Çetkin et al. [6] defined the neutrosophic submodules based on single valued neutrosophic sets and discussed their elementary properties.

Received: 2019-01-15, Accepted: 2019-07-16 . Communicated by: Ali Taghavi
∗Address correspondence to Banu Pazar Varol; E-mail: banupazar@kocaeli.edu.tr

In this paper, we introduce neutrosophic matrix in a completely different direction from Dhar et al. [8] and give some algebraic operations of it. Then, we prove that the neutrosophic matrix forms a vector space under component wise addition, multiplication and scalar multiplication.

This paper is organized as follows: Section 2 gives a brief summary of neutrosophic sets and operations on these sets. In section 3, we give the definition of neutrosophic matrix and investigate some of its algebraic operations. In section 4, we showed that neutrosophic matrix multiplication is associative and distributive. In addition, we proved that the set of all neutrosophic matrixes of order $n \times n$ is an algebra and form a vector space under complement wise addition, complement wise multiplication and scalar multiplication.

2. Preliminaries

In this chapter, we give some preliminaries about single valued neutrosophic sets and set operations, which will be called neutrosophic sets, for simplicity.

Definition 2.1 [16] A neutrosophic set A on the universe of X is defined as $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ where $t_A, i_A, f_A : X \to]^-0, 1^+[$ and $^-0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3^+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non standard subsets of $]^-0, 1^+[$. But in real life applications in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence throughout this work, the following specified definition of a neutrosophic set known as single valued neutrosophic set is considered.

Definition 2.2[19] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A on X is characterized by truth-membership function t_A, indeterminacy-membership function i_A and falsity-membership function f_A. For each point x in $X, t_A(x), i_A(x), f_A(x) \in [0, 1]$.

A neutrosophic set A can be written as

$$A = \sum_{i=1}^{n} < t(x_i), i(x_i), f(x_i) > /x_i, \ x_i \in X.$$

Example 2.3[19] Assume that $X = \{x_1, x_2, x_3\}, x_1$ is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, x_2 and x_3 are in $[0, 1]$. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a single valued neutrosophic set of X defined by

$A =< 0.3, 0.4, 0.5 > /x_1+ < 0.5, 0.2, 0.3 > /x_2+ < 0.7, 0.2, 0.2 > /x_3$.

Since the membership functions t_A, i_A, f_A are defined from X into the unit interval $[0, 1]$ as $t_A, i_A, f_A : X \to [0, 1]$, a (single valued) neutrosophic set A will be denoted by a mapping defined as $A : X \to [0, 1] \times [0, 1] \times [0, 1]$ and $A(x) = (t_A(x), i_A(x), f_A(x))$, for simplicity.

Definition 2.4 [14, 19] Let A and B be two neutrosophic sets on X. Then

(1) A is contained in B, denoted as $A \subseteq B$, if and only if $A(x) \leq B(x)$. This means that $t_A(x) \leq t_B(x), i_A(x) \leq i_B(x)$ and $f_A(x) \geq f_B(x)$. Two sets A and B is called equal, i.e., $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(2) the union of A and B is denoted by $C = A \cup B$ and defined as $C(x) = A(x) \vee B(x)$ where $A(x) \vee B(x) = (t_A(x) \vee t_B(x), i_A(x) \vee i_B(x), f_A(x) \wedge f_B(x))$, for each $x \in X$. This means that $t_C(x) = \max\{t_A(x), t_B(x)\}$, $i_C(x) = \max\{i_A(x), i_B(x)\}$ and $f_C(x) = \min\{f_A(x), f_B(x)\}$.

(3) the intersection of A and B is denoted by $C = A \cap B$ and defined as $C(x) = A(x) \wedge B(x)$ where $A(x) \wedge B(x) = (t_A(x) \wedge t_B(x), i_A(x) \wedge i_B(x), f_A(x) \vee f_B(x))$, for each $x \in X$. This means that $t_C(x) = \min\{t_A(x), t_B(x)\}$, $i_C(x) = \min\{i_A(x), i_B(x)\}$ and $f_C(x) = \max\{f_A(x), f_B(x)\}$.

(4) the complement of A is denoted by A^c and defined as

$A^c(x) = (f_A(x), 1 - i_A(x), t_A(x))$, for each $x \in X$. Here $(A^c)^c = A$.

Proposition 2.5[19] Let A, B and C be the neutrosophic sets on the common universe X. Then the following properties are valid.

(1) $A \cup B = B \cup A, A \cap B = B \cap A$.

(2) $A \cup (B \cup C) = (A \cup B) \cup C, A \cap (B \cap C) = (A \cap B) \cap C$.

(3) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C), A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(4) $A \cap \widetilde{\emptyset} = \widetilde{\emptyset}, A \cup \widetilde{\emptyset} = A, A \cup \widetilde{X} = \widetilde{X}, A \cap \widetilde{X} = A$, where

$t_{\widetilde{\emptyset}} = i_{\widetilde{\emptyset}} = 0, f_{\widetilde{\emptyset}} = 1$ and $t_{\widetilde{X}} = i_{\widetilde{X}} = 1, f_{\widetilde{X}} = 0$.

(5) $(A \cup B)^c = A^c \cap B^c, (A \cap B)^c = A^c \cup B^c$.

Definition 2.6 [6] Let A and B be two neutrosophic sets on X and Y, respectively. Then the cartesian product of A and B which is denoted by $A \times B$ is a neutrosophic set on $X \times Y$ and it is defined as $(A \times B)(x, y) = A(x) \times B(y)$ where $A(x) \times B(y) = (t_{A\times B}(x, y), i_{A\times B}(x, y), f_{A\times B}(x, y))$, i.e.,

$t_{A\times B}(x, y) = t_A(x) \wedge t_B(y)$, $i_{A\times B}(x, y) = i_A(x) \wedge i_B(y)$ and $f_{A\times B}(x, y) = f_A(x) \vee f_B(y)$.

3. Neutrosophic matrix

In this section, we introduce neutrosophic matrix and give some algebraic operations of it.

Definition 3.1 A neutrosophic matrix (NSM) of order $m \times n$ is defined as $A = [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]$ where $a_{ij}^T, a_{ij}^I, a_{ij}^F$ are called truth- membership, indeterminacy-membership, falsity-membership values of the ij-th element in A satisfying the condition $0 \leq a_{ij}^T + a_{ij}^I + a_{ij}^F \leq 3$ for all i,j.

For simplicity, we write $A = [a_{ij}]_{m\times n}$ where $a_{ij} =< a_{ij}^T, a_{ij}^I, a_{ij}^F >$.

Let $\mathbf{F}_{m\times n}$ denotes the set of all NSMs of order $m \times n$. In particular $\mathbf{F}_n$ denotes the set of all NSMs of order n.

Example 3.2 We would represent the Example 2.3 in matrix form of order 3×1 as

$$\begin{bmatrix} (0.3, 0.4, 0.5) \\ (0.5, 0.2, 0.3) \\ (0.7, 0.2, 0.2) \end{bmatrix}$$

Definition 3.3 Let a and b be two elements of a NSM such that $a =< a_{ij}^T, a_{ij}^I, a_{ij}^F >$, $b =< b_{ij}^T, b_{ij}^I, b_{ij}^F >$, then complement wise addition and multiplication is defined as

$a + b =< max\{a_{ij}^T, b_{ij}^T\}, max\{a_{ij}^I, b_{ij}^I\}, min\{a_{ij}^F, b_{ij}^F\} >$

$a \bullet b =< min\{a_{ij}^T, b_{ij}^T\}, min\{a_{ij}^I, b_{ij}^I\}, max\{a_{ij}^F, b_{ij}^F\} >$

We say $max\{a_{ij}^T, b_{ij}^T\} = a_{ij}^T + b_{ij}^T$ and $min\{a_{ij}^T, b_{ij}^T\} = a_{ij}^T.b_{ij}^T$

Definition 3.4 Some algebraic operaitons of NSMs

Let A and B be two NSMs such that $A = [a_{ij}]_{m\times n}, B = [b_{ij}]_{m\times n}$.

1) Matrix addition and subtraction are given by

$A + B =< max\{a_{ij}^T, b_{ij}^T\}, max\{a_{ij}^I, b_{ij}^I\}, min\{a_{ij}^F, b_{ij}^F\} >$ and

$A - B =< a_{ij}^T - b_{ij}^T, a_{ij}^I - b_{ij}^I, a_{ij}^F - b_{ij}^F >$ where

$$a_{ij}^T - b_{ij}^T = \begin{cases} a_{ij}^T, & \text{if } a_{ij}^T \geq b_{ij}^T; \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ij}^I - b_{ij}^I = \begin{cases} a_{ij}^I, & \text{if } a_{ij}^I \geq b_{ij}^I; \\ 0, & \text{otherwise} \end{cases}$$

$$a_{ij}^F - b_{ij}^F = \begin{cases} a_{ij}^F, & \text{if } a_{ij}^F < b_{ij}^F; \\ 0, & \text{otherwise} \end{cases}$$

2) Component wise matrix multiplicaiton is given by

$A \circ B =< min\{a_{ij}^T, b_{ij}^T\}, min\{a_{ij}^I, b_{ij}^I\}, max\{a_{ij}^F, b_{ij}^F\} >$

3) Let A and B be two NSMs of order $m \times n$ and $n \times p$, respectively. Then the matrix product AB is defined by

$AB =< \sum_{k=1}^{p} a_{ik}^T.b_{kj}^T, \sum_{k=1}^{p} a_{ik}^I.b_{kj}^I, \prod_{k=1}^{p} a_{ik}^F.b_{kj}^F > \in \mathbf{F}_{m\times n}$.

We can also write

$AB = [\max_k\{min\{a_{ik}^T, b_{kj}^T\}\}, \max_k\{min\{a_{ik}^I, b_{kj}^I\}\}, \min_k\{max\{a_{ik}^F, b_{kj}^F\}\}]$,

where $k = \overline{1,n}$, $i = \overline{1,m}$, $j = \overline{1,p}$.

The product AB is defined if and only the number of columns of A is the same as the number of rows of B. We say that A and B are comfortable for multiplication.

4) Transpose of A is given by

$A^T =< a_{ji}^T, a_{ji}^I, a_{ji}^F >$.

5) complement of A

$\overline{A} =< a_{ij}^F, 1 - a_{ij}^I, a_{ij}^T >$.

Definition 3.5 Let A be a $m \times n$ neutrosophic matrix. If all of its entries are $< 0, 0, 1 >$, then A is called zero neutrosophic matrix and denoted by $\mathbf{0}$.

If all of its entries are $< 1, 1, 0 >$, then A is called universal neutrosophic matrix and denoted by $\mathbf{J}$.

.

The $n \times n$ identity matrix I_n is defined by $< \lambda_{ij}^T, \lambda_{ij}^I, \lambda_{ij}^F >$ such that $\lambda_{ij}^T = \lambda_{ij}^I = 1,\ \lambda_{ij}^F = 0$, if $i = j$ and

$\lambda_{ij}^T = \lambda_{ij}^I = 0,\ \lambda_{ij}^F = 1$, if $i \neq j$.

Definition 3.6 Let $A = [< a_{ij}^T, a_{ij}^I, a_{ij}^F >] \in F_{m\times n}$ and $k \in F$. Then the neutrosophic scalar multiplication is defined as

$kA = [< min\{k, a_{ij}^T\}, min\{k, a_{ij}^I\}, max\{1 - k, a_{ij}^F\} >]$.

For the universal matrix $\mathbf{J}$,

$kJ = [< min\{k, 1\}, min\{k, 1\}, max\{1 - k, 0\} >] = [< k, k, 1 - k >]$.

Under component wise multiplication ,

$k\mathbf{J} \bullet A = [< min\{k, a_{ij}^T\}, min\{k, a_{ij}^I\}, max\{1 - k, a_{ij}^F\} >] = kA$.

Definition 3.7 Let $A = [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]$, $B = [< b_{ij}^T, b_{ij}^I, b_{ij}^F >] \in F_{m\times n}$, then we write $A \leq B$ if, $a_{ij}^T \leq b_{ij}^T$, $a_{ij}^I \leq b_{ij}^I$, $a_{ij}^F \geq b_{ij}^F$ for all i,j.

Example 3.8 $\mathbf{0} \leq A \leq \mathbf{J}$.

4. Main Results

In this section, we see that matrix multiplicaiton is associative and distributive. We prove that the neutrosophic matrix forms a vector space under component wise addition, multiplication and scalar multiplication

Theorem 4.1 For any NSMs $A \in F_{m\times n}$, $B \in F_{n\times p}$, $C \in F_{p\times q}$, $(AB)C = A(BC)$.

Proof $(AB)C$ and $A(BC)$ are defined and are type of $m \times q$.

Let $A = [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]$, $B = [< b_{jk}^T, b_{jk}^I, b_{jk}^F >]$ and $C = [< c_{kl}^T, c_{kl}^I, c_{kl}^F >]$ such that the ranges of the suffixes $i = \overline{1,m}$, $j = \overline{1,n}$, $k = \overline{1,p}$, $l = \overline{1,q}$.

(i,k)-th element of the product

$AB = [< \sum_{j=1}^{n} a_{ij}^T.b_{jk}^T, \sum_{j=1}^{n} a_{ij}^I.b_{jk}^I, \prod_{j=1}^{n} a_{ij}^F.b_{jk}^F >]$.

The (i,1)-th element in the product $(AB)C$ is the sum of products of the corresponding elements in the i-th row of AB, first column of C with k common. Hence, (i,1)-th element of

$$\begin{aligned}(AB)C &= [< \sum_{k=1}^{p}\left(\sum_{j=1}^{n} a_{ij}^T.b_{jk}^T\right)c_{kl}^T, \sum_{k=1}^{p}\left(\sum_{j=1}^{n} a_{ij}^I.b_{jk}^I\right)c_{kl}^I, \prod_{k=1}^{p}\left(\prod_{j=1}^{n} a_{ij}^F + b_{jk}^F\right) + c_{kl}^F >] \\ &= [< \sum_{k=1}^{p}\sum_{j=1}^{n} a_{ij}^T.b_{jk}^T.c_{kl}^T, \sum_{k=1}^{p}\sum_{j=1}^{n} a_{ij}^I.b_{jk}^I.c_{kl}^I, \prod_{k=1}^{p}\prod_{j=1}^{n}(a_{ij}^F + b_{jk}^F + c_{kl}^F) >].\end{aligned}$$

(j,1)-th element of the product

$BC = [< \sum_{k=1}^{p} b_{jk}^T.c_{kl}^T, \sum_{k=1}^{p} b_{jk}^I.c_{kl}^I, \prod_{k=1}^{p}(b_{jk}^F + b_{kl}^F) >]$.

Now, the (i,1)-th element of the product $A(BC)$ is the sum of products of the corresponding elements in the i-th row of A and first column of BC.

(i,l)-th element of

$$\begin{aligned}A(BC) &= [< \sum_{j=1}^{n} a_{ij}^T\left(\sum_{k=1}^{p} b_{jk}^T.c_{kl}^T\right), \sum_{j=1}^{n} a_{ij}^I\left(\sum_{k=1}^{p} b_{jk}^I.c_{kl}^I\right), \prod_{j=1}^{n} a_{ij}^F + \left(\prod_{k=1}^{p}(b_{jk}^F + c_{kl}^F)\right) >] \\ &= [< \sum_{k=1}^{p}\sum_{j=1}^{n} a_{ij}^T.b_{jk}^T.c_{kl}^T, \sum_{k=1}^{p}\sum_{j=1}^{n} a_{ij}^I.b_{jk}^I.c_{kl}^I, \prod_{k=1}^{p}\prod_{j=1}^{n}(a_{ij}^F + b_{jk}^F + c_{kl}^F) >].\end{aligned}$$

Thus, $(AB)C = A(BC)$.

Theorem 4.2 Let $A \in F_{m\times n}$, $B \in F_{n\times p}$ and $C \in F_{n\times p}$. Then $A(B + C) = AB + AC$.

Proof Let $A = [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]$, $B = [< b_{jk}^T, b_{jk}^I, b_{jk}^F >]$ and $C = [< c_{jk}^T, c_{jk}^I, c_{jk}^F >]$ such that the ranges of the suffixes $i = \overline{1,m}$, $j = \overline{1,n}$, $k = \overline{1,p}$.

(j,k)-th element of

$$\begin{aligned} B+C &= [< max\{b^T_{jk}, c^T_{jk}\}, max\{b^I_{jk}, c^I_{jk}\}, min\{b^F_{jk}, c^F_{jk}\} >] \\ &= [< b^T_{jk}+c^T_{jk}, b^I_{jk}+c^I_{jk}, b^F_{jk}.c^F_{jk} >]. \end{aligned}$$

(i,k)-th element in the product of A and $B+C$, that is of $A(B+C)$ is the sum of the products of the corresponging elements in the i-th row A and k-th column of $B+C$

$$A(B+C) = [< \sum_{j=1}^{n} a^T_{ij}(b^T_{jk}+c^T_{jk}), \sum_{j=1}^{n} a^I_{ij}(b^I_{jk}+c^I_{jk}), \prod_{j=1}^{n}(a^F_{ij}+b^T_{jk}.c^T_{jk}) >].$$

(i,k)-th element of $(AB+AC)$

$$\begin{aligned} AB+AC &= [< \sum_{j=1}^{n} a^T_{ij}.b^T_{jk}, \sum_{j=1}^{n} a^I_{ij}.b^I_{jk}, \prod_{j=1}^{n}(a^F_{ij}+b^F_{jk}) > + \sum_{j=1}^{n} a^T_{jk}.c^T_{jk}, \sum_{j=1}^{n} a^I_{jk}.c^I_{jk}, \prod_{j=1}^{n}(a^F_{ij}+c^F_{jk}) >] \\ &= [< \sum_{j=1}^{n}(a^T_{ij}.b^T_{jk}+a^T_{ij}.c^T_{jk}), \sum_{j=1}^{n}(a^I_{ij}.b^I_{jk}+a^I_{ij}.c^I_{jk}), \prod_{j=1}^{n}(a^F_{ij}+b^F_{jk}) \prod_{j=1}^{n}(a^F_{ij}+c^F_{jk}) >] \\ &= [< \sum_{j=1}^{n} a^T_{ij}(b^T_{jk}+c^T_{jk}), \sum_{j=1}^{n} a^I_{ij}(b^I_{jk}+c^I_{jk}), \prod_{j=1}^{n}(a^F_{ij}+b^F_{jk}.c^F_{jk}) >]. \end{aligned}$$

This completes the proof.

Theorem 4.3 Let $A, B \in \mathbf{F}_{m\times n}$. If $A \leq B$, then for any $C \in \mathbf{F}_{n\times p}$, $AC \leq BC$ and for any $D \in \mathbf{F}_{p\times m}$, $DA \leq DB$.

Proof Let $A \leq B$. Then $a^T_{ik} \leq b^T_{ik}$, $a^I_{ik} \leq b^I_{ik}$, $a^F_{ik} \geq b^F_{ik}$ for $i = \overline{1,m}$, $k = \overline{1,n}$.

By fuzzy multiplicaiton $a^T_{ik}.c^T_{kj} \leq b^T_{ik}.c^T_{kj}$, $a^I_{ik}.c^I_{kj} \leq b^I_{ik}.c^I_{kj}$ and $a^F_{ik}.c^F_{kj} \geq b^F_{ik}.c^F_{kj}$ for $j = \overline{1,p}$.

By fuzzy addition $\sum_{k=1}^{n} a^T_{ik}.c^T_{kj} \leq \sum_{k=1}^{n} b^T_{ik}.c^T_{kj}$,

$\sum_{k=1}^{n} a^I_{ik}.c^I_{kj} \leq \sum_{k=1}^{n} b^I_{ik}.c^I_{kj}$ and $\sum_{k=1}^{n} a^F_{ik}.c^F_{kj} \geq \sum_{k=1}^{n} b^F_{ik}.c^F_{kj}$.

Hence, $AC \leq BC$.

Similarly, we can see $DA \leq DB$.

Theorem 4.4 The set $\mathbf{F}_{m\times n}$ is a commutative semiring with identity $\mathbf{0}$ and $\mathbf{J}$.

Proof We have $A + \mathbf{0} = A$ and $A \bullet \mathbf{J} = A$ for all $A \in \mathbf{F}_{m\times n}$. So, we say that zero neutrosophic matix $\mathbf{0}$ is the aditive identitiy and universal neutrosophic matrix $\mathbf{J}$ is the multiplicative identity.

$A + \mathbf{J} = \mathbf{J}$ and $A \bullet \mathbf{0} = \mathbf{0}$.

Let $A, B, C \in \mathbf{F}_{m\times n}$ such that $A = [< a^T_{ij}, a^I_{ij}, a^F_{ij} >]$,
$B = [< b^T_{ij}, b^I_{ij}, b^F_{ij} >]$, $C = [< c^T_{ij}, c^I_{ij}, c^F_{ij} >]$.

$$\begin{aligned} A+(B+C) &= [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]+[< max\{b_{ij}^T, c_{ij}^T\}, max\{b_{ij}^I, c_{ij}^I\}, min\{b_{ij}^F, c_{ij}^F\} >] \\ &= [< max\{a_{ij}^T, b_{ij}^T, c_{ij}^T\}, max\{a_{ij}^I, b_{ij}^I, c_{ij}^I\}, min\{a_{ij}^F, b_{ij}^F, c_{ij}^F\} >]. \end{aligned}$$

and

$$\begin{aligned} (A+B)+C &= [< max\{a_{ij}^T, b_{ij}^T\}, max\{a_{ij}^I, b_{ij}^I\}, min\{a_{ij}^F, b_{ij}^F\} >]+[< c_{ij}^T, c_{ij}^I, c_{ij}^F >] \\ &= [< max\{a_{ij}^T, b_{ij}^T, c_{ij}^T\}, max\{a_{ij}^I, b_{ij}^I, c_{ij}^I\}, min\{a_{ij}^F, b_{ij}^F, c_{ij}^F\} >]. \end{aligned}$$

Hence, $A+(B+C)=(A+B)+C$. Similarly, we can show that $A \bullet (B \bullet C) = (A \bullet B) \bullet C$.

So, we obtain associativity law under $+$ and $\bullet$.

Now, we show that $A+(A \bullet B) = A \bullet (A+B)$.

$$\begin{aligned} A+(A \bullet B) &= [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]+[< min\{a_{ij}^T, b_{ij}^T\}, min\{a_{ij}^I, b_{ij}^I\}, max\{a_{ij}^F, b_{ij}^F\} >] \\ &= [< max\{a_{ij}^T, min\{a_{ij}^T, b_{ij}^T\}, max\{a_{ij}^I, min\{a_{ij}^I, b_{ij}^I\}, min\{a_{ij}^F, max\{a_{ij}^F, b_{ij}^F\} >] \\ &= [< a_{ij}^T, a_{ij}^I, a_{ij}^F >] \\ &= A. \end{aligned}$$

Similarly, we see $A \bullet (A+B) = A$. Hence, absorption is satisfied.

Now, we prove $A \bullet (B+C) = (A \bullet B)+(A \bullet C)$.

Suppose that $A \leqslant B, C$.

$$\begin{aligned} A \bullet (B+C) &= [< min\{a_{ij}^T, max\{b_{ij}^T, c_{ij}^T\}, min\{a_{ij}^I, max\{b_{ij}^I, c_{ij}^I\}, max\{a_{ij}^F, min\{b_{ij}^F, c_{ij}^F\} >] \\ &= [< a_{ij}^T, a_{ij}^I, a_{ij}^F >] \\ &= A. \end{aligned}$$

$(A \bullet B)+(A \bullet C) = [< min\{a_{ij}^T, b_{ij}^T\}, min\{a_{ij}^I, b_{ij}^I\}, max\{a_{ij}^F, b_{ij}^F\} >]$

$+[< min\{a_{ij}^T, c_{ij}^T\}, min\{a_{ij}^I, c_{ij}^I\}, max\{a_{ij}^F, c_{ij}^F\} >]$

$= [< max\{min\{a_{ij}^T, b_{ij}^T\}, min\{a_{ij}^T, c_{ij}^T\}\} >], max\{min\{a_{ij}^I, b_{ij}^I\}, min\{a_{ij}^I, c_{ij}^I\}\},$

$min\{max\{a_{ij}^F, b_{ij}^F\}, max\{a_{ij}^F, c_{ij}^F\}\}$

$= [< a_{ij}^T, a_{ij}^I, a_{ij}^F >]$

$= A$

So, we obtained the desired equality.

If $A \geqslant B, C$, then we have two cases.

If $A \geqslant B \geqslant C$, then we obtain $A \bullet (B+C) = (A \bullet B)+(A \bullet C)$ from the above equalities, and if $A \geqslant C \geqslant B$, similarly we see $A \bullet (B+C) = (A \bullet B)+(A \bullet C)$. Hence, distributivity law is hold.

Theorem 4.5 The set $F_{m \times n}$ is a vector space under the operations neutrosophic matrix addition and scalar multiplication.

Proof Let $A, B, C \in \mathbf{F}_{m \times n}$. We have $A+B = B+A$ and $A+(B+C) = (A+B)+C$. Commutative law and associative low hold in $\mathbf{F}_{m \times n}$.

For all $A \in \mathbf{F}_{m \times n}$, there exist an element $\mathbf{0} \in \mathbf{F}_{m \times n}$ such that

$A+\mathbf{0} = A$.

For $k \in F$, by Definition 3.6 and Theorem 4.4, we obtain

$$\begin{aligned} k(A+B) &= k\mathbf{J} \bullet (A+B) \\ &= k\mathbf{J} \bullet A + k\mathbf{J} \bullet B. \\ &= kA + kB \end{aligned}$$

Again, for $k_1, k_2 \in F$,

$$\begin{aligned} (k_1+k_2)A &= (k_1+k_2)\mathbf{J} \bullet A \\ &= (k_1\mathbf{J} + k_2\mathbf{J}) \bullet A. \\ &= k_1\mathbf{J} \bullet A + k_2\mathbf{J} \bullet A \\ &= k_1A + k_2A \end{aligned}$$

Hence, $F_{m\times n}$ is a vector space over F.

5. Conclusion

It is well-known that matrices play an important role in computer science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment Thomas [18] introduce fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory. According to this idea, we introduce the notion of a neutrosophic matrix to handle the computer science problems involving neutrosophic inputs which is an extension of the intuitionistic fuzzy matrix [17].

Acknowledgments

The authors wish to thank the referee for his/her valuable suggestions

References

[1] D. D. Anderson and E. Smith, *Weakly prime ideals*, Houston J. Mathematics, **29** (2003), 831–840.

[2] R. Y. Sharp, *Steps in commutative algebra*, Cambridge: Cambridge University Prees.

[3] I. Arockiarani, I. R. Sumathi, J. Martina Jency, *Fuzzy neutrosophic soft topological spaces*, International Journal of Mathematical Arhchive, **4 (10)** (2013) 225–238.

[4] R. A. Borzooei, H. Farahani, M. Moniri, *Neutrosophic deductive filters on BL-algebras*, Journal of Intelligent and Fuzzy Systems, **26 (6)** (2014) 2993–3004.

[5] V.Çetkin, H. Aygün, *An approach to neutrosophic subgroup and its fundamental properties*, Journal of Intelligent and Fuzzy Systems, **29** (2015) 1941–1947.

[6] V.Çetkin, B.P. Varol, H. Aygün, *On neutrosophic submodules of a module*, Hacettepe Journal of Mathematics and Statistics, **46 (5)** (2017) 791–799

[7] V. Çetkin, H. Aygün, *A new approach to neutrosophic subrings*, Sakarya University Journal of Science, **23(3)**, (20190) 472–477.
[8] M. Dhar, S. Broumi, F. Smarandache, *A note on square neutrosophic fuzzy matrices*, Neutrosophic Sets anSystems **3** (2014) 37–41.
[9] T. Eswarlal, R. Ramakrishma, *Vague fields and vague vector spaces*, International Journal of Pure and Applied Mathematics **94 (3)** (2014) 295–305.
[10] T. W. Hungerford, *Algebra, Graduate Texts in Mathematics* 73, Springer (1974).
[11] Vasantha Kandasamy W.B., Florentin Smarandache, *Some neutrosophic algebraic structures and neutrosophic N-algebraic structures*, Hexis, Phoenix, Arizona, 2006.
[12] P. Majumdar, S. K. Samanta, *On similarity and entropy of neutrosophic sets*, Journal of Intelligent and Fuzzy Systems- **26 (3)** (2014) 1245–1252.
[13] S. Nanda, *Fuzzy fields and fuzzy linear spaces*, Fuzzy Sets and Systems, **19** (1986) 89–94.
[14] A. A. Salama, S. A. Al-Blowi, Neutrosophic set and neutrosophic topological spaces, IOSR Journal of Math. **3 (4)** (2012) 31–35.
[15] M. Shabir, M. Ali, M. Naz, F. Smarandache, *Soft neutrosophic group*, Neutrosophic Sets and Systems, **1** (2013) 13–25.
[16] F. Smarandache, *A unifying field in logics. Neutrosophy/ Neutrosophic Probability, Set and Logic*, Rehoboth: American Research Press (1998) http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf (last edition online).
[17] S. Srivastava, P. Murugadas, *On semiring of intuitionistic fuzzy matrices*, Applied Mathematical Sciences, **4** (2010) 1099–1105.
[18] M. G. Thomas, *Convergence of powers of a fuzzy matrix*, J. Math. Annal Appl., **57** (1977) 476–480.
[19] H. Wang et al., *Single valued neutrosophic sets*, Proc. of 10th Int. Conf. on Fuzzy Theory and Technology, Salt Lake City, Utah, July 21-26 (2005).
[20] K.-M. Zhang, Y. Bai, X.-L. Li, Y.-F. Qin, *Intuitionistic fuzzy subfield and its characterizations*, 2010 Second International Conference on Intelligent Human-Machine Systems and Cybernetics, (2010) 58–61.

Banu Pazar Varol
Department of Mathematics, Kocaeli University, 41380, Kocaeli, Turkey
Email: `banupazar@kocaeli.edu.tr`

Vildan Çetkin
Department of Mathematics, Kocaeli University, 41380, Kocaeli, Turkey
Email: `vcetkin@gmail.com`

Halis Aygün
Department of Mathematics, Kocaeli University, 41380, Kocaeli, Turkey
Email: `halis@kocaeli.edu.tr`