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SOFT SEMIHYPERRINGS - AN INTRODUCTION

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ABSTRACT. The purpose of this paper is to introduce and study soft semihyperrings by giving importance both on attributes and functional value. In this paper the notions of soft semihyperring and its ideals are introduced and studied systematically. Prime (semiprime) soft hyperideals are defined and their characterizations are obtained. The regularity criterion for soft semihyperrings is characterized by using the properties of (left, right) soft hyperideals.

Key Words: Semihyperrings, hyperideal, m-system, prime hyperideal, regular, soft semihyperring.

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1. Introduction

Most of the problems in economics, engineering and environment have various uncertainties. We cannot successfully use classical methods because of various uncertainties typical for those problems. There are several theories, for examples, theory of fuzzy sets, intuitionistic fuzzy sets, rough sets etc. which can be considered as mathematical tools for dealing with uncertainties. But all these theories have their inherent difficulties as what were pointed by Molodtsov [4]. Molodtsov introduced the concept of soft set for dealing with uncertainty. Maji et al.

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[2] studied several operations on the theory of soft sets and described the application of soft set theory to decision making problem. There are also several authors who enriched the theory of soft sets.
The algebraic hyperstructures, a suitable generalization of classical algebraic structure, was introduced by Marty [3]. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements it is a set. Since then, hundreds of papers and several books have been written on this topic. Among these semihyperrings have been found useful for dealing with problems in different areas of algebraic structures. In this paper, we introduce and study semihyperrings using soft set theory and obtain some of its characterizations using its soft hyperideals.

2. Preliminaries

The hyperstructures are algebraic structures equipped with at least one set-valued operation, called hyperoperation. The largest classes of the hyperstructures are the ones called Hv - structures. A hyperoperation " $\cdot$ " on a non-empty set H is a mapping from $H \times H$ to the power set $P^*(H)$, where $P^*(H)$ denotes the set of all non-empty subsets of H, that is $\cdot : H \times H \mapsto P^*(H) : (x, y) \mapsto x \cdot y \subseteq H$. If A and B are two non-empty subsets of H, then

$$A \cdot B = \bigcup_{\substack{a \in A \\ b \in B}} a \cdot b$$

Definition 2.1. A semihyperring is an algebraic structure $(R, +, \cdot)$ which satisfies the following axioms:

(1) $(R, +)$ is a commutative hypermonoid, that is;
   (a) $(x + y) + z = x + (y + z)$ for all $x, y, z \in R$
   (b) There is $0 \in R$, such that $x + 0 = 0 + x = 0$ for all $x \in R$
   (c) $x + y = y + x$ for all $x, y \in R$.
(2) $(R, \cdot)$ is semigroup, that is $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in R$

(3) The multiplication is distributive with respect to hyperoperation '+' that is; $x \cdot (y + z) = x \cdot y + x \cdot z$ and $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$

(4) The element $0 \in R$, is an absorbing element, that is; $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$.

Definition 2.2. A semihyperring $(R, +, \cdot)$ is called a semihyperring with identity if there exists an element $1_R \in R$ such that $x \cdot 1_R = 1_R \cdot x = x$ for all $x \in R$.

Definition 2.3. An element $x \in R$ is called unit if and only if there exists $y \in R$ such that $1_R = x \cdot y = y \cdot x$. The set of all unit elements of a semihyperring R is denoted by $U(R)$.

Definition 2.4. A non-empty subset S of a semihyperring $(R, +, \cdot)$ is called a subsemihyperring of R if

(i) $a + b \subseteq S$ for all $a, b \in S$.

(ii) $a \cdot b \in S$ for all $a, b \in S$.

Definition 2.5. A left hyperideal of a semihyperring R is a non-empty subset I of R, satisfying

(i) For all $x, y \in I,\ x + y \subseteq I$

(ii) For all $a \in I$ and $x \in R,\ x \cdot a \in I$.

A right hyperideal of a semihyperring R is defined in a similar way.

Definition 2.6. [4] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.
For a soft set (F, A), the set Supp $(F, A) = \{x \in A | F(x) \neq \phi\}$ is called the support of the soft set (F, A). Thus a null soft set is indeed a soft set with an empty support and we say that a soft set (F, A) is non-null if Supp $(F, A) \neq \phi$.

Definition 2.7. For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(i) $A \subseteq B$ and

(ii) $F(x) \subseteq G(x)$ for all $x \in A$.

In this case (G, B) is said to be a soft super set of (F, A).

Definition 2.8. [2] Let (F, A) and (G, B) be two soft sets over a common universe U. The union of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i) $C = A \cup B$; (ii) for all $e \in C$,

$$H(e) = \begin{cases} F(e) \text{ if } e \in A \setminus B \\ G(e) \text{ if } e \in B \setminus A \\ F(e) \cup G(e) \text{ if } e \in A \cap B \end{cases}$$

This relation is denoted by $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.9. [1] Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. The intersection of (F, A) and (G, B) is denoted by $(F, A) \cap (G, B)$, and is defined as $(F, A) \cap (G, B) = (H, C)$, where $C = A \cap B$ and, for all $c \in C, H(c) = F(c) \cap G(c)$.

3. Soft Semihyperrings

In this section, we introduce and study the concept of soft semihyperring over a semihyperring H. Throughout this section unless otherwise mentioned H denotes a semihyperring. To give more importance on the attributes, we also assume that there is one and only one property of attributes which maps to the identity element of the semihyperring.
With abuse of notation in some cases we assume (F, A) and $F(A)$ are the same.

Definition 3.1. Let (F, A) be a non-null soft set over a semihyperring H. Then (F, A) is called *soft semihyperring* over H if $F(x)$ is a subsemihyperring of H for all $x \in \text{Supp } (F, A)$.

Example 3.2. Consider $H = \mathbb{N}^+ \cup \{0\}$ with an addition " $+$ " and multiplication " $\cdot$ " defined by

$$a + b = \max\{a, b\}, \; a \cdot b = \min\{a, b\}.$$

Let (F, A) be a soft set over H, with $A = \mathbb{N}^+,\ F(x) = \{0, x, 2x\}$ for all $x \in A$. Then (F, A) is a soft semihyperring.

Definition 3.3. Let (F, A) be a soft semihyperring over H. A non-empty soft subset (G, B) of (F, A) is called soft subsemihyperring of (F, A) if

(i) $G(a_1) + G(a_2) \subseteq (G, B)$ for all $a_1, a_2 \in$ Supp (G, B)
(ii) $G(a_1) \cdot G(a_2) \subseteq (G, B)$ for all $a_1, a_2 \in$ Supp (G, B).

Definition 3.4. A non-empty soft subset (E, I) of (F, A) over a semi-hyperring H is called left soft hyperideal of (F, A) if

(i) For all $i_1, i_2 \in$ Supp (E, I), $E(i_1) + E(i_2) \subseteq (E, I)$
(ii) For all $a \in$ Supp (E, I) and $x \in$ Supp (F, A), $F(x) \cdot E(a) \subseteq (E, I)$.

A right soft hyperideal of a soft semihyperring (F, A) over H is defined in a similar way.

Example 3.5. Let H be the set of all polynomials with real coefficients and define addition " $+$ " and multiplication " $\cdot$ " by

$$f + g = f, \text{ if deg } f(x) \geq \text{ deg } g(x)$$

$$f \cdot g = f \text{ if deg } f(x) \leq \text{ deg } g(x).$$

Let $A = \mathbb{N}^+ \cup \{0\}$ and consider the soft set (F, A) over H in which $F(n)$ is the set of all polynomials of degree $\leq n$ for all $n \in A$. Then (F, A) forms a soft semihyperring.
Let (E, I) be the soft subset of (F, A) over H, in which $I = \mathbb{N}^+ \cup \{0\}$ and $E(m)$ is the set of all polynomial with integer coefficient of degree $\leq m$ for all $m \in I$. Then (E, I) is a soft hyperideal of (F, A).

Proposition 3.6. *Every soft hyperideal of a soft semihyperring is a soft subsemihyperring.*

Definition 3.7. Let (F, A) be a soft semihyperring over H. Then for every $x \in$ Supp (F, A), there exists one and only one $y \in$ Supp (F, A) such that $0 \in F(x) + F(y)$. We shall write $\widehat{x}$ for y and call it opposite

of x.
Denote the set of all opposite elements of (F, A) over H by $V_H(F, A)$, that is;

$$V_H(F, A) = \{a \in \text{ Supp } (F, A) | \exists b \in \text{ Supp } (F, A) \text{ s.t. } 0 \in F(a) + F(b)\}.$$

Definition 3.8. A non-empty soft subset (G, B) of a soft semihyperring (F, A) over H is called soft semihyper subtractive if and only if $a \in$ Supp $(G, B) \cap V_H(F, A)$ implies that $\widehat{a} \in$ Supp $(G, B) \cap V_H(F, A)$.

Definition 3.9. A non-empty soft subset (G, B) of a soft semihyperring (F, A) over H is called soft hyper subtractive if and only if $a \in$ Supp (G, B) and $G(a) + b \subseteq (G, B)$ implies $b \subseteq (G, B)$.

Definition 3.10. A soft semihyperring (F, A) over H is called additively reversive if it satisfies reversive property with respect to the hyperoperation of addition that is, for $a \subseteq b + c$ implies $b \subseteq a + \widehat{c}$ and $c \subseteq a + \widehat{b}$.

Theorem 3.11. *A soft semihyperring is a soft hyperring if and only if* $V_H(F, A) = A$ *and* (F, A) *is additively reversive.*

Proof. Suppose (F, A) is a soft hyperring. Clearly $V_H(F, A) \subseteq A$ (i). For any $a \in$ Supp (F, A), since (F, A) is a soft hyperring so there exists $\widehat{a} \in A$, such that $0 \in F(a) + F(\widehat{a})$. Thus $a \in V_H(F, A)$. So $A \subseteq V_H(F, A)$ (ii). Hence $V_H(F, A) = A$. Also as (F, A) is a soft hyperring, it is additively reversive.
Conversely, suppose that (F, A) is an additively reversive soft semihyperring and $V_H(F, A) = A$. Let $s \in A = V_H(F, A)$. Then, there exists an element $\widehat{s} \in V_H(F, A) = A$ such that $0 \in F(s) + F(\widehat{s})$. Also since (F, A) is additively reversive, it follows from $F(a) \subseteq F(b) + F(c)$ that implies $F(b) \subseteq F(a) + F(\widehat{c})$ and $F(c) \subseteq F(a) + F(\widehat{b})$. Hence (F, A) is a canonical soft hypergroup, so (F, A) is a soft hyperring. □

Definition 3.12. A soft semihyperring (F, A) over H is called zero sumfree if and only if $0 \in F(r) + F(r^{'})$ implies that $r = r^{'} = 0$.

Theorem 3.13. *A soft semihyperring* (F, A) *is zero sumfree if and only if* $V_H(F, A) = \{0\}$.

Proof. Suppose $V_H(F, A) = \{0\}$. To prove that (F, A) is zero sumfree, let $a, b \in A$ such that $0 \in F(a) + F(b)$. Then $a, b \in V_H(F, A) = \{0\}$, that is $a = b = 0$. Therefore (F, A) is zero sumfree.
Conversely suppose that (F, A) is zero sumfree. Since $0 \in F(0) + F(0)$, $\{0\} \subseteq V_H(F, A)$. Now let $r \in V_H(F, A)$, then there exists $r^{'} \in$ Supp (F, A) such that $0 \in F(r) + F(r^{'})$. But (F, A) is zero sumfree, so $r = r^{'} = 0$, this implies that $V_H(F, A) \subseteq \{0\}$. Hence $V_H(F, A) = \{0\}$. □

Theorem 3.14. *Let (F, A) be a soft semihyperring over H and $\{(E_i, I_i)\}_{i\in\Lambda}$ be a family of soft hyperideals of (F, A). Then $\underset{i\in\Lambda}{\cap}(E_i, I_i)$ is also a soft hyperideal of (F, A).*

Proof. Assume that $\{(E_i, I_i)\}_{i\in\Lambda}$ is a family of soft hyperideals of (F, A). By definition, $\cap(E_i, I_i) = (\cap E_i, \cap I_i)$. Let $a, b \in$ Supp $(\cap E_i, \cap I_i)$. Then $a, b \in$ Supp (E_i, I_i) for all $i \in \Lambda$. Since each (E_i, I_i) is a soft hyperideal so $E_i(a) + E_i(b) \subseteq (E_i, I_i)$ for all $i \in \Lambda$. Thus $E_i(a) + E_i(b) \subseteq \underset{i\in\Lambda}{\cap}(E_i, I_i)$. Now let $x \in$ Supp (F, A) and $a \in$ Supp $(\cap E_i, \cap I_i)$. Since $a \in$ Supp $(\cap E_i, \cap I_i)$, $a \in$ Supp (E_i, I_i) for all $i \in \Lambda$ and each (E_i, I_i) is a soft hyperideal; so $F(x) \cdot E_i(a) \subseteq (E_i, I_i)$ for all $i \in \Lambda$ implies $F(x) \cdot E_i(a) \subseteq \underset{i\in\Lambda}{\cap}(E_i, I_i)$. Again for $x \in$ Supp (F, A) and $a \in$ Supp $(\cap E_i, \cap I_i)$, as each (E_i, I_i) is a soft hyperideal, $E_i(a) \cdot F(x) \subseteq (E_i, I_i)$ for all $i \in \Lambda$ implies $E_i(a) \cdot F(x) \subseteq \underset{i\in\Lambda}{\cap}(E_i, I_i)$. Hence $\underset{i\in\Lambda}{\cap}(E_i, I_i)$ is a soft hyperideal of (F, A). □

Theorem 3.15. *Let (E_1, I_1) and (E_2, I_2) be soft hyperideals of a soft semihyperring (F, A) over H. If I_1 and I_2 are disjoint, then $(E_1, I_1)\widetilde{\cup}(E_2, I_2)$ is a soft hyperideal of (F, A).*

Proof. Let (E_1, I_1) and (E_2, I_2) be soft hyperideals of a soft semihyperring (F, A) over H. Then by Definition 2.8 $(E_1, I_1)\widetilde{\cup}(E_2, I_2) = (E, I)$

where $I = I_1 \cup I_2$ and for every $x \in I$

$$E(x) = \begin{cases} E_1(x) \text{ if } x \in I_1 \setminus I_2 \\ E_2(x) \text{ if } x \in I_2 \setminus I_1 \\ E_1(x) \cup E_2(x) \text{ if } x \in I_1 \cap I_2 \end{cases}$$

Clearly, we have $I \subseteq A$. Suppose that I_1 and I_2 are disjoint, i.e., $I_1 \cap I_2 = \phi$. Then for every $x \in$ Supp (E, I), we know that either $x \in I_1 \setminus I_2$ or $x \in I_2 \setminus I_1$ If $x \in I_1 \setminus I_2$, then $E(x) = E_1(x) \neq \phi$ and so (E, I) is an hyperideal of (F, A) since (E_1, I_1) is an hyperideal of (F, A). Similarly, $x \in I_2 \setminus I_1$, then $E(x) = E_2(x) \neq \phi$ and so (E, I) is an hyperideal of (F, A) since (E_2, I_2) is an hyperideal of (F, A). Thus we conclude that (E, I) is an hyperideal of (F, A) and so $(E_1, I_1)\widetilde{\cup}(E_2, I_2)$ is a soft hyperideal of (F, A). □

Theorem 3.16. *If (E_1, I_1) and (E_2, I_2) are soft hyperideals of a soft semihyperring (F, A) over H, then $(E_1, I_1) + (E_2, I_2)$ is the smallest soft hyperideal of (F, A) containing both (E_1, I_1) and (E_2, I_2), where*

$$(E_1, I_1) + (E_2, I_2) = \bigcup_{a_i \in \text{ Supp } (E_i, I_i)} (E_1(a_1) + E_2(a_2)).$$

Proof. Let $x, y \subseteq (E_1, I_1) + (E_2, I_2)$.
Then there exist $a_1, b_1 \in$ Supp (E_1, I_1) and $a_2, b_2 \in$ Supp (E_2, I_2) such that $x \subseteq E_1(a_1) + E_2(a_2)$, $y \subseteq E_1(b_1) + E_2(b_2)$ and by using associativity and commutativity of (F, A), we have $x + y \subseteq (E_1(a_1) + E_2(a_2)) + (E_1(b_1) + E_2(b_2)) = (E_1(a_1) + E_1(b_1)) + (E_2(a_2) + E_2(b_2))$. Since (E_i, I_i), $i = 1, 2$, are soft hyperideals of (F, A), so, $E_1(a_1) + E_1(b_1) \subseteq (E_1, I_1)$, $E_2(a_2) + E_2(b_2) \subseteq (E_2, I_2)$. This implies that $(E_1(a_1) + E_1(b_1)) + (E_2(a_2) + E_2(b_2)) \subseteq (E_1, I_1) + (E_2, I_2)$ and so $x + y \subseteq (E_1, I_1) + (E_2, I_2)$. Now let $r \in$ Supp (F, A) and $x \subseteq (E_1, I_1) + (E_2, I_2)$. Then there exist $a \in$ Supp (E_1, I_1) and $b \in$ Supp (E_2, I_2) such that $x \subseteq E_1(a) + E_2(b)$. Now since (E_1, I_1) and (E_2, I_2) are soft hyperideals $F(r) \cdot x \subseteq F(r) \cdot (E_1(a) + E_2(b)) = F(r) \cdot E_1(a) + F(r) \cdot E_2(b) \subseteq (E_1, I_1) + (E_2, I_2)$. Similarly we have $x \cdot F(r) \subseteq (E_1, I_1) + (E_2, I_2)$. Hence $(E_1, I_1) + (E_2, I_2)$ is a soft hyperideal of (F, A).

Now we will show that $(E_1, I_1) + (E_2, I_2)$ is the smallest soft hyperideal of (F, A) containing $(E_1, I_1)\widetilde{\cup}(E_2, I_2)$.
First we will prove that $(E_1, I_1)\widetilde{\cup}(E_2, I_2) \subseteq (E_1, I_1) + (E_2, I_2)$. For this let $x \subseteq (E_1, I_1)\widetilde{\cup}(E_2, I_2)$. Then either $x \subseteq (E_1, I_1)$ or $x \subseteq (E_2, I_2)$. Now if $x \subseteq (E_1, I_1)$, then since $0 \subseteq (E_2, I_2)$ we have $x = x + 0 \subseteq (E_1, I_1) + (E_2, I_2)$. Similarly if $x \subseteq (E_2, I_2)$, then $x \subseteq (E_1, I_1) + (E_2, I_2)$. Hence $x \subseteq (E_1, I_1) + (E_2, I_2)$. Hence $(E_1, I_1)\widetilde{\cup}(E_2, I_2) \subseteq (E_1, I_1) + (E_2, I_2)$.
Now to prove $(E_1, I_1) + (E_2, I_2)$ is the smallest soft hyperideal of (F, A), let (E, I) be another soft hyperideal of (F, A) containing both (E_1, I_1) and (E_2, I_2). Let $x \subseteq (E_1, I_1) + (E_2, I_2)$. Then there exist
$a \in$ Supp (E_1, I_1) and $b \in$ Supp (E_2, I_2) such that $x \subseteq E_1(a) + E_2(b)$. Now since $E_1(a) \subseteq (E_1, I_1) \subseteq (E_1, I_1)\widetilde{\cup}(E_2, I_2) \subseteq (E, I)$ and $E_2(b) \subseteq (E_2, I_2) \subseteq (E_1, I_1)\widetilde{\cup}(E_2, I_2) \subseteq (E, I)$, we have $E_1(a) + E_2(b) \subseteq (E, I)$ as (E, I) is soft hyperideal of (F, A). Hence $(E_1, I_1) + (E_2, I_2) \subseteq (E, I)$. Hence $(E_1, I_1) + (E_2, I_2)$ is the smallest hyperideal of (F, A) containing both (E_1, I_1) and (E_2, I_2). □

Proposition 3.17. *Let (F, A) be a soft semihyperring over H. Assume that (G, B) is a soft subsemihyperring of (F, A) and (E, I) is a soft hyperideal of (F, A). Then*

(i) $(G, B) + (E, I)$ *is a soft subsemihperring of* (F, A)
(ii) $(G, B) \cap (E, I)$ *is a soft hyperideal of* (G, B).

Proof. (i) $(G, B) + (E, I) = \cup\{(G(s) + G(i)|s \in$ Supp $(G, B),\ i \in$ Supp $(E, I)\}$. Obviously, since $G(0) \subseteq (G, B)$ and $E(0) \subseteq (E, I)$, we have $G(0) + E(0) \subseteq (G, B) + (E, I)$. Hence $(G, B) + (E, I) \neq \phi$. Let $x, y \subseteq (G, B)+(E, I)$. Then there exist $b_1, b_2 \in$ Supp (G, B) and $i_1, i_2 \in$ Supp (E, I) such that $x \subseteq G(b_1) + E(i_1)$ and $y \subseteq G(b_2) + E(i_2)$. Hence $x + y \subseteq (G(b_1) + E(i_1)) + (G(b_2) + E(i_2)) = (G(b_1) + G(b_2)) + (E(i_1) + E(i_2)) \subseteq (G, B) + (E, I)$. Now $x \cdot y \subseteq (G(b_1) + E(i_1)) \cdot (G(b_2) + E(i_2)) = G(b_1) \cdot G(b_2) + (G(b_1) \cdot E(i_2) + E(i_1) \cdot G(b_2) + E(i_1) \cdot E(i_2)) \subseteq (G, B) + (E, I)$, since (G, B) is a soft subsemihyperring and (E, I) is a soft hyperideal. Hence $(G, B) + (E, I)$ is a soft subsemihperring of (F, A).
(ii) Straightforward. □

Definition 3.18. A soft semihyperring (F, A) over H is called a regular soft semihyperring if for each $x \in$ Supp (F, A), there exists $y \in$ Supp (F, A) such that $F(x) = F(x) \cdot F(y) \cdot F(x)$.

Definition 3.19. Let (F, A) be a soft semihyperring over H and (G_1, B_1), (G_2, B_2) are any two non-empty soft subset of (F, A). Then the product of (G_1, B_1) and (G_2, B_2) is denoted by $(G_1, B_1) * (G_2, B_2)$ and defined as $(G_1, B_1) * (G_2, B_2) = \cup\{\sum\limits_{finite} G_1(a_i) \cdot G_2(b_i) | a_i \in$ Supp $(G_1, B_1), b_i \in$ Supp $(G_2, B_2)\}$.

Theorem 3.20. *A soft semihyperring (F, A) over H with identity is regular if and only if $(G_1, B_1) \cap (G_2, B_2) = (G_1, B_1) * (G_2, B_2)$ for every right soft hyperideal (G_1, B_1) and left soft hyperideal (G_2, B_2) of (F, A).*

Proof. Suppose that the soft semihyperring (F, A) over H is regular. Let $x \subseteq (G_1, B_1) * (G_2, B_2)$. Then $x \subseteq (G_1, B_1) * (G_2, B_2) = \cup\{\sum\limits_{finite} G_1(a_i) \cdot G_2(b_i) | a_i \in$ Supp $(G_1, B_1), b_i \in$ Supp $(G_2, B_2)\} \subseteq (G_1, B_1), (G_2, B_2)$ which implies $(G_1, B_1) * (G_2, B_2) \subseteq (G_1, B_1) \cap (G_2, B_2)$. Now suppose $x \subseteq (G_1, B_1) \cap (G_2, B_2)$. Then there exists $a \in$ Supp (G_1, B_1) and $b \in$ Supp (G_2, B_2) such that $x = G_1(a) = G_2(b)$. Since (F, A) is soft regular there exists $y \in$ Supp (F, A) such that $x = x \cdot F(y) \cdot x = G_1(a) \cdot F(y) \cdot G_2(b) \subseteq (G_1, B_1) * (G_2, B_2)$. Therefore $(G_1, B_1) * (G_2, B_2) = (G_1, B_1) \cap (G_2, B_2)$.
Conversely, let $a \in$ Supp (F, A). Now consider $(G_1, B_1) = F(a) \cdot (F, A)$ and $(G_2, B_2) = (F, A) \cdot F(a)$. Then (G_1, B_1) and (G_2, B_2) are respectively right and left soft hyperideals of (F, A). Since $e \subseteq (F, A)$, obviously, $F(a) \subseteq (G_1, B_1)$ and $F(a) \subseteq (G_2, B_2)$ and so $F(a) \subseteq (G_1, B_1) \cap (G_2, B_2) = (G_1, B_1) * (G_2, B_2) = (F(a) \cdot (F, A)) * ((F, A) \cdot F(a)) = F(a) \cdot (\sum\limits_{finite} F(b^{'}))(\sum\limits_{finite} F(b^{''})) \cdot F(a) = F(a) \cdot F(b) \cdot F(a)$. Therefore (F, A) is soft regular. $\square$

Definition 3.21. A soft hyperideal (P, I) of a soft semihyperring (F, A) over H is called prime if for soft hyperideals (E_1, I_1), (E_2, I_2) satisfying

$E_1(x)E_2(y) \subseteq (P,I)$ implies $E_1(x) \subseteq P(x)$ or $E_2(y) \subseteq P(y)$, where $I_1 \cup I_2 \subseteq I$, $x \in$ Supp (E_1, I_1), and $y \in$ Supp (E_2, I_2).

Definition 3.22. A soft hyperideal (P,I) of a soft semihyperring (F,A) over H is called semiprime if for soft hyperideals (E_1, I_1) satisfying $E_1(x)E_1(x) \subseteq (P,I)$ implies $E_1(x) \subseteq P(x)$, where $I_1 \subseteq I$ and $x \in$ Supp (E_1, I_1).

Theorem 3.23. *The following conditions on soft hyperideal (E,I) of a soft semihyperring (F,A) over H with identity are equivalent.*

(i) *(E,I) is a prime soft hyperideal of (F,A)*

(ii) *For all $a \in$ Supp (E_1, I_1), $b \in$ Supp (E_2, I_2), $\{E_1(a) \cdot F(r) \cdot E_2(b) :\ r \in$ Supp $(F,A)\} \subseteq (E,I)$ if and only if $E_1(a) \subseteq (E,I)$ or $E_2(b) \subseteq (E,I)$, where $(E_i, I_i), i = 1,2$, are soft hyperideals with $I_1 \cup I_2 \subseteq I$*

(iii) *If $a, b \in$ Supp (F,A) satisfy $< F(a) > \cdot < F(b) > \subseteq (E,I)$, then either $F(a) \subseteq (E,I)$ or $F(b) \subseteq (E,I)$.*

Proof. $(i) \Rightarrow (ii)$ If $E_1(a) \subseteq (E,I)$ or $E_2(b) \subseteq (E,I)$ then, by definition of soft hyperideals for every $r \in$ Supp (F,A), $E_1(a) \cdot F(r) \cdot E_2(b) \subseteq (E,I)$ which implies that $\{E_1(a) \cdot F(r) \cdot E_2(b) :\ r \in$ Supp $(F,A)\} \subseteq (E,I)$.
Conversely, suppose that $(E_1, I_1) =< E_1(a) >$ and $(E_2, I_2) =< E_2(b) >$ are soft hyperideals of (F,A). Now $\{E_1(a) \cdot F(r) \cdot E_2(b) :\ r \in$ Supp $(F,A)\}$ $\subseteq ((F,A) \cdot E_1(a) \cdot (F,A)) \cdot ((F,A) \cdot E_2(b) \cdot (F,A)) \subseteq (E_1, I_1) \cdot (E_2, I_2)$. Since $E_1(a) \cdot (F,A) \cdot E_2(b) \subseteq (E,I)$ implies $(F,A) \cdot E_1(a) \cdot (F,A) \cdot E_2(b) \cdot (F,A) \subseteq (F,A) \cdot (E,I) \cdot (F,A) \subseteq (E,I)$. So, $(E_1, I_1) \cdot (E_2, I_2) \subseteq (E,I)$ which implies $(E_1, I_1) \subseteq (E,I)$ or $(E_2, I_2) \subseteq (E,I)$. This implies that $E_1(a) \subseteq (E,I)$ or $E_2(b) \subseteq (E,I)$.
$(ii) \Rightarrow (iii)$ Let $a, b \in$ Supp (F,A) such that $< F(a) > \cdot < F(b) > \subseteq (E,I)$. Since $< F(a) >= (F,A) \cdot F(a) \cdot (F,A)$ and $< F(b) >= (F,A) \cdot F(b) \cdot (F,A)$, so $((F,A) \cdot F(a) \cdot (F,A)) \cdot ((F,A) \cdot F(b) \cdot (F,A)) =< F(a) > \cdot < F(b) > \subseteq (E,I)$. Also since $\{F(a) \cdot F(r) \cdot F(b) | r \in$ Supp $(F,A)\} \subseteq (F,A) \cdot F(a) \cdot (F,A) \cdot (F,A) \cdot F(b) \cdot (F,A) \subseteq (E,I)$ implies either $F(a) \subseteq (E,I)$ or $F(b) \subseteq (E,I)$.

$(iii) \Rightarrow (i)$ Let (E_1, I_1), (E_2, I_2) be soft hyperideals of (F, A) such that $E_1(a) \cdot E_2(b) \subseteq (E, I)$, where $a \in$ Supp (E_1, I_1), $b \in$ Supp (E_2, I_2) with $I_1 \cup I_2 \subseteq I$. Suppose $E_1(a) \nsubseteq E(a)$. We show that $E_2(b) \subseteq E(b)$. For this consider $< E_1(a) >$ and $< E_2(b) >$, the soft hyperideals generated by $E_1(a)$ and $E_2(b)$ respectively. Now by the given condition, $< E_1(a) > \cdot < E_2(b) > \subseteq < F(a) > \cdot < F(b) > \subseteq (E, I)$. Since (E_1, I_1) is a soft hyperideal of (F, A), $E_1(a) \subseteq F(a)$. Also as $E_1(a) \nsubseteq E(a) \subseteq (E, I)$, $F(a) \nsubseteq (E, I)$ and so $E_2(b) \subseteq F(b) \subseteq (E, I) \Rightarrow E_2(b) \subseteq E(b)$. Hence the proof is complete. □

Definition 3.24. A non empty soft subset (G, B) of a soft semihyperring (F, A) over H is a soft m-system if for $a, b \in$ Supp (G, B) there exists an $r \in$ Supp (F, A) such that $G(a) \cdot F(r) \cdot G(b) \subseteq (G, B)$.

Definition 3.25. A non empty soft subset (G, B) of a soft semihyperring (F, A) over H is a soft p-system if for $a \in$ Supp (G, B) there exists an $r \in$ Supp (F, A) such that $G(a) \cdot F(r) \cdot G(a) \subseteq (G, B)$.

Theorem 3.26. *A soft hyperideal (E, I) of a soft semihyperring (F, A) over H is prime (respectively, semiprime) if and only if $(F, A) \setminus (E, I)$ is an m-system (respectively, p-system), where $(F, A) \setminus (E, I) = (F, A \setminus I)$.*

Proof. Suppose that (E, I) is a prime soft hyperideal of (F, A). Let $a, b \in$ Supp $(F, A \setminus I)$. Then $F(a), F(b) \nsubseteq (E, I)$ and so, by Theorem 3.23, $\{F(a) \cdot F(r) \cdot F(b) | r \in$ Supp $(F, A)\} \nsubseteq (E, I)$. So there exists $r \in$ Supp (F, A) such that $F(a) \cdot F(r) \cdot F(b) \nsubseteq (E, I) \Rightarrow F(a) \cdot F(r) \cdot F(b) \subseteq (F, A \setminus I) \Rightarrow (F, A) \setminus (E, I)$ is an m-system.

Conversely, suppose that $(F, A) \setminus (E, I)$ is an m-system. Let $a, b \in$ Supp (F, A) with $\{F(a) \cdot F(r) \cdot F(b) | r \in$ Supp $(F, A)\} \subseteq (E, I)$. If $F(a) \nsubseteq E(a)$ and $F(b) \nsubseteq E(b)$, then $F(a) \subseteq (F, A \setminus I)$ and $F(b) \subseteq (F, A \setminus I)$. Since $(F, A) \setminus (E, I)$ is an m-system there exists $r_1 \in$ Supp (F, A) such that $F(a) \cdot F(r_1) \cdot F(b) \subseteq (F, A \setminus I) \Rightarrow F(a) \cdot F(r_1) \cdot F(b) \nsubseteq (E, I)$. So, $\{F(a) \cdot F(r) \cdot F(b) | r \in$ Supp $(F, A)\} \nsubseteq (E, I)$ which is a contradiction. Hence $F(a) \subseteq E(a)$ or $F(b) \subseteq E(b)$, that is (E, I) is a prime soft hyperideal.

Similarly we can prove the result for p-system and semiprime hyperideal also. □

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