Journal of Hyperstructures 12 *(1)* (*2023*), *103-110.*
ISSN: 2322-1666 print/2251-8436 online
Research Paper

STUDY OF TEMPORAL INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. In order to create a dynamic measure that describes distances between spatiotemporal points whose positions change over time as well as between the data represented by these points, a temporal intuitionistic fuzzy metric space is created in this paper. The notions of temporal fuzzy t-norm, temporal fuzzy t-conorm, and temporal fuzzy negation which have not previously been discussed in the literature are defined, and some of their fundamental characteristics are investigated, in order to define this new method.The idea that the degrees of nearness and non-nearness change with time is the basis for a novel definition of the concept of temporal intuitionistic fuzzy metric spaces. However, the basic topological characteristics of the temporal intuitionistic fuzzy metric space are also looked at. We demonstrate how this new temporal metric space preserves the basic characteristics offered by both classical and fuzzy metric spaces. As a result, a new, more flexible, and dynamic metric topology is created while maintaining the fundamental topological characteristics of fuzzy and intuitionistic fuzzy metric spaces.

Key Words:fuzzy sets, fuzzy metric spaces, intuitionistic fuzzy metric spaces, temporal intuitionistic sets, temporal intuitionistic spaces
2010 Mathematics Subject Classification: Primary: 13A15; Secondary: 13F30, 13G05.

Received: 30 October 2022, Accepted: 09 May 2023. Communicated by Hoger Ghahramani;
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1. Introduction

The need to reevaluate all concepts with fuzzy set theory arose after Zadeh [1] changed the direction of mathematics by first defining the idea of fuzzy logic and, subsequently, the fuzzy set in 1965. The concept of distance has undergone many revisions with the new methods that fuzzy sets have introduced. The fuzzy metric created by Kramosil and Michalek [2] in 1975 is one of the most well-known of them. A new idea termed the degree of nearness is used in this metric to define the distance between points. George and Veeramani [3] made modifications to this fuzzy metric space to obtain Hausdorff topology.

The fuzzy set theory has been expanded using a variety of methods.One of these is the Atanassov-described intuitionistic fuzzy set theory [4]. Along with the degree of membership, the degrees of non-membership and uncertainty are also defined in intuitionistic fuzzy set theory. In recent years, numerous studies across all disciplines have demonstrated the high effectiveness of the intuitionistic fuzzy set theory as a technique for handling bipolar circumstances. [5,10]. In contrast to the fuzzy set theory, the intuitionistic fuzzy set theory employs more useful definitions of mathematical concepts.It has made it easier to construct more expansive and realistic mathematical models, particularly by using uncertainty and negative information while defining ideas.

The temporal intuitionistic fuzzy set, which was created by adding the time component to the intuitionistic fuzzy set, is a significant extension of fuzzy sets [11]. It is clear that the concept of shifting membership and non-membership levels with time and place has given rise to a thriving field of study in spatio-temporal research areas like meteorology, economics, image processing, and video processing. On the other hand, successful outcomes on the aforementioned topics have been attained in numerous approaches [12-17]. A serious flaw in the literature is the fact that the idea of the temporal intuitionistic fuzzy metric space that can be derived by accounting for the time component is still not specified.The idea of dynamising distance measures by defining them with spatiotemporal dynamics is the primary driving force behind this project. The fact that the locations of the points between which we want to measure the distance may change over time emphasises the need for the metric we use to measure the distance to be tied to both the concept of time and the positions of the points. Another crucial element is the benefits of this mathematical framework's adaptable and changing structure, which we utilise to measure distance. As a result, the idea of a temporal intuitionistic fuzzy metric will be a more versatile and adaptable metric when the time parameter is added. In this study, we define and analyse the fundamental topological properties of temporal intuitionistic fuzzy metric spaces. The study's primary goal is to present a novel notion of a metric that can quantify the separation between moving points and sets quickly and change adaptively over time. The temporal intuitionistic fuzzy metric space is defined and its characteristics are discussed in the sections that follow, along with the fundamental terminology utilised in the study.

2. Preliminaries

Definition 2.1 A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions:

(a) $*$ is commutative and associative;
(b) $*$ is continuous;
(c) $\mathrm{a}^*1 = \mathrm{a}$ for all $\mathrm{a} \in [0,1]$;
(d) $\mathrm{a}^*\ \mathrm{b} \leq \mathrm{c}^*\ \mathrm{d}$ whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for each $\mathrm{a,b,c,d} \in [0,1]$.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is a continuous t-conorm if it satisfies the following conditions:

(a) is commutative and associative;
(b) is continuous;
(c) $\mathrm{a} \diamond 0 = \mathrm{a}$ for all $\mathrm{a} \in [0,1]$;
(d) $\mathrm{a} \diamond \mathrm{b} \leq \mathrm{c} \diamond \mathrm{d}$ whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for each $\mathrm{a,b,c,b} \in [0,1]$:

Definition 2.3. A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm, $\diamond$ a continuous t-conorm and M, N are fuzzy sets on $X \times X \times (0, \infty)$, satisfying the following conditions for all $x, y, z \in X$ and $s, t > 0$:

a. $M(x,y,t) + N(x,y,t) \leq 1$;
b. $M(x,y,t) > 0$;
c. $M(x,y,t) = 1$ if and only if $x = y$;
d. $M(x,y,t) = M(y,x,t)$;
e. $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$;
f. $M(x,y,) : (0,\infty) \to (0,1]$ is continuous;
g. $N(x,y,t) \geq 0$;
h. $N(x,y,t) = 0$ if and only if $x = y$;
i. $N(x,y,t) = N(y,x,t)$;
j. $N(x,y,t) \diamond N(y,z,s) \geq N(x,z,t+s)$;
k. $N(x,y,) : (0,\infty) \to (0,1]$ is continuous.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t respectively [11].

Definition 2.4. Let T be a time set. If the mapping $*_t : ([0,1]\times[0,1])\times T \to [0,1]$ is satisfied following the condition for a fixed time moment $t \in T$, it is called temporal fuzzy triangular norm (temporal t-norm) at time moment t :

1. $*_t((x,1),t) = x$ (bounded condition);
2. $*_t((x,y),t) = *_t((y,x),t)$ (commutativity);
3. $*_t\left((x,{}_t((y,z),t)),t\right) = *_t\left((*_t((x,y),t),\text{z}),t\right)$ (associativity)
4. $*_t((a,b),t) \le *_t((c,d),t)$ if $a \le c, b \le d$ (monotonicity).

Definition 2.5. Let T be a time set. If the mapping $\diamond_t : ([0,1]\times[0,1])\times T \to [0,1]$ is satisfied following the condition for a fixed time moment $t \in T, \diamond_t$ is called temporal intuitionistic fuzzy triangular conorm (temporal $s-$ norm) at time moment t :

1. $\diamond_t((x,0),t) = x$ (bounded condition);
2. $\diamond_t((x,y),t) = \diamond_t((y,x),t)$ (commutativity);
3. $\diamond_t\left((x,\diamond_t((y,z),t)),t\right) = \diamond_t\left((\diamond_t((x,y),t),z),t\right)$ (associativity);
4. $\diamond_t((a,b),t) \le \diamond_t((c,d),t)$ if $a \le c, b \le d$ (monotonicity).

Unlike the t-norm and t-conorm defined in the fuzzy set theory, these temporal fuzzy t-norm and t-conorm gain variable and adaptive structure with the time parameter. This provides the opportunity for conjunctions and disconjunctions represented by t-norms and t-conorms to change over time.

Definition 2.6. Let T be a time set. If $N_t : [0,1]\times T \to [0,1]$ satisfies the following conditions, N_t is called temporal fuzzy negation at fixed time moment $t \in T$:

i. N_t is non-increasing mapping for the first variable and continuous for the second variable;

ii. $N_t(0,t) = 1$ and $N_t(1,t) = 0$ at fixed time moment $t \in T$.

If a temporal fuzzy negation is monotonously decreasing at the first component $(x < y \Rightarrow N(x,t) > N(y,t))$ and continuous at both terms, it is called a strict negation. If a strict negation N is an involution $(N(N(x)) = x$ for all $x \in X$), it is called a strong negation.

Definition 2.7. Let $*_t$ be a temporal intuitionistic fuzzy t-norm, $\succ_t$ be a temporal intuitionistic fuzzy t-conorm and N_t be a temporal intuitionistic fuzzy strong negation. If the ordered triplet $(\diamond_{s,\,t}, N_t)$ satisfies De Morgan's condition:

$$\Delta_t((x,y),t) = N_t\left(*_t\left((N_t(x,t),N_t(y,t)),t\right),t\right),$$

the triplet is called temporal intuitionistic fuzzy De Morgan triplet.

Definition 2.8. A 6-tuple $(X, T, M_t, N_t, *_t, \diamond_t)$ is said to be a temporal intuitionistic fuzzy metric space if X is an arbitrary non-empty set and T is the time set, $\overset{*}{t}$ is a temporal continuous $t-$ norm, 0_t is a temporal continuous t-conorm, M_t and N_t are functions on $X \times X \times (0, \infty) \times T$ to $[0, 1]$, satisfying the following conditions for fixed $t \in T$ and all $x, y, z \in X$ and $n, n_1, n_2 > 0$:

i. $M_t(x, y, n, t) + N_t(x, y, n, t) \leq 1$;
ii. $M_t(x, y, n, t) \geq 0$;
iii. $M_t(x, y, n, t) = 1$ if and only if $x = y$;
iv. $M_t(x, y, n, t) = M_t(y, x, n, t)$;
v. $*_t\left(\left(M_t\left(x, y, n_1, t\right), M_t\left(y, z, n_2, t\right)\right), t\right) \leq M_t\left(x, z, n_1 + n_2, t\right)$;
vi. $M_t(x, y, ., t) : (0, \infty) \to (0, 1]$ is continuous;
vii. $N_t(x, y, n, t) \leq 1$
viii. $N_t(x, y, n, t) = 0$ if and only if $x = y$;
ix. $N_t(x, y, n, t) = N_t(y, x, n, t)$
x. $\diamond_t\left(\left(N_t\left(x, y, n_1, t\right), N_t\left(y, z, n_2, t\right)\right), t\right) \geq N_t\left(x, z, n_1 + n_2, t\right)$
xi. $N_t(x, y, ., t) : (0, \infty) \to (0, 1]$ is continuous;
xii. $M_t(x, y, n,) : .T \to (0, 1]$ and $N_t(x, y, n,) : .T \to (0, 1]$ are continuous.

Then (M_t, N_t) is called a temporal intuitionistic fuzzy metric on X. The functions $M_t(x, y, n, t)$ and $N_t(x, y, n, t)$ denote the degree of nearness and the degree of non-nearness between x and y at time moment t respectively.

3. Main Results

Theorem 3.1. Let $(X, T, M_t, N_t, *_t, \diamond_t)$ be an intritionistic fuzzy metric space such that every cauchy sequence in X has a convergent subsequence. Then $(X, T, M_t, N_t, *_t, \diamond_t)$ is complete.

Proof: let $\{a_n\}$ be a cauchy sequence and $\{a_m\}$ be a subsequence of $\{a_n\}$. Which converge to limit l.

Now we have to show that $a_n \to l$. Let $m > 0$ and $\varepsilon > 0$. Choose $s \in (0, 1)$ such that $(1 - s) * (1 - s) \geqslant 1 - \varepsilon$. and $s \diamond s \leq \varepsilon$ but $\{a_n\}$ is a cauchy sequence then their exist a positive integer n_0 such that

$$M_t\left(a_n, l, \frac{m}{2}, t\right) > 1 - s$$

and $N_t\left(a_n, l, \frac{m}{2}, t\right) < s$, for all $m, n \geqslant n_0$.

Since $a_n \to l$, then, there is a positive integer m such that $m > n_0$, $M_t\left(a_m, l, \frac{m}{2}, t\right) > 1-s$ and $N_t\left(a_m, l, \frac{m}{2}, t\right) < s$. Now, if $n \geq n_0$,

$$M_t\left(a_n, a_m, m, t\right) \geqslant *_t\left\{\left(M_t\left(a_n, l, \frac{m}{2}, t\right), \; M_t\left(a_m, l, \frac{m}{2}, t\right)\right), t\right\}$$
$$> (1-s) * (1-s)$$
$$\geq 1-\varepsilon.$$

and

$$N_t\left(a_n, a_m, m, t\right) \leq \diamond_t\{\left\{\left(N_t\left(a_n, l, \frac{m}{2}, t\right), N_t\left(a_m, l, \frac{m}{2}, t\right)\right), t\right\}$$
$$< s \diamond s$$
$$\leq \varepsilon.$$

Hence $(X, T, M_t, N_t, *_t, \diamond_t)$ is complete.

Theorem 3.2. Every separable temporal intuitionistic fuzzy metric space is second countable.

Proof. Let $(X, T, M_t, N_t, *_t, \diamond_t)$ be the separable temporal intuitionistic fuezy metric space. Let $B(t) = \{a_m; m \in N\}$ be a countable dense subset of y. Consider a family $\alpha = \left\{A_{(M_t, N_t)}\left(a_i, \frac{1}{j}, \frac{1}{j}, t\right)\right.$ then α is countable. We claim that α is a base for the family of all open sets in Y. Let $v(t)$ be any open set in y and let $l \in V(t)$. Then there exist $t \in T$ and $s \in (0,1)$ such that.

$$A_{(M_t, N_t)}(l, s, m, t) \subset V(t).$$

Since $s \in (0,1)$, we can choose an $p \in (0,1)$ such that
$*_t(((1-p), (1-p)), t) > 1-s$ and $\diamond_t((p,p), t) < s$
since $n \in R$, such that $\frac{1}{n} < \min\left\{p, \frac{m}{2}\right\}$. Since B is dense in y, there exists $a_i \in B$ such that

$$a_i \in A_t\left(l, \tfrac{1}{n}, \tfrac{1}{n}, t\right).$$

Now if $x \in A_t\left(a_i, \frac{1}{n}, \frac{1}{n}, t\right)$, then

$$M(l, x, m, t) \geqslant *_t\left(\left(M_t\left(l, a_i, \frac{m}{2}, t\right), \; M_t\left(x, a_i, \frac{m}{2}, t\right)\right), t\right)$$
$$\geqslant M_t\left(l, a_i, \frac{1}{n}, t\right) * M_t\left(x, a_i, \frac{1}{n}, t\right)$$
$$\geqslant *_t\left(\left(\left(1-\tfrac{1}{n}\right), \left(1-\tfrac{1}{m}\right), t\right)\right), t)$$
$$\geqslant *_t(((1-p), (1-p)), t) > s$$

and

$$
\begin{aligned}
N_t(l,x,m,t) &\le N_t\left(l,a_i,\frac{m}{2},t\right)\diamond N_t\left(x,a_i,\frac{m}{2},t\right)\\
&\le N_t\left(l,a_i,\frac{1}{n},t\right)\diamond N_t\left(x,a_i,\frac{1}{n}t\right)\\
&\qquad\qquad \le \diamond_t\left(\left(\frac{1}{n},\frac{1}{n}\right),t\right)\\
&\qquad\qquad \le \diamond_t((p,p),t)\\
&\qquad\qquad < r
\end{aligned}
$$

Thus. $x \in A_t(l,s,m,t) \subset V(t)$ and hence α is a base.

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