Journal of Hyperstructures 12 *(1) (2023), 91-102.*
ISSN: 2322-1666 print/2251-8436 online
Research Paper

SOME NOTES ON SOFT NEIGHBORHOOD

MUSTAFA BURÇ KANDEMİR AND GÜL DURSUN

Abstract. In this paper, the concept of soft neighborhood of an element in a soft topological space is introduced, and its basic properties are studied.

Key Words: Neighborhood, Soft neighborhood, Soft set, Soft topology, Topology.
2010 Mathematics Subject Classification: Primary: 54Axx, 54A20; Secondary: 03E99, 54Cxx.

1. Introduction and Preliminaries

Decisions are the building blocks of our lives. They shape our destiny, influence the paths we tread, and determine the outcomes we achieve. From the seemingly inconsequential choices in our daily routines to the weighty decisions that shape the course of businesses, governments, and societies, the act of making decisions is a fundamental and ubiquitous aspect of human existence. Whether we recognize it or not, our choices carry profound significance, impacting our individual journeys and the world around us. In this age of complexity and rapid change, understanding the importance of decision-making is not just a matter of personal empowerment; it is the key to navigating the intricate web of life and contributing to the progress of the collective human experience.

Decision making is the process of choosing a course of action from multiple available options. It typically involves identifying a problem or opportunity, generating alternatives, evaluating these options based on

Received: 1 April 2022, Accepted: 20 May 2023. Communicated by Ahmad Yousefian Darani;
∗Address correspondence to Mustafa Burç Kandemir; E-mail: mbkandemir@mu.edu.tr.

certain criteria, selecting the best alternative, implementing the decision, and monitoring its outcomes. Decisions can be influenced by individual preferences, organizational goals, and the need to balance benefits and risks. Various decision-making styles exist, including intuitive, analytical, and group decision making. Effective decision making is a critical skill in personal and professional life, helping individuals and organizations navigate complex situations and achieve desired outcomes. It often involves recognizing and managing cognitive biases and heuristics that can impact the quality of decisions.

The relationship between decision-making and mathematics is profound and multifaceted. Mathematics provides a structured and analytical framework that plays a crucial role in various aspects of decision-making such as quantification and measurement, probability and risk assessment, optimization, modelling and simulation, data analysis, logic and deductive reasoning, resource allocation, game theory, supply chain and operations management and project planning and scheduling. In summary, mathematics provides decision-makers with a structured, logical, and quantitative framework to analyze, assess, and optimize decisions. It enhances the precision and rigor of decision-making, enabling individuals, businesses, and governments to make more informed and strategic choices across a wide array of domains.

Soft set theory is a versatile and innovative mathematical framework that offers a novel approach to handling uncertainty and vagueness in decision-making and data analysis. Introduced by Molodtsov in 1999, soft set theory provides a flexible and intuitive method for dealing with imprecise information, which is prevalent in various real-world scenarios, including artificial intelligence, economics, medicine, and social sciences. By providing a means to model and manipulate uncertain information, soft set theory has the potential to enhance decision support systems and provide solutions to complex problems in a more realistic and inclusive manner. Soft set theory has been studied and developed by many scientists [1, 2, 3, 6, 14, 15, 16, 17] since the day it was first introduced, and it still continues to develop.

In 1999, Molodtsov [19] introduced the theory of soft sets in order to come through the uncertainties in any fundamental sciences. He showed that this theory can be applied in many directions. The formal definition of soft set in any universal set is as follows.

Definition 1.1. [19] Let U be an initial universal set, E be a parameters set, $A \subseteq E$. The pair (F, A) is called a *soft set* over U if $F : A \to \mathcal{P}(U)$ is a set-valued mapping where $\mathcal{P}(U)$ is a power set of U.

The set-theoretic operations between soft sets defined by Maji et. al. and Ali et. al. in [17, 3]. Let's give these statements now.

Let (F, A) and (G, B) be soft sets over U. (G, B) is called a *soft subset* of (F, A) if $B \subseteq A$ and $G(p) \subseteq F(p)$ for each $p \in B$, and denoted by $(G, B)\widetilde{\subset}(F, A)$. If $(F, A)\widetilde{\subset}(G, B)$ and $(G, B)\widetilde{\subset}(F, A)$, then it is called that (F, A) is *equal* to (G, B). The *soft union* of (F, A) and (G, B) is defined as the soft set (H, C) over U, and denoted by $(F, A)\widetilde{\cup}(G, B) = (H, C)$ such that $C = A \cup B$ and

$$H(p) = \begin{cases} F(p) & , p \in A - B \\ G(p) & , p \in B - A \\ F(p) \cup G(p) & , p \in A \cap B \end{cases}$$

for each $p \in C$. The *soft intersection* of (F, A) and (G, B) is defined as the soft set (H, C) over U, and denoted by $(F, A)\widetilde{\cap}(G, B) = (H, C)$ such that $C = A \cap B \neq \varnothing$ and $H(p) = F(p) \cap G(p)$ for each $p \in C$. The *complement* of (F, A) is defined and denoted as $(F, A)^c = (F^c, A)$ where $F^c : A \to \mathcal{P}(U)$ such that $F^c(p) = U - F(p)$ for each $p \in A$. It is called that (F, A) is a *relative whole soft set* if $F(p) = U$ for each $p \in A$ and denoted by $\widetilde{\mathbb{U}}_A$. If $A = E$, (F, E) is called *absolute soft set* and denoted by $\widetilde{\mathbb{U}}$. Similarly, (F, A) is called a *relative null soft set* if $F(p) = \varnothing$ for each $p \in A$ and denoted by $\widetilde{\Phi}$.

As it is known, the concepts of soft element belonging to a soft set has been redefined by many people [4, 8, 9, 10, 21, 23] and its operations have been carried out similarly to the classical theory. This causes the soft set theory to lose its spirit. However, since the system (the soft set) is obtained by using the elements of the problem universe, it becomes necessary to give a description of the belonging of the elements of the problem universe to the system.

Additionally, it is understood from the studies [6, 7, 13, 14, 15, 16, 18, 22] that soft set theory is a very useful mathematical tool in decision-making problems.

In [24], Shabir and Naz proposed that an element of the initial universe belongs to a soft set on this universe as follows:

Definition 1.2. [24] Let $U \neq \varnothing$ be an initial universe, $E \neq \varnothing$ be a set of parameters and (F, A) be a soft set over U where $A \subseteq E$. Given any

$x \in U$. If $x \in F(p)$ for each $p \in A$, then x is member of (F, A), and it is denoted by $x \widetilde{\in} (F, A)$. Otherwise, it is denoted by $x \widetilde{\notin} (F, A)$.

The above definition is reasonable but not useful. Because, for example, in a house purchase problem, it is expected that Mr. X will choose the house that has the maximum features according to Mr. X's wishes. In other words, the house to be purchased is selected from this set, as the problem universe is the set of houses with certain characteristics, and this house meets the wishes of Mr. X at the maximum level. Now suppose there are two houses that meet almost all of Mr. X's wishes. However, one of these houses is expensive and the other is cheap. Naturally, Mr. X is expected to choose the house that almost satisfies all his wishes and is cheaper than the other. In this case, we cannot expect the elements belonging to the system (flexible set) to provide all the properties. As a result, the house that Mr. X will choose will be chosen again thanks to this system. In other words, this house will belong to this system. So, the definition of belonging to the soft set given in Definition 1.2 should be given in accordance with this situation. For this, let us give the following definition.

Definition 1.3. Let $U \neq \varnothing$ be an initial universe, $E \neq \varnothing$ be a set of parameters and (F, A) be a soft set over U where $A \subseteq E$. Given any $x \in U$. If there exist a $p \in A$ such that $x \in F(p)$, then x is *member* of (F, A), and it is denoted by $x \widetilde{\in} (F, A)$. Moreover, If $x \in F(p)$ for each $p \in A$, then x is *strong member* of (F, A), and it is denoted by $x \widetilde{\in}_s (F, A)$.

If $x \notin F(p)$ for each $p \in A$, then x is *not a member* of (F, A) and it is denoted by $x \widetilde{\notin} (F, A)$.

Proposition 1.4. *Let (F, A) and (G, B) be a soft sets over U where $A, B \subseteq E$ and $x \in U$.*

(1) *If $x \widetilde{\in} (G, B)$ and $(G, B) \widetilde{\subset} (F, A)$, then $x \widetilde{\in} (F, A)$.*
(2) *Let $x \widetilde{\in}_s (G, B)$ and $(G, B) \widetilde{\subset} (F, A)$. If $A = B$, then $x \widetilde{\in}_s (F, A)$.*

The theory of soft topology has been investigated and studied by many scientists. Shabir and Naz [24] gave the definition of soft topology for the first time as follows.

Definition 1.5. [24] Let $\mathbb{T}$ be the collection of soft sets over U, then $\mathbb{T}$ is said to be a *soft topology* on U if

(1) $\widetilde{\Phi}$, $\widetilde{\mathbb{U}}$ are in $\mathbb{T}$,

(2) the union of any number soft sets in $\mathbb{T}$ is in $\mathbb{T}$,
(3) the intersection of any two soft sets in $\mathbb{T}$ is in $\mathbb{T}$.

The triplet $(U, \mathbb{T}, E)$ is called a soft topological space over U.

In [24], the concepts of soft open set, soft closed set, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms have been introduced. In [5] and [11], other basic concepts of topology have been given.

In [12], the concept of soft topology viewed from a different perspective. We believe that soft topology should carry the spirit of soft set. Therefore, the concept of soft topology as a parametrization of the subspaces of a topological space has been given, preserving Molodtsov' s sense in [12].

Definition 1.6. [12] Let $(U, \mathcal{T})$ be a topological space, E be a parameters set and (F, A) be a soft set over U where $A \subseteq E$. It is called that (F, A) is a *soft topology* over U if $(F(p), \mathcal{T}_{F(p)})$ is a subspace of $(U, \mathcal{T})$ for each $p \in A$. $(F, A, \mathcal{T})$ is called a *soft topological space* over U.

Some basic topological concepts such as Hausdorffness, compactness, connectedness for soft topological spaces have also been given and their some properties have been discussed in [12]. In [13], we have given the notions of interior, closure, etc., introduced the concept of soft continuity of a function given between soft topological spaces and studied their properties. In addition, we also proposed a method of decision making using soft topological concepts.

In the concept of soft topology defined in [24] (or similar), it is not possible to relate to the points of space.

In this paper, we will define the notion of soft neighborhood of a point of the space via soft topological space, and investigate its basic properties.

2. The Concept of Soft Neighborhood of an Element

Details of the concept of neighborhood are discussed in [20]. However, this concept is defined through the concept soft topology given in [24]. The concept of soft neighborhood to be given here will be given based on the points of a problem space using the soft topology defined in [12]. This definition is thought to be more applicable and meaningful compared to other definitions.

So, let's start with the following definition.

Definition 2.1. Let $(F, A, \mathcal{T})$ be a soft topological space, $x \in U$ and $(G, B)\widetilde{\subset}(F, A)$. If there exits $O_p \in \mathcal{T}_{F(p)}$ for some $p \in B$ such that $x \in O_p \subseteq G(p)$, then (G, B) is called a *weak soft neighborhood* of x.

For each $x \in U$, the family of all soft neighborhood of x is denoted by $\widetilde{\mathcal{N}}_w(x)$.

Definition 2.2. Let $(F, A, \mathcal{T})$ be a soft topological space, $x \in U$ and $(G, B)\widetilde{\subset}(F, A)$. If there exits $O_p \in \mathcal{T}_{F(p)}$ for each $p \in B$ such that $x \in O_p \subseteq G(p)$, then (G, B) is called a *strong soft neighborhood* of x.

For each $x \in U$, the family of all soft neighborhood of x is denoted by $\widetilde{\mathcal{N}}_s(x)$.

Note that, If there exists $O_p \in \mathcal{T}_{F(p)}$ for some $p \in B$ such that $x \in O_p$ and $G(p) = O_p$ then (G, B) is called a *weak open soft neighborhood* of x. Moreover, if there exits $O_p \in \mathcal{T}_{F(p)}$ for each $p \in B$ such that $x \in O_p$ and $G(p) = O_p$, then (G, B) is called a *strong open soft neighborhood* of x. We denote the family of all weak open soft neighborhoods of a point and the family of strong open soft neighborhoods as $\widetilde{\mathcal{O}}_w(x)$ and $\widetilde{\mathcal{O}}_s(x)$, respectively

Example 2.3. Let $U = \{a, b, c\}$ be an initial topological universe where $\mathcal{T} = \{\varnothing, U, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$, $E = \{1, 2, 3, 4\}$ be the set of parameters and $(F, A) = \{1 = \{a, b\}, 2 = \{a, c\}\}$ be a soft topology over U where $A = \{1, 2\} \subset E$ and, $\mathcal{T}_{F(1)} = \{\varnothing, \{a, b\}, \{a\}\}$ and $\mathcal{T}_{F(2)} = \{\varnothing, \{a, c\}, \{a\}, \{c\}\}$.

Consider the soft subset $(G, B) = \{1 = \{a\}, 2 = \{a, c\}\}$ of (F, A). Since, there is $\{a\} \in \mathcal{T}_{F(1)}$ for $1 \in B$ such that $a \in \{a\} \subseteq G(1) = \{a\}$ and there is $\{a\} \in \mathcal{T}_{F(2)}$ for $2 \in B$ such that $a \in \{a\} \subseteq G(2) = \{a, c\}$, (G, B) is a strong soft neighborhood of a, i.e. $(G, B) \in \widetilde{\mathcal{N}}_s(a)$.

However, since there is $2 \in B$ such that there is an open set $\{c\} \in \mathcal{T}_{F(2)}$ and $c \in \{c\} \subseteq \{a, c\} = G(2)$, then (G, B) is a weak neighborhood of c, i.e. $(G, B) \in \widetilde{\mathcal{N}}_w(c)$.

Note that, (G, B) is neither a strong soft neighborhood nor a weak soft neighborhood of b.

Clearly, each strong soft neighborhood of a point is a weak soft neighborhood, and if $(F, A, \mathcal{T})$ is a soft topological space over $(U, \mathcal{T})$, then $(F, A) \in \widetilde{\mathcal{N}}_w(x)$ for each $x\widetilde{\in}(F, A)$.

Suppose that $(G, B) \in \widetilde{\mathcal{N}}_w(x)$. Then, we have that there is a $p \in B$ and $O_p \in \mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$. So, $x\widetilde{\in}(G, B)$, obviously. Thus, we can give the following proposition.

Proposition 2.4. *Let $(F, A, \mathcal{T})$ be a soft topological space over $(U, \mathcal{T})$, $x \in U$ and $(G, B)\widetilde{\subset}(F, A)$. If $(G, B) \in \widetilde{\mathcal{N}}_w(x)$, then $x\widetilde{\in}(G, B)$.*

Remark 2.5. In classical topology theory, the intersection of two neighborhoods of a point is also neighborhood of that point. However, this may not be provided for weak soft neighborhoods. For example, let $U = \{a, b, c, d, e\}$ and $\mathcal{T} = \{\varnothing, U, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Let $(F, A, \mathcal{T})$ be the soft topological space over $(U, \mathcal{T})$ such that

$$(F, A) = \{1 = \{a, b, c, d\}, 2 = \{a, b, d, e\}, 3 = \{c, e\}\}$$

and

$\mathcal{T}_{F(1)} = \{\varnothing, \{a, b, c, d\}, \{a\}, \{a, b\}, \{b, c\}, \{b\}, \{a, b, c\}\}$,
$\mathcal{T}_{F(2)} = \{\varnothing, \{a, b, c, e\}, \{a\}, \{a, b\}, \{b\}\}$,
$\mathcal{T}_{F(3)} = \{\varnothing, \{c, e\}, \{c\}\}$.

Consider the soft subsets of (F, A), $(G, B) = \{1 = \{b, c\}, 2 = \{a, d, e\}\}$ and $(H, C) = \{2 = \{a, b, d\}, 3 = \{c\}\}$. Obviously, $(G, B), (H, C) \in \widetilde{\mathcal{N}}_w(c)$, and then

$$(G, B)\widetilde{\cap}(H, C) = \{2 = \{a, d\}\} \notin \widetilde{\mathcal{N}}_w(x).$$

Proposition 2.6. *Let $(F, A, \mathcal{T})$ be a soft topological space over $(U, \mathcal{T})$, $x \in U$ and $(G, B), (H, C)\widetilde{\subset}(F, A)$. If $(G, B), (H, C) \in \widetilde{\mathcal{N}}_w(x)$ and either $(G, B)\widetilde{\subset}(H, C)$ or $(H, C)\widetilde{\subset}(G, B)$, then $(G, B)\widetilde{\cap}(H, C) \in \widetilde{\mathcal{N}}_w(x)$.*

Proof. It is obvious. □

Proposition 2.7. *Let $(F, A, \mathcal{T})$ be a soft topological space over $(U, \mathcal{T})$, $x \in U$ and $(G, B), (H, C)\widetilde{\subset}(F, A)$. If $(G, B) \in \widetilde{\mathcal{N}}_w(x)$ and $(G, B)\widetilde{\subset}(H, C)$, then $(H, C) \in \widetilde{\mathcal{N}}_w(x)$.*

Proof. Suppose that $(G, B) \in \widetilde{\mathcal{N}}_w(x)$ and $(G, B)\widetilde{\subset}(H, C)$. So, we have $G(p) \subseteq H(p)$ for each $p \in B \subseteq C$. Since $(G, B) \in \widetilde{\mathcal{N}}_w(x)$, there is a $p \in B$ and $O_p \in \mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$. Therefore, we obtain that $x \in O_p \subseteq H(p)$. Thus, $(H, C) \in \widetilde{\mathcal{N}}_w(x)$. □

Proposition 2.8. *Let $(F, A, \mathcal{T})$ be a soft topological space over $(U, \mathcal{T})$, $x \in U$ and $(G, B), (H, C)\widetilde{\subset}(F, A)$. For each $(G, B) \in \widetilde{\mathcal{N}}_w(x)$, there is $(H, C) \in \widetilde{\mathcal{N}}_w(x)$ such that for each $y\widetilde{\in}(H, C)$ implies $(G, B) \in \widetilde{\mathcal{N}}_w(y)$*

Proof. Suppose that $(G, B) \in \widetilde{\mathcal{N}}_w(x)$. Then, by definition of weak soft neighborhoods, there is $O_p \in \mathcal{T}_{F(p)}$ for some $p \in B$ such that $x \in O_p \subseteq G(p)$. Noe, if we define that $C = \{p\} \subset B$ for fixed $p \in B$ and

$H: C \to \mathcal{P}(U), H(p) = O_p$, we obtain that $(H,C) \in \widetilde{\mathcal{N}}_w(x)$. Thus, for each $y \in H(p) = O_p \in \mathcal{T}_{F(p)}$ we get $y \in O_p \subseteq G(p)$. Hence, $(G,B) \in \widetilde{\mathcal{N}}_w(y)$. □

Theorem 2.9. *Let $(F,A,\mathcal{T})$ be a soft topological space over U, $x \in U$ and $(G,B)\widetilde{\subset}(F,A)$. (G,B) is a strong soft neighborhood of x if and only if there is an open soft set (H,B) such that $x\widetilde{\in}(H,B)\widetilde{\subset}(G,B)$.*

Proof. Suppose that $(G,B) \in \widetilde{\mathcal{N}}_s(x)$. Then, there is an open set $O_p \in \mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$ for each $p \in B$. If we define the soft set (H,B) as $H(p) = O_p$ for each $p \in B$, then (H,B) is an open soft set in $(F,A,\mathcal{T})$, obvioulsly. Hence we have $x\widetilde{\in}(H,B)\widetilde{\subset}(G,B)$.

On the other hand, suppose that there is an open soft set (H,B) in $(F,A,\mathcal{T})$ such that $x\widetilde{\in}(H,B)\widetilde{\subset}(G,B)$. Then we have $x \in H(p) \subseteq G(p)$ for each $p \in B$. With this, since (H,B) is open soft set, then $H(p) \in \mathcal{T}_{F(p)}$ for each $p \in B$. Hence, we have $(G,B) \in \widetilde{\mathcal{N}}_s(x)$. □

Theorem 2.10. *Let $(F,A,\mathcal{T})$ be a soft topological space over U, $x \in U$ and $\widetilde{\mathcal{N}}_s(x)$ be the family of strong soft neighborhoods of x. Followings are satisfied.*

(1) *If $(G,B) \in \widetilde{\mathcal{N}}_s(x)$, then $x\widetilde{\in}_s(G,B)$.*
(2) *If $(G,B),(H,C) \in \widetilde{\mathcal{N}}_s(x)$, then $(G,B)\widetilde{\cap}(H,C) \in \widetilde{\mathcal{N}}_s(x)$.*
(3) *If $(G,B) \in \widetilde{\mathcal{N}}_s(x)$ and $(G,B)\widetilde{\subset}(H,B)$, then $(H,B) \in \widetilde{\mathcal{N}}_s(x)$.*
(4) *For each $(G,B) \in \widetilde{\mathcal{N}}_s(x)$, there exits $(H,C) \in \widetilde{\mathcal{N}}_s(x)$ and $(H,C)\widetilde{\subset}(G,B)$ such that $(G,B) \in \widetilde{\mathcal{N}}_s(y)$ for each $y\widetilde{\in}(H,C)$.*

Proof. (1) It is straightforward.

(2) Suppose that $(G,B),(H,C) \in \widetilde{\mathcal{N}}_s(x)$ and say $(K,D) = (G,B)\widetilde{\cap}(H,C)$. So, we have $K(p) = G(p) \cap H(p)$ for each $p \in D$. Since $(G,B),(H,C) \in \widetilde{\mathcal{N}}_s(x)$, there are open sets O_p and O'_p in $\mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$ and $x \in O'_p \subseteq H(p)$. Hence, we obtain that

$$x \in O_p \cap O'_p \subseteq G(p) \cap H(p) = K(p)$$

where $O_p \cap O'_p \in \mathcal{T}_{F(p)}$. Since p is an arbitrary element of D, then $(K,D) \in \widetilde{\mathcal{N}}_s(x)$.

(3) It is straightforward.

(4) Suppose that $(G,B) \in \widetilde{\mathcal{N}}_s(x)$. Then we know that for each $p \in B$ there is an $O_p \in \mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$. Now, we take $B = C$ and define $H: B \to \mathcal{P}(U), H(p) = O_p$. Obviously, for each $y\widetilde{\in}(H,C)$,

$(H,C) \in \widetilde{\mathcal{N}}_s(y)$ and $(H,C)\widetilde{\subset}(G,B)$. Hence, we obtain that $(G,B) \in \widetilde{\mathcal{N}}_s(y)$. □

Remark 2.11. The soft intersection of two strong soft neighborhoods of a point is the strong soft neighborhood of that point. However, the soft intersection of a weak soft neighborhood and a strong soft neighborhood of a point may not be a strong soft neighborhood of that point.

Also, note that, let $(F, A, \mathcal{T})$ be a soft topological space over U, $x \in U$ and $(G,B),(H,C)\widetilde{\subset}(F,A)$. If $(G,B) \in \widetilde{\mathcal{N}}_s(x)$, $(H,C) \in \widetilde{\mathcal{N}}_w(x)$ and $(G,B)\widetilde{\subset}(H,C)$ then $(G,B)\widetilde{\cap}(H,C) \in \widetilde{\mathcal{N}}_s(x)$, obviously.

Moreover, the soft union of strong soft neighborhood of a point is also strong soft neighborhood of that point.

In [13], in a soft topological space, the notion of open soft set is given in the following form.

Let $(F, A, \mathcal{T})$ be a soft topological space over U and $(G,B)\widetilde{\subset}(F,A)$. If $G(p) \in \mathcal{T}_{F(p)}$ for each $p \in B$, then (G,B) is an open soft set in $(F, A, \mathcal{T})$ [13]. Using this definition, we can give the following proposition.

Proposition 2.12. *Let $(F, A, \mathcal{T})$ be a soft topological space over U and $(G,B)\widetilde{\subset}(F,A)$. (G,B) is an open soft set in $(F, A, \mathcal{T})$ if and only if $(G,B) \in \widetilde{\mathcal{N}}_w(x)$ for each $x\widetilde{\in}(G,B)$.*

Proof. Suppose that (G,B) is an open soft set in $(F, A, \mathcal{T})$. Then we know that $G(p) \in \mathcal{T}_{F(p)}$ for each $p \in B$. Now let's choose an arbitrary member x from (G,B). Then, since there is a $p \in B$ with $x \in G(p)$ and it can be written that $x \in G(p) \subseteq G(p)$, we obtain that $(G,B) \in \widetilde{\mathcal{N}}_w(x)$.

Conversely, suppose that any $x\widetilde{\in}(G,B)$ and $(G,B) \in \widetilde{\mathcal{N}}_w(x)$. Then, there is a $p \in B$ and $O_p \in \mathcal{T}_{F(p)}$ such that $x \in O_p \subseteq G(p)$. Since x is an arbitrary member of (G,B), then the condition $x \in O_p \subseteq G(p)$ is satisfied for all member x in (G,B). Therefore, we have

$$\bigcup_{x \in G(p)} \{x\} \subseteq \bigcup_{x \in G(p)} O_p \subseteq G(p).$$

So, $G(p) \in \mathcal{T}_{F(p)}$. Since the arbitrariness of x will bring the arbitrariness of p. Thus, (G,B) becomes an open soft set in $(F, A, \mathcal{T})$. □

Proposition 2.13. *Let $(F, A, \mathcal{T})$ be a soft topological space over U, $x \in U$ and $(G,B)\widetilde{\subset}(F,A)$. If there is an open soft set (H,C) such that $x\widetilde{\in}(H,C)\widetilde{\subset}(G,B)$, then $(G,B) \in \widetilde{\mathcal{N}}_w(x)$.*

Proof. It is straightforward. □

Definition 2.14. Let $(F, A, \mathcal{T})$ be a soft topological space over U, $x \in U$ and $(G, B)\widetilde{\subset}(F, A)$. If $(G, B) \in \widetilde{\mathcal{N}}_w(x)$, then x is a *soft interior point* of (G, B).

If $(G, B) \in \widetilde{\mathcal{N}}_s(x)$, then x is a *strong soft interior point* of (G, B).

In [13], The interior of a soft set is a soft set defined by taking the interiors of the relevant subset in the relevant subspace according to each parameter. Formally, let $(F, A, \mathcal{T})$ be a soft topological space over U and $(G, B)\widetilde{\subset}(F, A)$. The interior of (G, B) defined as

$$\widetilde{\mathfrak{i}}(G, B) = \{p = \mathfrak{i}(G(p)) \mid p \in B\}$$

where $\mathfrak{i}(G(p))$ is the interior of $G(p)$ in $\big(F(p), \mathcal{T}_{F(p)}\big)$.

If x is a soft interior point of (G, B), let's denote it by $x\widetilde{\in}\widetilde{\mathfrak{i}}(G, B)$. If x is a strong soft interior point of (G, B), let's denote it by $x\widetilde{\in}\widetilde{\mathfrak{i}}_s(G, B)$. Thus, from this notation, $x\widetilde{\in}\widetilde{\mathfrak{i}}(G, B)$ is equivalent to $(G, B) \in \widetilde{\mathcal{N}}_w(x)$. Because by the definition of soft neighborhood, there is a $p \in B$ such that there is an $O_p \in \mathcal{T}_{F(p)}$ that satisfies the condition $x \in O_p \subseteq G(p)$. So, this provide that there is a $p \in B$ such that $x \in \mathfrak{i}(G(p))$ in the relevant subspace. Formally, we can express it as $x\widetilde{\in}\widetilde{\mathfrak{i}}(G, B)$ if and only if there is a $p \in B$ such that $x \in \mathfrak{i}(G(p))$.

The interior definition given in [13] is about finding the interior of a subset in subspaces independently of the point. According to the expression given above, there is a direct relationship between the interior concept defined in [13] and the soft interior point defined here.

3. Conclusion

In this study, the concept of soft neighborhood of the elements of a topological space is defined and some of its basic properties are examined. Since soft set theory has the potential to be easily applied to many social problems such as decision-making problems, it is thought that this study may be beneficial to scientists working in this field. The concept of convergence has an important place in mathematics. For this reason, as a continuation of this study, the convergence of a sequence in a topological space will be investigated using soft neighborhood of an element and soft topology in the Kandemir sense [12].

Acknowledgments

The authors are grateful to the editors for all their help.

References

[1] U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Comput. Math. Appl., 59 (11) (2010) 3458-3463.
[2] H. Aktaş, N. Çağman, Soft sets and soft groups, Inform. Sci., 177 (13) (2007) 2726-2735.
[3] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Comput. Math. Appl., 57 (2009) 1547-1553.
[4] A.A. Allam, T.H. Ismail, R. Mohammed, A new approach to soft belonging, Ann. Fuzzy Math. Inform., 13 (1)(2017) 145-152.
[5] A. Aygüoğlu, H. Aygün, Some notes on soft topological spaces, Neural Comput. & Applic., 21 (1) (2012) 113-119.
[6] N. Çağman, S. Enginoğlu, Soft set theory and $uni-int$ decision making, European J. Oper. Res., 207 (2) (2010) 848-855.
[7] V. Çetkin, A. Aygüoğlu, H. Aygün, A new approach in handling soft decision making problems, J. Nonlinear Sci. Appl., 9 (2016) 231-239.
[8] S. Das, S.K. Samanta, Soft real sets ,soft real numbers and their properties, Journal of Fuzzy Mathematics, 20 (3) (2012) 551-576.
[9] S. Das, S.K. Samanta, On soft metric spaces, Journal of Fuzzy Mathematics, 21 (3) (2013) 1-28.
[10] N. Ghosh, D. Mandal, T. Samantha, Soft groups based on soft element, Jordan J Math Stat., 9(2) (2016) 141-159.
[11] S. Hussain, B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., 62 (11) (2011) 4058-4067.
[12] M.B. Kandemir, A new perspective on soft topology, Hittite J. Sci. Eng., 5 (2) (2018) pp.105–113.
[13] M.B. Kandemir, Some notes on soft topological concepts and an application method, Mathematical Sciences and Applications E-Notes, 8 (1) (2020) 105–116
[14] M.B. Kandemir, Monotonic soft sets and its applications, Annals of Fuzzy Mathematics and Informatics, 12 (2) (2016) 295-307.
[15] M.B. Kandemir, The concept of σ-algebraic soft set, Soft Comp., 22 (13) (2018) 4353-4360.
[16] M.B. Kandemir, Some notes on σ-algebraic soft sets, Annals of the University of Craiova - Mathematics and Computer Science Series, 47 (2) (2020) 387-396.
[17] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory. Comput. Math. Appl., 45 (2003) 555-562.
[18] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl., 44 (8-9) (2002) 1077-1083.
[19] D. Molodtsov, Soft set theory - first results. Comput. Math. Appl., 37 (1999), 19-31.
[20] S.K. Nazmul, S.K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform., 6 (1) (2013) 1-15.

[21] N. Çakmak Polat, G. Yaylalı, B. Tanay, Some results on soft element and soft topological space, Math. Meth. Appl. Sci., (2019) 5607-5614.

[22] K. Qin, J. Yang, X. Zhang, Soft set approaches to decision making problems, In. T. Li et al. (eds) Rough Sets and Knowledge Technology RSTK 2012. Lecture Notes in Computer Science, 7414 (2012), 456-464.

[23] M. Saeed, M. Hussain, A.A. Mughal, A studuy of soft sets with soft members and soft elements: A new approach, Punjab University Journal of Mathematics, 52 (8) (2020) 1-15.

[24] M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl., 61 (7) (2011) 1786-1799.

Mustafa Burç Kandemir
Department of Mathematics, Mugla Sıtkı Koçman University, 48170 Mugla, Turkey
Email: mbkandemir@mu.edu.tr

Gül Dursun
Department of Mathematics, Mugla Sıtkı Koçman University, 48170 Mugla, Turkey
Email: gulldursunnn@gmail.com