A Hybrid Algorithm for Optimal Location and Sizing of Capacitors in the presence of Different Load Models in Distribution Network

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ABSTRACT

In practical situations, distribution network loads are the mixtures of residential, industrial, and commercial types. This paper presents a hybrid optimization algorithm for the optimal placement of shunt capacitor banks in radial distribution networks in the presence of different voltage-dependent load models. The algorithm is based on the combination of Genetic Algorithm (GA) and Binary Particle Swarm Optimization (BPSO) algorithm. For this purpose, an objective function including the cost of energy loss, reliability, and investment cost of the capacitor banks is considered. Also, the impacts of voltage-dependent load models, considering annual load duration curve, is taken into account. In addition, different types of customers such as industrial, residential, and commercial loads are considered for load modeling. Simulation results for 33-bus and 69-bus IEEE radial distribution networks using the proposed method are presented and compared with the other methods. The results showed that this method provided an economical solution for considerable loss reduction and reliability and voltage improvement.

1. INTRODUCTION

The utilization of shunt capacitors in distribution networks is essential for many reasons, including power flow control, loss minimization, power factor correction, voltage profile management, and system stability improvement. However, to achieve these objectives, keeping in mind the overall economy, capacitor planning must determine the optimal number, location, and size of capacitors to be installed in the distribution system. Many approaches have been proposed to solving the capacitor planning problem. Duran [1] considered capacitor sizes as discrete variables and employed dynamic programming to the problem solving. Grainger and Lee [2] developed a nonlinear programming-based method, in which capacitor location and capacity were expressed as continuous

variables. Baran and Wu [3] presented a method with mixed integer programming. Also, some authors [4, 5] proposed genetic algorithm approach for determining the optimal placement of capacitors based on the mechanism of natural selection.

Srinivasas et al. [6] Suggested an approach consisting of two parts; in the first part, loss sensitivity factors were used to select the candidate locations for the capacitor placement and, in the second part, plant growth simulation algorithm was used to estimate the optimal size of capacitors at the optimal buses determined in part one. Tabatabaei and Vahidi [7] represented a methodology based on fuzzy decision making which used a new evolutionary method. In this method, the installation node was selected by the fuzzy reasoning supported by the fuzzy set theory in a step by step procedure. In addition, an evolutionary algorithm known as bacteria foraging algorithm was used for solving the multivariable objective optimization problem and the optimal node for capacitor placement was

determined. Federico et al. [8] proposed an extended dynamic programming approach that lifted Duran's restricting assumptions and restored the perspective of achieving global optimality for capacitor allocation on radial distribution feeders. Hamouda and Sayaha [9] represented a fast heuristic method to solve the capacitor sizing and locating problems. In the proposed method, the candidate locations of the capacitor were determined by means of node stability indices and capacitor optimal sizes were determined by solving a non-linear constrained problem. Abul'wafa [10] suggested a loss sensitivity technique for selecting the candidate locations for the capacitor placement. The size of the optimal capacitor at the compensated nodes was determined simultaneously by optimizing the loss saving equation with respect to the capacitor currents. The authors in [11] proposed a dynamic model, in which load growth rate, load factor, and cost of power and energy losses were incorporated to maintain the voltage profile, considering the multiperiod capacitor allocation problem of radial distribution system. Huang and Liu [12] reported a plant growth-based optimization approach for capacitor placement in power systems.

Reliability improvement is one of the important goals of power utilities in system planning; but, it has not been sufficiently investigated in the studies related to the problem of capacitor placement thus far. In the above-mentioned works, loss reduction and/or voltage improvement has been considered for the objective problem. Etemadi and Fotuhi [13] proposed a new objective function for capacitor placement problem in order to improve reliability; however, similar to most of the above-mentioned works, a constant power load model was considered in the distribution system, which is far away from the real situation in distribution systems.

In this paper, a combined optimization algorithm is proposed for determining the optimal number, location, and size of capacitor banks in distribution networks including different load models for loss reduction and reliability improvement. In this method, optimum location of capacitors is determined by GA algorithm and their optimum size is determined by BPSO. At first, the initial population for the capacitor size and location is randomly produced. Then, the cost function is calculated using distribution system load flow for each random size and location and the size of capacitor is optimized using BPSO algorithm. In the next step, the new location of capacitor is optimized using GA by cost function minimization. In other words, in the proposed algorithm in this paper, the size of capacitor was optimized by BPSO which reduced the search area for the GA. Then, the location of capacitor is optimized by GA. The results demonstrated that the proposed combined GA/BPSO method is better than GA and BPSO methods when they are used individually.

The remaining part of the paper is organized as follows: In Sec. 2, the allocation and sizing of capacitors are formulated. The optimization algorithm and its implementation are explained in Sec. 3. Results and conclusions are given in Sec. 4 and 5, respectively.

2. FORMULATION OF THE CAPACITOR PLACEMENT

2.1. Reliability analysis of distribution system

Analysis of the customer failure statistics of most utilities shows that distribution system makes the greatest individual contribution to the unavailability of supply to customers [13]. Most distribution systems have been operated as radial networks; consequently, the principles of series systems can be directly applied to them. Three basic reliability indices of the system, as average failure rate, λ_s, average outage time, r_s, and annual outage time U_s, are given by:

$$\lambda_s = \sum_i \lambda_i \tag{1}$$

$$U_s = \sum_i \lambda_i r_i \tag{2}$$

$$r_s = \frac{U_s}{\lambda_s} \tag{3}$$

Where λ_i, r_i, and $\lambda_i r_i$ are average failure rate, average outage time, and annual outage time of the ith component, respectively. Energy not Supplied (ENS) as one of the most important reliability indices of a distribution system is evaluated and is included as a part of the objective function. This index reflects total ENS by the system due to the

faults in the system components. ENS can be calculated for each load point i using the following equation:

$$ENS_i = L_{a(i)} U_i \qquad (4)$$

where, $L_{a(i)}$ is the active load connected to load point i.

2.2. Impact of capacitor placement on reliability enhancement

Customer interruptions are caused by a wide range of phenomena including equipment failure, animals, trees, severe weather, and human error. Feeders in distribution systems deliver power from distribution substations to distribution transformers. A considerable portion of customer interruptions is caused by equipment failure in distribution systems consisting of underground cables and overhead lines [13]. Resistive losses increase the temperature of feeders, which is proportional to the square of the current magnitude flowing through the feeder. For underground cables, there is a maximum operating temperature which, if exceeded, would cause the insulation problem and an increase in component failure rates [13].

Life expectancy of the insulation material exponentially decreases as the operating temperature is raised [14]. On the other hand, a major reliability concern pertaining to the underground cables is water treeing. Severity of treeing is strongly correlated with thermal age since moisture absorption occurs more rapidly at high temperatures [15].

Temperature also has impacts on the reliability of overhead lines. High currents will cause lines to sag, reduce ground clearance, and increase the probability of phase conductors swinging into contact. Higher currents can cause conductors to anneal, reduce tensile strength, and increase the probability of a break occurrence [16].

Capacitor placement can supply a part of the reactive power demands. Therefore, due to the reduction in the current magnitude, the resistive losses would decrease. As a result, the destructive effects of temperature on the reliability of overhead lines and underground cables are moderated. These impacts on reliability are taken into consideration as the failure rate reduction of distribution feeder

components. Before capacitor placement, any feeder i has an uncompensated failure rate of λ_i^{uncomp}. If the reactive component of a feeder is fully compensated, its failure rate is reduced to λ_i^{comp}. If the reactive component of current is not completely compensated for, a failure rate is defined with linear relationship to the percentage of compensation. Thus, the compensation coefficient of the *ith* branch is defined as:

$$\alpha_i = \frac{I_r^{new}}{I_r^{old}} \qquad (5)$$

where, I_r^{new} and I_r^{old} are the reactive component of the ith branch current after and before compensation, respectively. The new failure rate of the *i*th branch is computed as follows:

$$\lambda_{i-new} = \alpha_i (\lambda_i^{uncomp} - \lambda_i^{comp}) + \lambda_i^{comp} \qquad (6)$$

2.3. Objective function and constraints

The objective of capacitor placement in the distribution system is the minimization of the annual cost of the system. In this paper, capacitor placement problem was formulated as the minimization of the total defined system cost including the capacitor investment cost, cost of energy losses, and cost of reliability improvement. The objective function which is minimized subject to the constraints (Sec. 2.3.2) is defined as follows:

$$TCOST = K_e \sum_{j=1}^{L} T_j P_{T,j} + K \sum_{k=1}^{L} ENS_k + \sum_{i=1}^{ncap} K_c Q_{ci} \qquad (7)$$

where, *TCOST* is the total annual cost of the system ($/year), $P_{T,j}$ is the power loss for each load level *j*, T_j is the time duration of the jth load level, ENS_k is total ENS because of the occurrence of faults in overhead lines and underground cables for each load level (kWh), Q_{ci} is the size of the capacitor at node i, ncap is the number of candidate locations for capacitor placement, *L* is the number of load level, K_c is the cost of capacitor per kVAr, *K* is the price of ENS ($/kWh), and K_e is the factor to convert energy loss into dollar ($/kWh).

2.3.1. Power losses

Generally, distribution systems are fed at one point and have a radial structure. The load flow equations of radial distribution network are computed by forward/backward method because of its low

memory requirements, computational efficiency, and robust convergence characteristic. The active and reactive power losses of the line section connecting buses i and $i+1$ may be computed as:

$$P_{Loss}(i, i + 1) = R_{i,i+1}I^2_{i,i+1} \qquad (8)$$

$$Q_{Loss}(i, i + 1) = X_{i,i+1}I^2_{i,i+1} \qquad (9)$$

Where $I_{i,i+1}$ is the magnitude of the current of the line section connecting buses i and $i+1$, $R_{i,i+1}$, and $X_{i,i+1}$ are resistance and reactance of the line section connecting buses i and $i+1$, respectively. Also, $P_{Loss}(i, i + 1)$ and $Q_{Loss}(i, i + 1)$ are active and reactive power losses of the line section connecting buses i and $i+1$, respectively. The total active and reactive power losses of the feeders in the system are determined as follows:

$$P_{T,Loss} = \sum_{i=1}^{NB} P_{Loss}(i, i + 1) \qquad (10)$$

$$Q_{T,Loss} = \sum_{i=1}^{NB} Q_{Loss}(i, i + 1) \qquad (11)$$

where, NB is the number of line section in the distribution system and $P_{T,Loss}$ and $Q_{T,Loss}$ are total active and reactive power losses in the distribution system, respectively.

2.3.2. Operational constraints

The magnitude of bus voltage for all buses and current magnitude for all branches in distribution system should be maintained within the acceptable range. These constraints are expressed as follows:

$$V_{min} < |V_i| < V_{max} \qquad (12)$$

$$|I_i| < I_{i,max} \qquad (13)$$

Where $|V_i|$ is the voltage magnitude of bus i, V_{min} and V_{max} are minimum and maximum bus voltage limits, respectively, $|I_i|$ is current magnitude, and $I_{i,max}$ is maximum current limit of branch i.

2.4. Load model

Practical voltage-dependent load models, i.e., residential, industrial, and commercial ones, were considered in this paper. The load models can be mathematically expressed as [17]:

$$P_i = P_{oi} \left(\frac{V_i}{V_{oi}}\right)^{\alpha} \qquad (14)$$

$$Q_i = Q_{oi} \left(\frac{V_i}{V_{oi}}\right)^{\beta} \qquad (15)$$

where, V_i is the voltage at bus i, V_{oi} is the nominal operating voltage at bus i, P_i and Q_i are active and reactive power for load point i with bus voltage V_i, respectively, P_{oi} and Q_{oi} are the active and reactive power for load point i with bus voltage V_{oi}, respectively, and α and β are real and reactive power exponents, respectively. In the constant power model conventionally used in power flow studies, $\alpha = \beta = 0$ is assumed. The values of the active and reactive exponents used in this paper for industrial, residential, and commercial loads are given in Table 1 [17].

In practical situations, loads are the mixtures of different load types, depending on the nature of the area being supplied. Thus, in this study, three different types of load consisting of residential, industrial, and commercial loads are considered, in which every bus of the system had one type of load. On the other hand, distribution system load varies in different seasons of the year. Thus, in this paper, load condition is considered in three stages as low level for summer night period, medium level for summer day and winter night periods, and peak-load-level for winter day period. Load levels in different periods of the year for determining size and location are presented in Fig. 1 [17].

Table 1. Typical load types and exponent values.

Season	Load type	Day		Night	
		α	β	α	β
Summer	Residential	0.72	2.96	0.92	4.04
	Commercial	1.25	3.50	0.99	3.95
	Industrial	0.18	6.00	0.18	6.00
Winter	Residential	1.04	4.19	1.30	4.38
	Commercial	1.50	3.15	1.51	3.40
	Industrial	0.18	6.00	0.18	6.00

Load duration data is listed in Table 2.

Table 2. Load duration data.

Level	Network situation	Duration time (h)
1	Low load	2190
2	Medium load	4380
3	Peak load	2190

It should be noted that power loss and ENS index for each load level are calculated considering its duration time. In other words, the sum of power loss

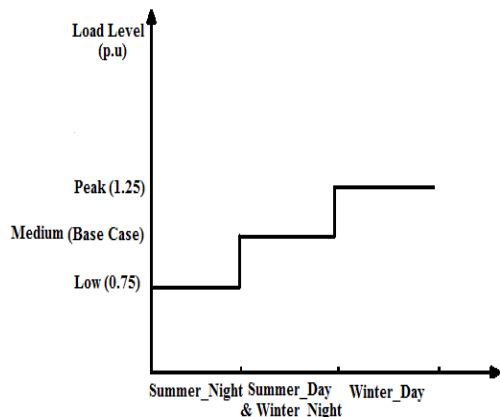and ENS index for three load levels is considered in the objective function.

Fig. 1. Load levels in different periods of the year.

3. PROPOSED HYBRID OPTIMIZATION ALGORITHM FOR CAPACITOR ALLOCATION

3.1. Genetic algorithm

GA is an effective search technique for solving the optimization problem, which provides a solution for an optimization problem using the population of individuals representing a possible solution [18]. Each possible solution is termed a "chromosome". New points of the search space are generated through GA operations, known as reproduction, crossover and mutation. These operations consistently produce fitter offspring through successive generations, which rapidly lead the search to global optima.

3.2. Particle swarm optimization algorithm
3.2.1 Classical approach abstract

The PSO method is an optimization technique which is motivated by the social behaviors of organisms such as fish schooling and bird flocking [19]. It provides a population-based search procedure, in which individuals called "particles" change their positions (states) over time. In a PSO system, particles fly around in a multidimensional search space. During the flight, each particle adjusts its position according to its own experience and the experience of neighboring particles, making use of the best position encountered by itself and its neighbors. The swarm direction of a particle is defined by the set of particles neighboring the particle and its history experience.

3.2.2. Binary particle swarm optimization

In order to solve optimization problems in discrete search spaces, Kennedy and Eberhart (1997) developed a binary version of PSO [20]. In this version, the particle is characterized by a binary solution representation and the velocity must be transformed into the change of probability for each binary dimension to take the value of one.

3.3. Proposed hybrid algorithm

This searching technique is developed for optimal capacitor locating and sizing. The problem consists of two parts; the first part is to determine the optimal location of capacitor and the second one is to determine the optimal sizing of capacitor. Location of capacitor which is one of buses in the distribution system is an integer parameter. Therefore, an integer-based optimization algorithm such as GA is needed. In this paper, the structure of each chromosome coding for the location of capacitor used in GA is shown in Fig. 2.
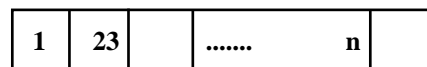

Fig. 2. Structure of each chromosome coding for location of capacitor used in GA.

It can be seen in this figure that the chromosome was formed of "n" bit strings, each of which corresponded to a system bus for capacitor installation. It should be noted that each bit showed the situation of the capacitor bank installation on the specific bus so that the values of 0 and 1 demonstrated the absence and presence of capacitor at the specific bus, respectively. The location of capacitor determined by GA is used in BPSO algorithm to optimize the sizing of capacitor. BPSO has the fast convergence ability that is a suitable method for a large iterative and time-consuming problem. Fig. 3 shows the structure of the particles used for BPSO.
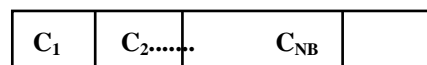

Fig. 3. Structure of a particleused for BPSO.

It is observed in Fig. 3 that this particle is composed of NB cells with the value of C_i, which is the rating of capacitor (NB is the number of

candidate buses determined by GA.). In fact, each candidate bus for capacitor installation determined by GA is assigned by a cell in BPSO and the rating of the capacitor at the relative candidate bus is the value of the corresponding cell.

For example, C_i is the rating of the capacitor installed at bus i.

The parameters of GA/BPSO method used for solving the optimization problems are presented in Table 3. Flowchart of the GA/BPSO method for optimal location and sizing of capacitor is shown in Fig. 4 in the following steps:

1. Initializing: Set the time counter $t=0$ and randomly generate "n" chromosomes which represent "n" initial candidates for the location of capacitor.

2. Calculating fitness function using BPSO: Evaluate each chromosome in terms of determining the optimal sizing of capacitor.

- Initialize particle population for the sizing of capacitor.
- Calculate the objective function.
- Determine the minimum value of the objective function as the overall, global best of the group and record the best candidate of particle for the sizing of capacitor.
- Update velocity (v) and position parameters of BPSO.
- Check the stop criterion.

3. Time updating: Update the time counter $t=t+1$.

4. New population: Create a new population for the location of capacitor using the following operations by GA: Selection - Crossover – Mutation.

5. Calculating fitness function using BPSO and time updating.

6. Checking the stop criterion.

4. TEST RESULTS

Two standard distribution systems consisting of 33-bus and 69-bus test systems are used to validate the effectiveness of the proposed optimization algorithm. For this purpose, the GA, BPSO, and proposed GA/BPSO algorithms are used for determining the optimal sitting and sizing of capacitor in the test systems, the results of which are compared and discussed in this section. For calculating the reliability indices and determining optimal capacitor placement, it is assumed that the section with the highest resistance had the biggest failure rate of 0.5 f/year and the section with the smallest resistance had the least failure rate of 0.1 f/year [13]. Based on this assumption, failure rates of other sections are linearly calculated proportional to these two values according to their resistance.

Furthermore, it is assumed that, if the reactive component of a section's current is fully compensated for, its failure rate would reduce to 85% of its uncompensated failure rate [13]. Also, for partial compensation, the failure rate is calculated using (6). Moreover, in both test systems, it was assumed that there is only one breaker at the beginning of the main feeder and also there is one sectionalizer at the beginning of each section.

Besides, for each line, the repair time and total isolation and switching time are considered 8 and 0.5 h, respectively. Also, other components such as transformers, busbars, breakers and disconnects are assumed to be fully reliable in this paper. The parameters of the test systems are listed in Table 4.

Table 3. GA/BPSO, GA and BPSO parameters.

| Method	Pop. size		Selection method	Crossover	Mutation		Algorithm stopcriterion
GA/BPSO	Chromosome	30	Normalized geometric selection	Simple Xover	Binary mutation		Maximum number of generation(200)
	Particle	20					
GA	40		Normalized geometric selection	Simple Xover	Binary mutation		Maximum number of generation(300)
BPSO	30		C_1	C_2	r_1	r_2	Maximum number of generation(250)
			2	2	1	1	
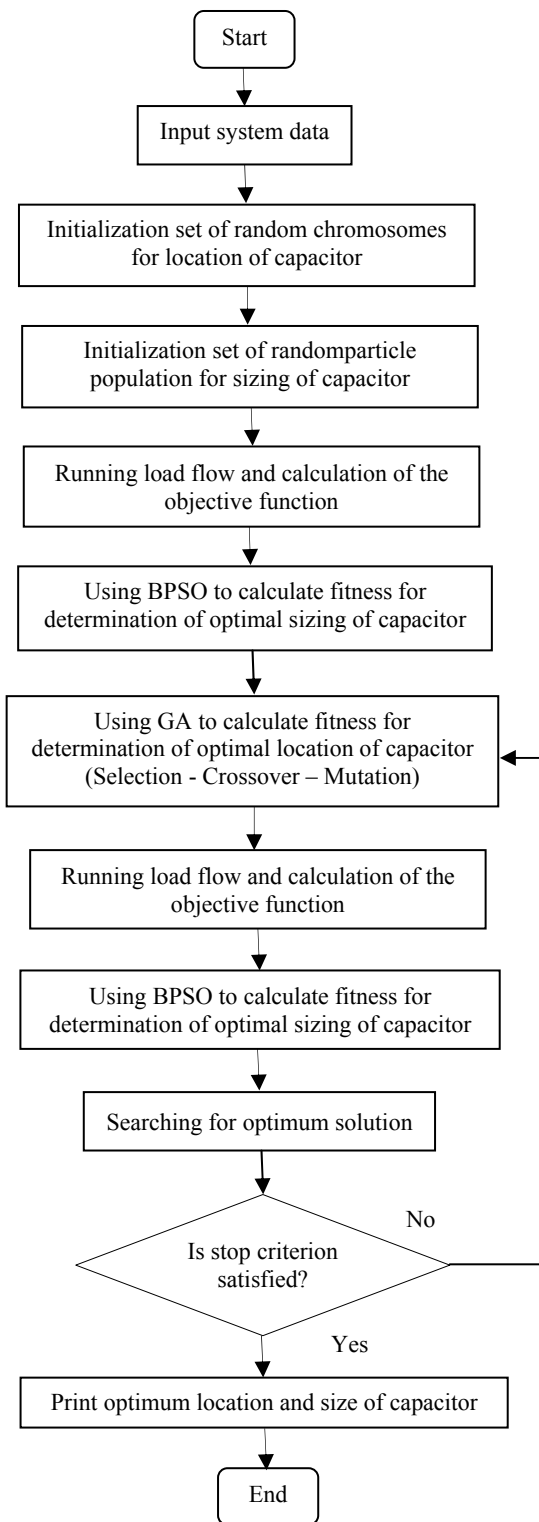
Fig. 4. Flowchart of the GA/BPSO method for optimal sitting and sizing of capacitor.

Table 4. The value of constant coefficients [21], [22].

Parameter	Value
K_e	0.06
K	0.1
K_c	3

4.1. 33 Bus radial distribution system

Figure 5 shows the single line diagram of the 12.66 kV, 33-bus, 4-lateral radial distribution system. Total load of the system in the base case is (3715+ j2300) kVA. The load flow data of the system were taken from [23] and load type data of each bus are presented in Table A.1. in the Appendix.
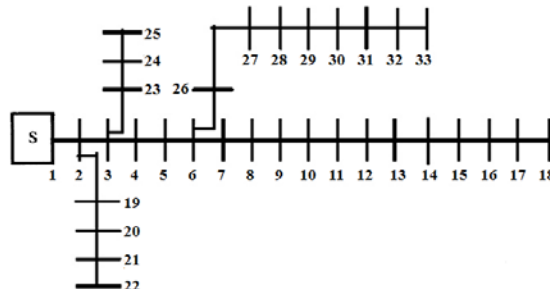

Fig. 5. Single line diagram of 33-bus radial distribution system.

Results of the optimal placement of capacitor banks using GA, BPSO, and GA/BPSO algorithms in 33-bus test system are presented in Table 5.

Table 5. The results of optimal placement of capacitor banks in 33-bus test system.

Method	Bus no.	Size(kVAr)
GA/BPSO	5	972.8
	8	101.4
	21	101
GA	2	140.6
	3	876.2
	5	129.6
BPSO	2	305.6
	5	952.5
	30	127.9

In this case, optimal number of capacitor is obtained to be three. Table 6 demonstrates the effects of optimal capacitor placement by GA, BPSO, and GA/BPSO algorithms on active power loss, reactive power loss, ENS, and total annual cost in 33-bus test system. It can be seen from this table that determination of optimum size and location of capacitors by GA, BPSO, and GA/BPSO algorithms had a considerable effect on active and reactive power loss, ENS, and total annual cost. For example, it is observed in Table 6 that the installation of capacitors decreased total cost from 45884.5 $/year in the base case to 42404.3 $/year,

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Table 6. The results of optimal placement of capacitor banks using GA, BPSO and GA/BPSO algorithms in 33-bus test system.

Case		Total Cost ($/year)	ENS (kWh/year)	$P_{T,Loss}$ (kW)	$Q_{T,Loss}$ (kVAr)
Before installation		45884.5	57286	76.4	50.7
After installation	GA/BPSO	42404.3	53454	63.8	42.2
	GA	43953.5	54568	66.7	44.3
	BPSO	43119.4	53755	63.9	42.3

Table 7. The percentage of loss reduction, reliability improvement and total cost reduction using three optimization algorithms in 33-bus test system.

	Total Cost (%)	ENS (%)	$P_{T,Loss}$ (%)	$Q_{T,Loss}$ (%)
GA/BPSO	7.5847	6.6892	16.4921	16.7653
GA	4.2084	4.7446	12.6963	12.6232
BPSO	6.0262	6.1638	16.3612	16.5680

43953.5 $/year, and 43119.4 $/year by GA/BPSO, GA and BPSO algorithms, respectively. Moreover, it is observed in Table 7 that optimum capacitor installation by combined GA/BPSO algorithm caused more loss reduction, reliability improvement, and total cost reduction compared to GA and BPSO algorithms. For example, it can be seen in Table 7 that capacitor installation caused 7.5847%, 4.2084%, and 6.0262% reduction in total cost by GA/BPSO, GA and BPSO algorithms, respectively, compared to the base case.

In addition to minimizing power losses and reliability improvement in distribution networks, proper capacitor planning can improve the overall network voltage profiles. For instance, Fig. 6 shows the voltage profile improvement before and after capacitor installation by GA/BPSO, GA and BPSO algorithms for summer day and winter night load levels in the 33-bus system.
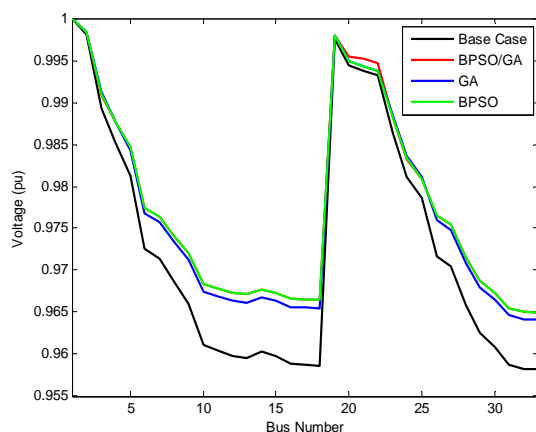
69-bus radial distribution system with the total load of 3.80 MW and 2.69 MVar in the base case is considered as another test system, which has 7 laterals, 69 buses, and 68 branches. The load flow data of the system are taken from [24] and load type data of each bus are presented in Table A.2. in the Appendix. The results for the optimal number of four capacitor banks of GA/BPSO, GA, and BPSO algorithms are demonstrated in Table 8.

Table 8. The results of optimal placement of capacitor banks in 69-bus test system.

Method	Bus no.	Size(kVAr)
GA/BPSO	17	110.5
	31	95.2
	47	368.4
	61	76.2
GA	2	300
	14	399.2
	29	357.1
	52	237.6
BPSO	28	140.7
	36	132.5
	38	336
	52	277.4

The effects of optimal capacitor placement by GA, BPSO and GA/BPSO algorithms on active and reactive power losses, ENS, and total annual cost in 69-bus test system are presented in Table 9. Also, improvement percent of these parameters after capacitor installation is shown in Table 10.

Similar to 33-bus test system, it is illustrated in Tables 9 and 10 that optimal capacitor placement by GA/BPSO, GA and BPSO algorithms led to loss

Fig. 6. Voltage profile for Summer-Day and Winter-Night load levels in 33-bus system.

Table 9. The results of optimal placement of capacitor banks using GA, BPSO and GA/BPSO algorithms in 69-bus test system.

Case		Total Cost ($/year)	ENS (kWh/year)	$P_{T,Loss}$ (kW)	$Q_{T,Loss}$ (kVAr)
Before installation		63630.7	92837	103.4	50.1
After installation	GA/BPSO	20507.9	85180	19.1	11.1
	GA	22952	85583	20	11.9
	BPSO	21068.7	85644	18.73	11

Table 10. The percentage of loss reduction, reliability improvement and total cost reduction using three optimization algorithms in 69-bus test system.

	Total Cost (%)	ENS (%)	$P_{T,Loss}$ (%)	$Q_{T,Loss}$(%)
GA/BPSO	67.770	8.247	81.528	77.844
GA	63.929	7.813	80.657	76.247
BPSO	66.889	7.748	81.885	78.044

reduction and reliability improvement of 69-bus distribution system. However, it is observed in these tables that optimum capacitor installation by combined GA/BPSO algorithm caused more total cost reduction than GA and BPSO algorithms.

Results of Table 10 show that capacitor installation caused 67.770%, 63.929% and 66.889% reduction in total cost by GA/BPSO, GA and BPSO algorithms, respectively, compared to the base case.

Also, optimal placement of capacitors led to the voltage improvement of 69-bus distribution system. Figure 8 shows the voltage profile improvement before and after capacitor installation by GA/BPSO, GA, and BPSO algorithms for summer day and winter night load levels.

Totally, while comparing the effect of capacitor installation in the two test systems, it can be concluded that optimal sitting and sizing of capacitor in the systems have considerable effects on loss reduction, reliability enhancement, and voltage

improvement. In particular, in combined GA/BPSO algorithm, the search space is reduced and a tight distribution was obtained for the search results. The combined method is converted into a solution in the minimum number of iterations and the BPSO was the least. However, the running time for the BPSO is faster than the other two and was the least for GA.

4.3. Comparing variances of objective functions calculated by GA/BPSO, GA, and BPSO algorithms

Figure 9 shows the variance of the objective function (per unit) determined by three optimization algorithms in 33-bus and 69-bus test systems. The variances are calculated for the 30 initial populations. It can be seen in this figure that the variances of the objective function for GA and BPSO are 0.0831 and 0.032 and also 0.0826 and 0.0317 in 33-bus and 69-bus systems, respectively, while variances of the objective function for
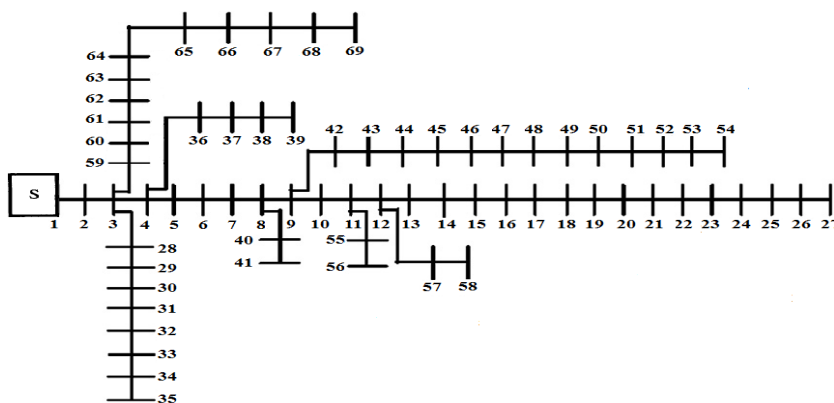

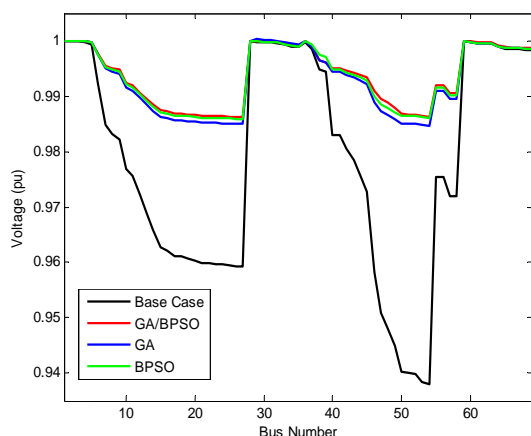**Fig. 7.** Single line diagram of69-bus radial distribution system.

Fig. 8. Voltage profile for Summer-Day and Winter-Night load levels in 69-bus system.

GA/BPSO are 0.0017 and 0.0016 in 33-bus and 69-bus systems, respectively. Very small variance in the results of GA/BPSO algorithm. demonstrated that this method led to more uniformity and reliable results than GA and BPSO algorithms.
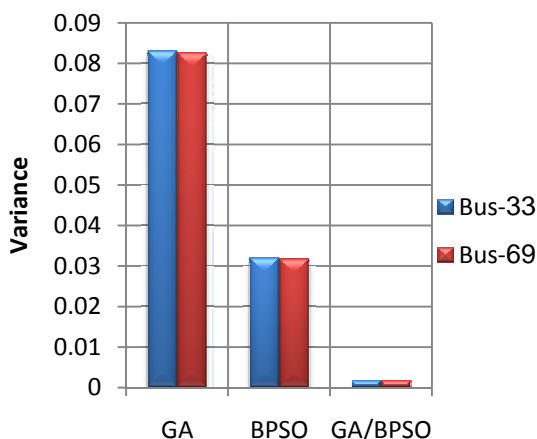

Fig. 9. Variances of objective function calculated by GA/BPSO, GA and BPSO algorithms.

5. CONCLUSIONS

In the paper, a combined optimization method is presented for determining the optimum location and capacity of shunt capacitors in distribution systems. The proposed method is based on the combination of GA and BPSO algorithms for optimal sitting and sizing of capacitors. Also, the objective function consisted of the cost of energy loss, reliability and investment cost of the capacitor banks for the optimization problem. In addition, the impacts of

types of customers for load modeling, voltage-dependent load models and annual load duration curve are considered for the optimization problem. The proposed GA/BPSO method is implemented on 33-bus and 69-bus test systems for the purpose of loss minimization and reliability improvement considering cost factor. Also, the obtained results are compared with those of GA and BPSO algorithms. Simulation results demonstrate that GA, BPSO and the proposed method (GA/BPSO) may result in considerable loss reduction, reliability and voltage profile improvement and annual cost reduction. However, the results indicate that optimum capacitor installation by combined GA/BPSO algorithm is caused more total cost reduction than GA and BPSO algorithms. In addition, the proposed method has less variance than GA and BPSO algorithms. Therefore, the results obtained from this method are more reliable than those of GA and BPSO algorithms in terms of finding optimum solutions.

APPENDIX

Table A.1. Load type dataof 33-bus system.

Bus no.	Customer type	Bus no.	Customer type
1	Commercial	18	Commercial
2	Commercial	19	Commercial
3	Commercial	20	Commercial
4	Residential	21	Commercial
5	Residential	22	Commercial
6	Industrial	23	Industrial
7	Industrial	24	Industrial
8	Residential	25	Residential
9	Residential	26	Residential
10	Residential	27	Residential
11	Residential	28	Commercial
12	Commercial	29	Industrial
13	Commercial	30	Commercial
14	Residential	31	Industrial
15	Residential	32	Commercial
16	Residential		
17	Residential		

Table A.2. Load type dataof 69-bus system.

Bus no.	Customer type	Bus no.	Customer type
1	-	35	-
2	-	36	Industrial
3	-	37	Commercial
4	-	38	Commercial
5	Residential	39	Commercial
6	Residential	40	Industrial
7	Residential	41	Industrial
8	Residential	42	Industrial
9	-	43	Industrial
10	Residential	44	-
11	Residential	45	-
12	Residential	46	-
13	Residential	47	Residential
14	-	48	-
15	Residential	49	Industrial
16	Residential	50	Industrial
17	Residential	51	-
18	-	52	Commercial
19	Industrial	53	Commercial
20	Commercial	54	Industrial
21	Commercial	55	Industrial
22	-	56	Residential
23	Commercial	57	Residential
24	-	58	Commercial
25	Residential	59	Commercial
26	Residential	60	-
27	Residential	61	Industrial
28	Residential	62	Industrial
29	-	63	Industrial
30	-	64	-
31	-	65	Industrial
32	Industrial	66	-
33	Residential	67	Residential
34	Industrial	68	Residential

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