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Analysis of Structural Reliability of Complex Coefficients Fractional-Order System Using Plane Transformation

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Abstract- *The main goal of the paper is to achieve the structural reliability of the failure components in the system that can be modelled as a Transfer Function (TF). The classical reliability of the power system has been a major field for research in the past decades, which has resulted in the reliability of the power grid by integrating the failure rates of the system components. As a result, a gap analysis is carried out by modelling the failure components into TF, and a comparison of structural and classical reliability is explained in the paper. The paper expands on methodology of the mapping technique for transforming a system from one domain to another. By doing so, the transformation of the Complex Coefficients Integer Order (CCIO) and the Complex Coefficients Fractional Order (CCFO) system transfer function becomes the Non-complex Coefficients Integer Order (NCCIO) in nature. Therefore, the root locus plot for the transformed system is observed as the symmetrical structure about the real axis. Therefore, the plane transformation becomes advantageous in the field of structural reliability analysis. The root locus plot for the transformed system into a w-plane becomes reliable as per the symmetrical structure. The reliability index Loss of Load Probability (LOLE) has been evaluated with different forced outage rates of the system components to analysis the classical reliability of the system.*

Keyword: CCFO system, CCIO system, LOLE, reliability index, root locus

1. INTRODUCTION

1.1. Aim and motivation

The motivation for this observation includes the reliability states of different complex coefficients and fractional-order systems. Complex coefficients in the characteristics equation of a transfer function make the Root Locus (RL) plot unsymmetrical along the real axis. This asymmetrical behaviour of the graph leads to unreliable scenarios of the RL plot. The remedial operation for a tackle with such an issue discusses in this paper. The transformation of the domain of the existing TF into another domain is mapped and the RL plotted for the new transformed TF becomes NCCIO in nature. Such a system's graphic structure will be symmetrical along the real axis [1]. Therefore, the reliable RL plot can be obtained through the required plane transformation. The recent trend in system behaviour can easily be modeled in fractional order (FO) differential equation. Therefore, analysis of the FO system has become the research trend from the past few years [2, 3]. The analysis of several control strategies for system behavior can be observed through the RL plot. The most common use of the RL plot is to obtain the stability condition of the system in different operating zones. The reliability study of such a system becomes a more thinkable issue for the modern future [4, 5]. Now, the reliability can be understood through several aspects such as energy indices, failure scenarios, structural symmetry of various system analyses, etc [6, 7]. Because RH-criteria cannot be used to analyse the stability and reliability of a root locus plot, the analysis for stability and reliability of a root locus plot becomes more important for FO systems.

1.2. Backgrounds

The transfer function contains the complex coefficient, fractional order, and irrational characteristics due to the large-scale distributed elements of the electrical network. The stability analysis of the irrational transfer function based fractional order controller is elaborated with several advantages like controller robustness and stability constraints including real, complex, and infinite crossing boundaries with numerical examples [8]. Micro power grids have different fractional controller designs for distributed generation and remote control for providing the system stability and robustness within a stipulated

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delay margin. Microgrid time-delayed modelling is integrated with the controller to prevent system stability in the frequency domain during the time delay margin [9, 10]. A fractional order transfer function is formulated with electrical circuit parameters for testing the realizability influences on it. The complex analysis-based open mapping method is used to assess the realizability of polor plots theorem [11]. Transmission line switching plays an effective contribution to the optimal operation of the power system network to make the system reliable and secure. The changes in the transmission network are acceptable during the minimum loading condition, and by doing so, the network transfer characteristics follow unreliable structural behaviour. Therefore, it is necessary to maintain the classical and structural reliability [12, 13]. Synchrophasor technology provides the integration of several components into the existing power grid network with the most reliable and stable operation. This could lead to tracing the network's transfer characteristics in real time in order to model them for reliability. Generally, the root locus characteristics were used for stability analysis, but the plane transformation of the transfer characteristics may contribute to the structural reliability of the modelled network during uncertainties. The control strategies are applicable in designing the controller based on the transfer characteristics following complex coefficient, fractional order, and irrational behaviour. Grid synchronisation is used in various scenarios to analyse small signal stability by providing an efficient fractional order controller [14]. Transmission network resilience assessment is gaining more research due to cyber threats or abnormalities that occur with grid operations. Therefore, grid reliability needs to be maintained in such cyber operations. The resilience analysis of the transmission grid with a hybrid approach, including a reliability matrix and a local topological framework, is presented in the paper [15, 16]. For the real-time modelling and assessing the dynamic behaviour of the smart grid, a model of reduced order transfer function is implemented based on Jordan's continued fraction expansion. The modelling of such a design controlled the stability performance of the grid by integrating several converters into the system. Therefore, monitoring the reliability of such control designs becomes a focus area for future perspective and goals [17]. The complex coefficients operation in the TF makes the RL graphical representation unsymmetrical in the s-plane. The graphical structure of the transformed system has the same behavior as in the s-plane. The fractional-order system will not create any asymmetry in the system. The transformation of such a system into an integer order system is due to the easier way to plot such a system RL. However, the involvement of complex coefficients in the characteristics equation makes the system unreliable. As a result, transforming any plane into a w-plane makes the system more reliable because the transformed system is no longer based on complex coefficients. As Routh-Howritz (RH) criteria are not applicable for fractional-order systems to predict the stability of the system. Therefore, the RL plot for the FO system becomes more useful in the study of stability and reliability [18, 19]. The FO system's breakaway point, asymptotes, and stability conditions will be different in comparison with the integer order system. The transfer function q-domain transformation has different stability conditions than the s-plane, but the system stability and instability scenarios are the same in both planes. If the system is stable or unstable in the s-plane, then it will also be stable or unstable correspondingly in the q-plane, although the stability condition will change in both the domains. Fig. 1 shows the block diagram of conventional power system reliability analysis using classical and structural approaches [12, 20].

Structural reliability is measured through the transfer function characteristics. When the parameters of the lines, such as resistance, inductance, and capacitance, are changed, the characteristics of the transfer function change. The transfer function may follow the complex coefficient fractional-order characteristics by changing the states of the transmission line modeling [17]. Therefore, reliability monitoring of the system during this condition should be modeled for healthy operation. The symmetrical plot about the real axis of the root locus drawn from the transfer function leads to a stable characteristics zone followed by reliable structural behavior [21, 22].

1.3. Contribution

The reliability of the RL plot is a more focused observation because past research shows the several techniques associated with TF to predict the stability of the system. Therefore, any operation in the characteristics equation will affect the reliability condition. We need to discuss this more [23]. Here, reliability prediction is made through the structural or graphical representation of a system that may follow the symmetrical operation for the corresponding plot. RL is a plot having symmetrical nature along the real axis. Therefore, if any disturbance will occur in the system, which leads to asymmetry of the system. The system plot becomes unreliable in such scenarios.

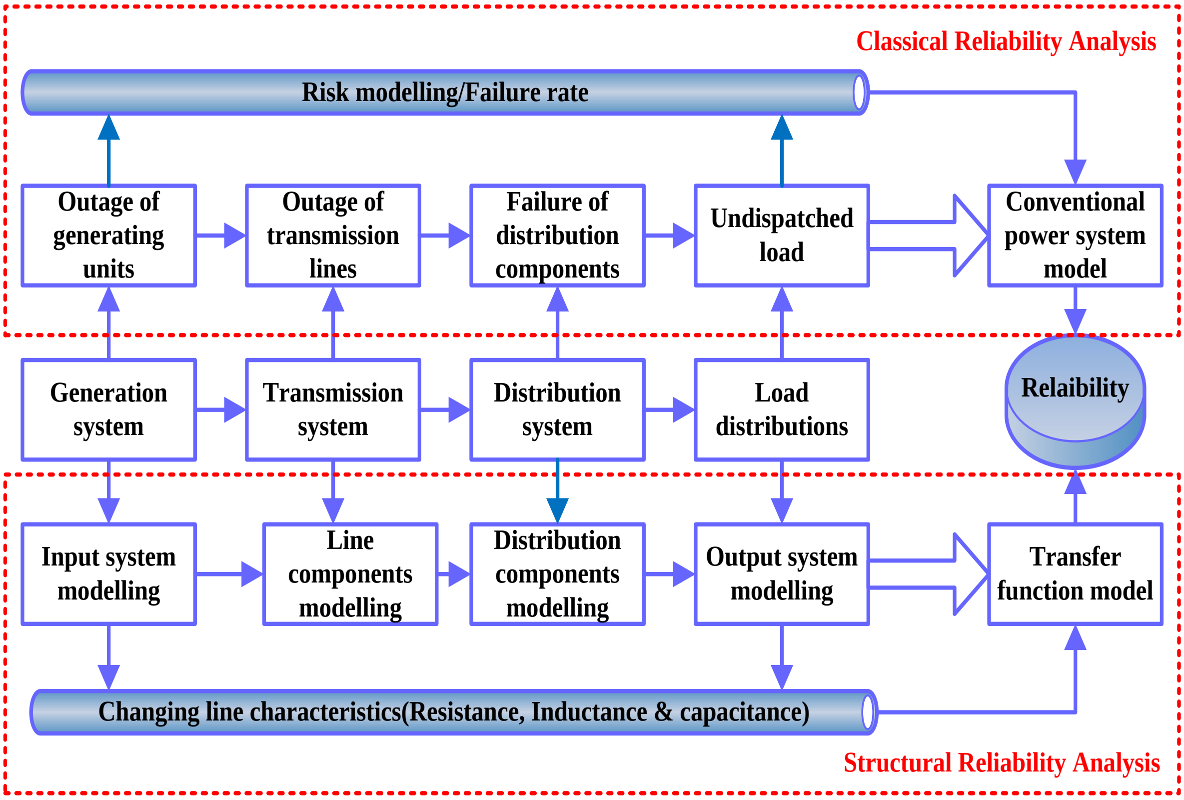

Fig. 1. Block diagram representing the modelling of structural and classical reliability

The structural reliability representation of the electrical power system has been a gap in research for several years due to the existence of stability analysis through structural representation by using bode plot, root locus plot, and polar plot. The paper contributes to the structural reliability analysis during the network components' failures by modelling the failure through transfer characteristics. The root locus plot of the changes is analysed by transforming the transfer characteristics from one plane to another plane. Fractional order system RL plot can be simply analyzed in different modeling planes instead of s-plane [24]. The mapping of the system from one domain to another domain is advantageous whenever there is an issue with the existing domain. The CCFO system has both complex coefficients and fractional order, so such a system RL plot can be drawn through two plane transformations. First, transform s-plane into q-plane that converts the CCFO system into a CCIO system, and further q-plane is transformed into w-plane that results in an NCCIO system. The analysis becomes reliable in the w-plane due to the symmetrical plot of RL about the real axis [25].

The paper is systematised as follows. Section 1 gives the basic introduction and motivation of this paper. Section 2 represents the methodology to transform the system from one to another domain through mapping. Section 3 contains a results analysis of various complex coefficients and fractional-order systems through-plane transformation. Section 4 shows the conclusion part of the paper.

2. METHODOLOGY

2.1. Root locus for complex coefficients integer order (CCIO) system

A method is proposed to plot the Root Locus (RL) of the CCIO system by transforming the Transfer Function (TF) into Non-complex Coefficients Integer Order (NCCIO) system. The root locus of the CCIO system is plotted in the s-plane and transform NCCIO system RL is plotted into w-plane. As the RL graph is symmetrical about the real axis. The presence of complex coefficients in the TF leads to an unreliable RL plot because complex coefficients in characteristics equation outcomes into an unsymmetrical plot about the real axis. Therefore, the system transformation from s-plane to w-plane is proposed which makes the system reliable by upholding the symmetrical RL about the real axis [26]. The mapping of s-plane into w-plane is determined by the following steps. S-domain $s=|s|e^{j\theta}$ can be mapped in w domain i.e. $f:s\rightarrow w$ by putting $s=\frac{w}{i}$ so we get w-plane RL mapping where, $w=|w|e^{j(\theta+\pi/2)}$. The clear observation shows the complex w-plane is $\pi/2$ angular rotation of the s-plane RL graph. Thus, by doing so, the new RL of the w-plane becomes reliable because of the symmetrical status of the graph about the real axis. This transformation is independent of any stability analysis. It will only change the reliability of the graph from its original plot. The method is more useful for dealing with complex coefficient-based transfer functions. The mapping of s-domain into w-domain is shown in Fig. 2.

2.2. Root locus for complex coefficients fractional order (CCFO) system

The RL of the fractional order system is very difficult to plot in the s-plane. Therefore, a fractional order system can be converted into an integer-order system by transforming the s-plane into the q-plane. The further q-plane system is transformed into a w-plane as its transfer function consists of complex coefficients. The result of the two-plane transformation is a Non-complex Coefficients Integer Order (NCCIO) or a Non-complex Coefficients Non-fractional Order (NCCNFO) system. The mapping of s-plane to q-plane ($f:s\rightarrow q$) is attained through putting $q=s^{1/v}$ which gives the integer-order transfer function. Again the system is transformed from q-plane to w-plane ($f:q\rightarrow w$) by putting $q=\frac{w}{i}$. The new domain is easy to analyze because TF having an NCCIO system. S -domain RL plot $s=|s|e^{j\theta}$ is mapped in q-domain by plotting its RL for $q=|q|e^{j\varphi}$. Here, the angular shift is changes from θ to ϕ which is proportional to the ratio of the fractional factor of the initial angular range. The transformation of s-plane into q-plane affects the stability condition for the new plane [27, 28]. The w-plane can make the system reliable under the complex coefficient situation of TF. Therefore, the system that requires both transformations simultaneously can change the stability and reliability conditions for the new transformed system [29, 30]. The two-plane transformation from s-plane to q-plane and further q-plane to w-plane are shown below in Fig. 4.

Riemann-Liouville linear fractional-order derivative system TF can be expressed in the equation (1-2).

$$G(s)=\frac{b_g s^{g/v}+b_{g-1}s^{g-1/v}+............+b_1 s^{1/v}+b_0}{a_h s^{h/v}+a_{h-1}s^{h-1/v}+............+a_1 s^{1/v}+a_0} \quad (1)$$

$$-\pi\le\arg(s)\le\pi \text{ Where, } \theta=\arg(s) \quad (2)$$

Where b_m and a_n are constant $(m=0,1..........g)$ and $(n=0,1,.......h)$ such that $g\le h$ and $v>1$

Now, the transformation of TF into q-plane can be performed by substituting $q=s^{1/v}$ then the transformed TF can be expressed as shown in equations (3-4).

$$G_q^t(q)=\frac{b_g q^g+b_{g-1}q^{g-1}+............+b_1 q^{1/v}+b_0}{a_h q^h+a_{h-1}q^{h-1}+............+a_1 q^{1/v}+a_0} \quad (3)$$

$$-\frac{\pi}{v}\le\arg(q)\le\frac{\pi}{v} \text{ where, } \phi=\arg(q) \quad (4)$$

2.3. Stability condition in q-plane

The transformation of TF into a q-plane is now plotted through RL graph analysis. The instability range is also plotted to predict the stability of the graph. The instability range for transformed TF will be $-\frac{\pi}{2v}\le\arg(q)\le\frac{\pi}{2v}$ which is in $-\frac{\pi}{2}\le\arg(s)\le\frac{\pi}{2}$ for s-plane. Figure 3 shows the stability and instability regions for the fractional-order system The stability zone for the fractional-order system will increase due to the fractional coefficients, but the reliability of the system will decrease due to these coefficients. Therefore, the plane transformation approach is presented to make the system reliable and stable along the real axis. As the symmetrical behavior of a graph about the real axis makes the system reliable, transforming the complex coefficient fractional-order system into a different plane makes the system approach the stability and reliability zone [31]. Therefore, stability condition for transformed TF into q-plane are defined as shown in equation (5):

$$|\arg(q_i)|>\frac{\pi}{2v} \quad (5)$$

Where, q_i is a root of the polynomial of the characteristics equation.

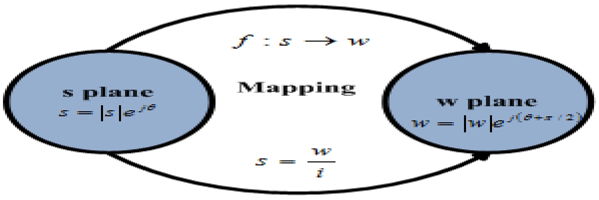

Fig. 2. Mapping of s- plane into w-plane domain

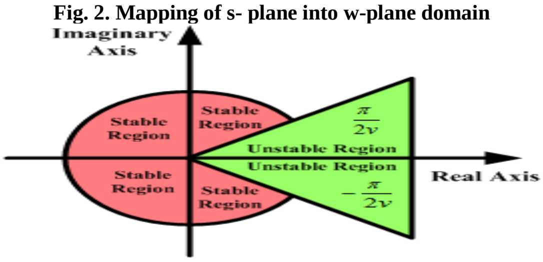

Fig. 3. Stability and unstability regions for fractional order system

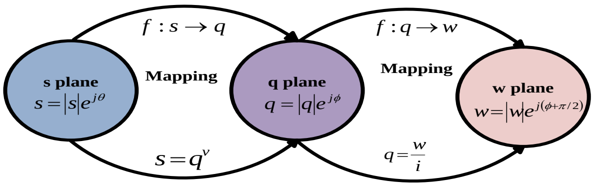

Fig. 4. Mapping of s-plane into q-plane and further q-plane into w-plane

2.4. Reliability condition in w-plane

The transformation of s-plane into w-plane is the NCCIO system. The RL of such a transformed TF should be symmetrical about the real axis for a reliable RL plot. Otherwise, it will be unreliable. The complex coefficients that exist in the system cause the unreliable RL plot because of its unsymmetrical behavior along the real axis.

Steps to plot RL of CCFO system:

- Express the power of fractional order TF system into the rational form.
- Transformation of s-plane into q-plane by substituting $q = s^{1/v}$. The transformed TF is now Integer Order (IO) system.
- Plot the RL of transformed IO TF into q-plane.
- The plot is unreliable due to the presence of complex coefficients.
- Further transformation of q-plane into w-plane is performed by substituting $q = \frac{w}{i}$
- The new w-plane is easy to plot the RL of the NCCIO system. This will lead to a reliable RL plot.
- Plot the unstable region in the new plane.
- Check the stability and reliability through graph symmetrical behavior about the real axis.

Evaluation of $K_{marginal}$: The transition of RL plot from stable to unstable region or unstable to stable region causes RL intersection with the imaginary axis. The value of gain K at such point of intersection is known as $K_{marginal}$. Consider a fractional-order system in generalized form

$$G(s) = \frac{b_1 s^{g_1} + b_2 s^{g_2} + ... + b_n}{a_1 s^{h_1} + a_2 s^{h_2} + ... + a_m} \tag{6}$$

Where, $b_1, b_2, ...b_n \; and \; a_1, a_2, ...a_m$ are constant having real positive values and $h_1 > g_1 \; and \; h_1 > h_2$. Characteristics equation for the fractional-order system having unity feedback system can be written as in equations (7-8):

$$1 + KG(s) = 0 \tag{7}$$

$$1 + \frac{K(b_1 s^{g_1} + b_2 s^{g_2} + ... + b_n)}{a_1 s^{h_1} + a_2 s^{h_2} + ... + a_m} = 0 \tag{8}$$

For the marginal stable point, RL must cut the imaginary axis at $s = jw$. By substituting the value and separating real and imaginary parts, we will get the value of $K_{marginal}$ which can be expressed as shown in equation (9):

$$K_{marginal} = \frac{a_1 w^{h_1} \sin\left(\frac{\pi h_1}{2}\right) + a_2 w^{h_2} \sin\left(\frac{\pi h_2}{2}\right) + ... + a_{m-1} w^{h_{m-1}} \sin\left(\frac{\pi h_{m-1}}{2}\right)}{b_1 w^{g_1} \sin\left(\frac{\pi g_1}{2}\right) + b_2 w^{g_2} \sin\left(\frac{\pi g_2}{2}\right) + ... + b_{n-1} w^{g_{n-1}} \sin\left(\frac{\pi g_{n-1}}{2}\right)} \tag{9}$$

2.5. Classical reliability analysis

A lot of work on the classical reliability of power systems has been published, including the risk modelling of components. Classical reliability is represented by several reliability parameters such as loss of load probability (LOLE), expected demand not supplied (EDNS), etc. Classical reliability is also known as quantitative reliability. Equations (10–11) show the mathematical expression of LOLP used to find the classical or quantitative reliability.

$$LOLE = \sum_{k=1}^{m} IP_k * D_k \tag{10}$$

$$LOLE = \sum_{k=1}^{m} \left(D_k - D_{k-1}\right) * CP_k \tag{11}$$

m is total generating units, IP_k is individual probability of k^{th} generating unit outage, D_k is duration of load loss during outage of k^{th} unit, CP_k is cumulative probability of k^{th} unit outage.

3. RESULTS ANALYSIS

3.1. Analysis of various complex coefficients and fractional order TF system

The nature of TF is studies in various graphical and analytical ways to gain the observation of systems in different response statuses. The stability analysis of the system by modeling it into the characteristics equation has various methods to solve and analyse it. The complex coefficients and fractional order exist in the system have some different way to analyze it. The most useful way to study the different status of such TF is through the transformation of the domain to another domain. The results of the analysis on various TF of the CCFO systems has been discussed using the different transformation operations.

- **Analysis of transfer function having complex coefficients (CCIO system)**

Consider a CCIO system having TF $G_1(s)$ as shown in equation (10)

$$G_1(s)=\frac{s^5+i}{3s^7+11s^3+5.5s+6i} \tag{10}$$

Transforming the above transfer function into w-plane by substituting $s=\frac{w}{i}$ then TF for a new domain will be an NCCIO system in nature and defined as in equation (11).

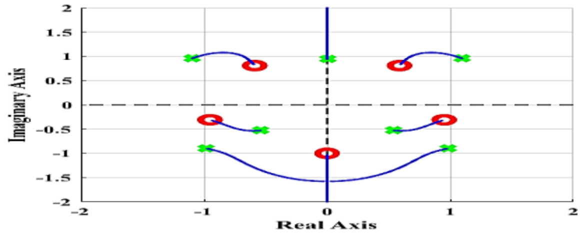

Fig. 5. RL plot for TF having complex coefficients drawn in s-plane

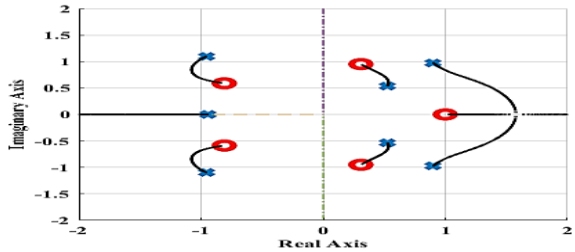

Fig. 6. RL plot for transformed TF   drawn in w-plane

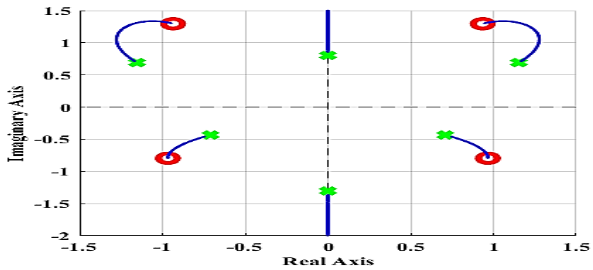

Fig. 7. RL plot for TF having complex coefficients drawn in s-plane

$$G_1^t(w)=\frac{w^5-1}{-3w^7-11w^3+5.5w-6} \tag{11}$$

As for the imaginary coefficients, involvement with the TF makes the unsymmetrical RL plot about the real axis. The unsymmetrical status of a graph leads to an unreliable system. Therefore, to change the status of TF having complex coefficients, a transformation of domain becomes an important method to wash out the problem associated with the existing plane. Now a CCIO system having TF $G_1(s)$ RL diagram is plotted as shown in Fig. 5. The plot is not symmetrical along with its real axis so an unreliable situation occurs with the RL graph of $G_1(s)$. Therefore, to change the status of the plot, the transformation of TF into a w-plane is performed. Now, the new TF has the nature of NCCIO. The RL of the transformed TF $G_1^t(w)$ is shown in Fig. 6. The RL plot is now symmetrical about its real axis as per the $\pi/2$ angular rotation in the new domain.

The symmetrical nature of the plot shows the reliability of the graph. The transformed domain of the transfer function will only change the reliable status of the graph instead of the stability status. The complex coefficients are involved in the TF in such a way that after its transformation there should be no complex coefficients remaining in the new domain. Otherwise, although the transformation is performed the existence of such complex coefficients always creates an unreliable situation in the system plot. The consider TF $G_1(s)$ is the odd degree in nature so it will leave with complex coefficients after its transformation in w-plane. Therefore, to washout the situation there must be a complex coefficients involvement with constants terms.

Consider a CCIO system having TF $G_2(s)$ as shown in equation (12).

$$G_2(s)=\frac{is^4+s^3+4i}{is^6+1.8s^3+0.2is^2+1.3i} \tag{12}$$

Now by transforming the above TF into w-plane by putting the mapping value $s=\frac{w}{i}$ then new NCCIO TF obtained which can be written as in equation (13):

$$G_2^t(w)=\frac{-w^4-w^3-4}{w^6-1.8w^3+0.2w^2-1.3} \tag{13}$$

Even degree of S always results the real value when domain transformation of s-plane into w-plane is performed by mapping the $s=\frac{w}{i}$. Therefore, TF $G_2(s)$ is a model such that it will have complex coefficients in its even degree order. The transformation plays a significant role in graph orientation to make it reliable. Fig. 7 shows the RL plot for the CCIO TF system. The plot is unstable and unreliable in its s-domain plane because of the unsymmetrical operation of RL along the real axis. The TF is taken for the study of the reliability status of the RL plot of the system. Instead of the instability of TF in its original form, the major concern of the paper is to make a reliable RL plot. After plane transformation in w-plane, new TF $G_2^t(w)$ is in nature of NCCIO.

Fig. **8** shows the plot of $G_2^t(w)$ which is now symmetrical about the real axis. Therefore, the transition from s-plane to w-plane becomes the change in status scenarios from unreliable to reliable. If we observe the stability status of the TF, there is the same status of instability after the transformation. The operation of mapping of the system from s-plane to w-plane gives the phase rotation of $\pi/2$. Therefore, this transformation is the change of its axis operation such that the initial graph is symmetrical about the imaginary axis, but now it is symmetrical about the real axis when it transforms into a w-plane.

- **Analysis of transfer function having complex coefficients and fractional order (CCFO system)**

Consider a CCFO system having TF $G_3(s)$ as shown in equation (14).

$$G_3(s)=\frac{2i}{s^{0.3}+i} \tag{14}$$

Now fractional-order systems are converted into non-fractional (integer) systems by substituting $q=s^{1/10}$ then the TF in q-domain is defined as in equation (15).

$$G_3^t(q)=\frac{2i}{q^3+i} \tag{15}$$

Further TF of q-domain is transformed into w-plane by substituting $q=\frac{w}{i}$ then-new TF of the corresponding domain can be described as in equation (16).

$$G_3^t(w)=\frac{2}{w^3+1} \tag{16}$$

The analysis of fractional-order TF is quite difficult to plot. The transformation of the fractional order system into an integer order system is advantageous and easy to plot. First, the transformed RL in the q-plane is plotted then it will map corresponds with the s-plane to judge the stability of the system. The presence of complex coefficients in the system makes the graph unsymmetrical. Therefore, one more transformation is performed from q-plane to w-plane to study the different statuses of the RL plot. Equation (14) shows the TF of CCFO system represented as $G_3(s)$, Equation (15) shows the TF of the transformed system in q-domain represented as, $G_3^t(q)$ and Equation (16) shows the TF of the transformed system in w-domain represented as $G_3^t(w)$. The fractional order system will change the stability condition for the new q-plane. Fig. 9 shows the RL plot of $G_3^t(q)$ in q-plane after transforming it by putting $q=s^{1/10}$. The graph clearly shows the status of the plot as stable and unreliable. The dashed dotted line shows the instability region for the transformed system. As from Fig. 9, it is observed that the RL branches lie in stable regions, so the plot becomes stable for such TF. However, due to unsymmetrical behavior along the real axis, the plot is unreliable. Therefore, one more transformation from q-plane to w-plane is performed which results from the plot shown in Fig. 10. The plot is now symmetrical about the real axis in the w-plane. The outcome of plot has a new operating status for stability and reliability.

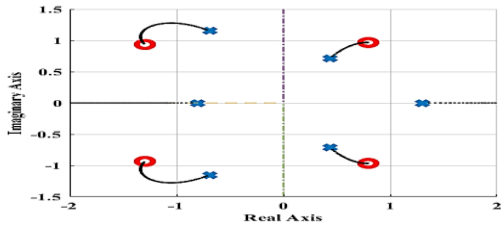

Fig. 8. RL plot for transformed TF drawn in w-plane

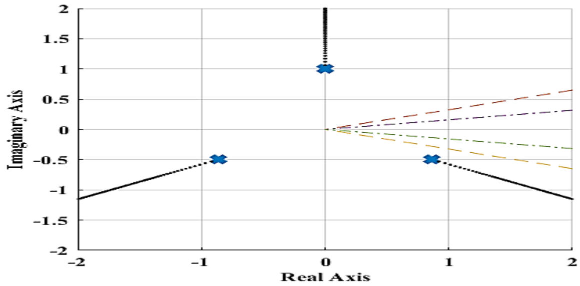

Fig. 9. RL plot for TF having CCFO in nature and the dashed dotted line shows the unstable region

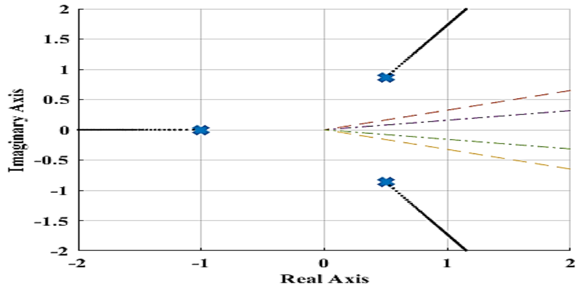

Fig. 10. RL plot for transformed TF into w-plane and the dashed dotted line shows the unstable region

By doing so, the RL plot becomes reliable for its operating range. Table 1 shows the initial states and transformation states of different CCFO systems. As the system, having complex coefficients that make the system unsymmetrical about the real axis, this will outcome in an unreliable plot. Therefore, to make the system orientation reliable, one plane transformation or two plane transformation becomes the focusing point for analysis of this paper. By transforming $G_3(s)$ in to $G_3^t(q)$, the following observation can be made for stability range: (Unstable region: $-\frac{\pi}{20}\le \arg(q)\le \frac{\pi}{20}$ as $(v=10)$)

Table 1: Transfer function states before and after transformation

Transfer function	Initial states		States after transformation	
	Stable	Reliable	Stable	Reliable
$G_1(s)$	No	No	No	Yes
$G_2(s)$	No	No	No	Yes
$G_3(s)$	Yes	No	Yes	Yes

Table 2: Comparison between the structural and classical reliability analysis

Structural reliability	Classical reliability
It is graphical based analysis.	It is quantitative based analysis.
The changes in the electrical network are modeled into the transfer function.	The changes in the electrical network are specified in the failure rate and its failure duration.
The symmetrical characteristics of the transfer function's root locus plot are determined by analysing it in various plane transformations.	There are several reliability indices used to find quantitative reliability, such as LOLE. It can determine the risk level of the system.
Check the reliability through graph symmetrical behavior about the real axis during transformation.	The reliability of the system will be greater if the risk level or LOLE is lower.

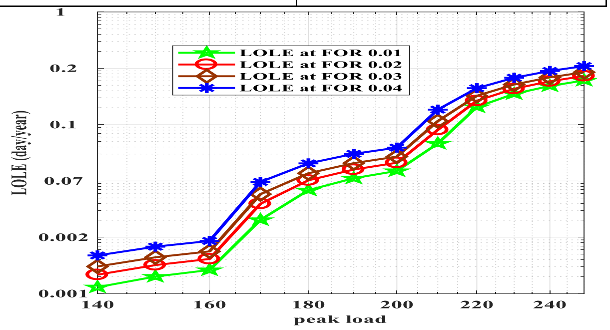

Fig. 11. Effect on LOLE with different FOR of generating units in the system

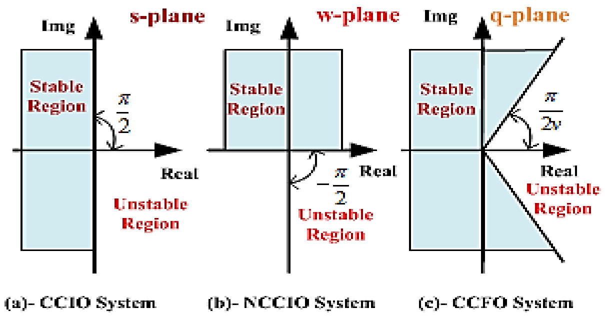

Fig. 12. Comparison of stability regions between CCIO, NCCIO, and CCFO systems

3.2. Classical reliability analysis of system by observing the risk factor

The classical reliability analysis can be performed by observing the risk level of the system, which can be determined through a reliability parameter named LOLE. The system will be more reliable if the risk level, or LOLE, of the system is lower. Fig. 11 illustrates the variation in the LOLE with an increment in the forced outage rate (FOR) of the system components. From the observations, it can be concluded that the risk level, or LOLE, of the system is increasing by increasing the FOR of the system components.

By doing so, the reliability can be compared with the base case conditions. The system is less reliable when the outage rate of the components is higher. Because classical reliability is a quantitative analysis, the risk level in each situation of network component failures can be determined. The comparison between structural reliability and classical reliability is shown in Table 2.

3.3. Comparison between the CCIO, NCCIO, and CCFO system

The CCIO system is very difficult to plot in the s-domain due to the complex characteristics of the transfer function. Therefore, the mapping of one domain to another will help in analysing the stability and reliability of the plotted graph. The conversion of a CCIO system into an NCCIO system results in a different stability region that may help in making the system reliable due to the symmetrical characteristics of the graph. The mapping of S-domain $s=|s|e^{j\theta}$ can be mapped in w domain i.e. $f:s\rightarrow w$ by putting $s=w/i$ therefore; the w-plane is mapped by changing the axis of the s-plane i.e. $w=|w|e^{j(\theta+\pi/2)}$ and stability zone shifted by $\pi/2$ factor with respect to s-plane. Similarly, a CCFO system is converted into an NCCIO system by the two-plane transformation method. Firstly, a CCFO system is transformed into a CCIO system by converting the s-plane into a q-plane. Further, a CCIO system is transformed into an NCCIO system by converting the q-plane into the w-plane. The mapping of s-plane to q-plane ($f:s\rightarrow q$) is achieved by substituting $q=s^{1/v}$. Again system is transformed from q-plane to w-plane ($f:q\rightarrow w$) by substituting $q=w/i$.

By doing so, the graph is symmetrical about the real axis, leading to a reliable plot of the system. Fig. 12 shows the comparison of stability regions between the CCIO, NCCIO, and CCFO systems. The concluding remarks state that a system becomes reliable structurally when it is transformed into an NCCIO system or w-plane from other plane states.

4. CONCLUSIONS

The paper discusses the structural reliability analysis of the power system by modelling the failures of components into TF. The transfer function has complex coefficient, fractional order, and irrational characteristics due to changes in the system's distributed components. Therefore, the RL plot becomes unreliable or unsymmetrical about the real axis due to the presence of these characteristics. The plane transformation methodology has been presented to convert the CCIO and the CCFO systems into an NCCIO system that may lead to a reliable structure of the locus plot due to symmetry by observing the real axis. A plot comparing the stability region between the CCIO, NCCIO, and CCFO systems is presented to show the stability zone for the transformation. The classical reliability has been evaluated by using the reliability index LOLE, which will give the quantitative reliability of the system during the components' failure. The comparison of structural and classical reliability has been discussed in order to demonstrate the paper's main finding and concluding remarks.

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