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Research Paper

MAPS PRESERVING THE MIXED LIE TRIPLE $*$-DERIVATIONS ON $*$-ALGEBRAS

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ABSTRACT

Let $\mathcal{A}$ be an unital $*$-algebra with the unit I. In this paper, under some mild conditions on $\mathcal{A}$, it is shown that a map $\Phi : \mathcal{A} \to \mathcal{A}$ is the mixed Lie triple $*$-derivations on $*$-algebras if and only if Φ is an additive $*$-derivations.

1. Introduction

Let $\mathcal{A}$ be a $*$-algebra over the complex field $\mathbb{C}$. For $A, B \in \mathcal{A}$ define the $\bullet$ product of A and B by $A \bullet B = AB + B^*A$ and the $\circ$ product of A and B by $A \circ B = AB - B^*A$. Recall that an additive map $\Phi : \mathcal{A} \to \mathcal{A}$ is said to be an additive derivation if $\Phi(AB) = \Phi(A)B + A\Phi(B)$ for all $A, B \in \mathcal{A}$. Furthermore, Φ is said to be an additive $*$-derivation when Φ is an additive derivation and satisfies $\Phi(A^*) = \Phi(A)^*$ for all $A \in \mathcal{A}$. Recently, many authors have studied derivations corresponding to some mixed products(see [12], [21-23]).

Set $\mathcal{A}_{sa} = \{A \in \mathcal{A} : A^* = A\}$ and $\mathcal{A}_{sk} = \{A \in \mathcal{A} : A^* = -A\}$.

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L. Abedini, A. Taghavi [1] proved if $\mathcal{A}$ is a factor von Neumann algebra with dim $\mathcal{A} \geq 2$, the map $\Phi : \mathcal{A} \longrightarrow \mathcal{A}$ satisfies

$$\Phi(A\blacklozenge B\lozenge C) = \Phi(A)\blacklozenge B\lozenge C + A\blacklozenge\Phi(B)\lozenge C + A\blacklozenge B\lozenge\Phi(C)$$

for all $A, B, C \in \mathcal{A}$ if and only if Φ is an additive $*$-derivation, where $A\blacklozenge B = A^*B + B^*A$ and $A\lozenge B = A^*B - B^*A$. Very recently, D. Zhang and C. Li [21] considered the nonlinear mixed Jordan triple $*$-derivations. Let $\mathcal{A}$ be a $*$-algebra. A map $\Phi : \mathcal{A} \to \mathcal{A}$ is said to be a nonlinear mixed Jordan triple $*$-derivation if

$$\Phi([A \bullet B, C]_*) = [\Phi(A) \bullet B, C]_* + [A \bullet \Phi(B), C]_* + [A \bullet B, \Phi(C)]_*$$

for all $A, B, C \in \mathcal{A}$.

C. Li, D. Zhang, [7] proved any map Φ from a factor von Neumann algebra $\mathcal{A}$ with $dim\mathcal{A} \geq 2$ to itself satisfies

$$\Phi([A, B]_* \bullet C) = [\Phi(A), B]_* \bullet C + [A, \Phi(B)]_* \bullet C + [A, B]_* \bullet \Phi(C)$$

for all $A, B, C \in \mathcal{A}$, if and only if Φ is an additive $*$-derivation, where $[A, B]_* = AB - BA^*$ and $A \bullet B = AB + BA^*$. Similarly, we can give the definition of the second nonlinear mixed Jordan triple $*$-derivations. A map $\Phi : \mathcal{A} \to \mathcal{A}$ is said to be the nonlinear mixed Lie triple $*$-derivation if

$$\Phi(A \circ B \bullet C) = \Phi(A) \circ B \bullet C + A \circ \Phi(B) \bullet C + A \circ B \bullet \Phi(C), \tag{1.1}$$

for all $A, B, C \in \mathcal{A}$. In this paper, it is shown that under some mild conditions on a $*$-algebra $\mathcal{A}$ containing a nontrivial projection P_1, if a mapping $\Phi : \mathcal{A} \to \mathcal{A}$ satisfies the conditions (1.1)) then Φ is additive on $\mathcal{A}_{sa}$ and $\mathcal{A}_{sk}$. Furthermore, the map Φ satisfies (1.1) for every $A, B \in \mathcal{A}$ and $C \in \mathcal{A}_{P_1}$ if and only if Φ is an additive $*$-derivation, where, $\mathcal{A}_{p_1} = \{\frac{I}{2}, \frac{iI}{2}, P_1, I - P_1, I - 2P_1\}$.

2. Main results and Proofs

Our main result in this paper reads as follows.

Theorem 2.1. *Let $\mathcal{A}$ be a unital $*$-algebra with the unit I. Suppose that $\mathcal{A}$ contains a nontrivial projection P_1 and satisfies*

$$X\mathcal{A}P_i = 0 \text{ implies } X = 0, \ i = 1, 2. \tag{2.1}$$

where, $P_2 = I - P_1$. Then a map $\Phi : \mathcal{A} \longrightarrow \mathcal{A}$ satisfies the conditions (1.1)) for all $A \in \mathcal{A}_{sa} \cup \{\frac{iI}{2}\}$, $B \in \mathcal{A}$ and $C \in \mathcal{A}_{p_1}$, then Φ is additive on $\mathcal{A}_{sa}$ and $\mathcal{A}_{sk}$. Furthermore, the map Φ satisfies (1.1) for every $A, B \in \mathcal{A}$ and $C \in \mathcal{A}_{P_1}$ if and only if Φ is an additive $*$-derivation.

Proof. Clearly, the necessity is obvious.

Let $P_1(= P)$ be a nontrivial projection in $\mathcal{A}$. Denote $\mathcal{A}_{ij} = P_i\mathcal{A}P_j (i, j = 1, 2)$, then $\mathcal{A} = \sum_{i,j=1}^{2} \mathcal{A}_{ij}$. Denote $\boldsymbol{A}_{12} = A_{12} + A_{12}^*$. So $\boldsymbol{A}_{12} = P_1AP_2 + P_2AP_1$ whenever $A \in \mathcal{A}_{sa}$. Hence, for every $A \in \mathcal{A}_{sa}$ we can write $A = A_{11} + \boldsymbol{A}_{12} + A_{22}$. In all that follows, when we write A_{ij}, it indicates that $A_{ij} \in \mathcal{A}_{ij} (i, j = 1, 2)$. To prove the additivity of Φ on $\mathcal{A}$ we shall use the above partition of $\mathcal{A}$ and we shall establish some Steps proving that Φ is additive on each $\mathcal{A}_{ij}$, $i, j = 1, 2$. Also, we denote $Z(\mathcal{A}) = \{A \in \mathcal{A} : AB = BA, \ B \in \mathcal{A}\}$. We organize the proof in a series of steps.

Step 1. $\Phi(0)=0$.

We have

$$\Phi(0)=\Phi(0\circ 0\bullet 0)=\Phi(0)\circ 0\bullet 0+0\circ\Phi(0)\bullet 0+0\circ 0\bullet\Phi(0)=0.$$

Step 2. For every $B\in\mathcal{A}$ we have

$$\Phi(\frac{B-B^*}{2})=\frac{\Phi(B)-\Phi(B)^*}{2},$$

and $\Phi\big(\pm\frac{I}{2}\big)=0$.

Let $A\in\mathcal{A}_{sa}\cup\{\frac{iI}{2}\}$ and $B\in\mathcal{A}$. From (1.1)), we get

$$\begin{aligned}\Phi(AB-B^*A) &= \Phi(A\circ B\bullet\frac{I}{2})\\ &= \Phi(A)\circ B\bullet\frac{I}{2}+A\circ\Phi(B)\bullet\frac{I}{2}+A\circ B\bullet\Phi(\frac{I}{2})\\ &= \Phi(A)B-B^*\Phi(A)+A\Phi(B)-\Phi(B)^*A\\ &+ (AB-B^*A)\Phi(\frac{I}{2})+\Phi(\frac{I}{2})^*(AB-B^*A).\end{aligned}\tag{2.2}$$

By replacing B by $\frac{I}{2}$ in Equation (2.2) and using Step 1; we can conclude that

$$\Phi(\frac{I}{2})^*=\Phi(\frac{I}{2})\in\mathcal{Z}(\mathcal{A}).\tag{2.3}$$

By choosing $A=\frac{I}{2}$, we obtain

$$\Phi(\frac{B-B^*}{2})=\Phi(\frac{I}{2})(B-B^*)+\frac{\Phi(B)-\Phi(B)^*}{2},\tag{2.4}$$

for every $B\in\mathcal{A}$. Let B be a skew self-adjoint. From (2.4) we obtain $\Phi(B)=2\Phi(\frac{I}{2})B+\frac{\Phi(B)-\Phi(B)^*}{2}$. By using (2.3), we can conclude that Φ preserves skew self-adjoint elements. Also, it can be easily follows from relation (2.4) that Φ preserves self-adjoint elements.

Again, if $B^*=-B$ from Equation (2.4) we get $\Phi(\frac{I}{2})B=0$ and so $\Phi(\frac{I}{2})=0$. This together with (2.4) we obtain

$$\Phi(\frac{B-B^*}{2})=\frac{\Phi(B)-\Phi(B)^*}{2},$$

for every $B\in\mathcal{A}$.

Similarly, we can show that $\Phi\big(-\frac{I}{2}\big)=0$.

Step 3. For every $A\in\mathcal{A}_{sa}$ we have $\Phi(iA)=i\Phi(A)$.

Let $A\in\mathcal{A}_{sa}$. From Step 2 we have

$$\begin{aligned}\Phi(iA) &= \Phi(A\circ\frac{iI}{2}\bullet\frac{I}{2})\\ &= \Phi(A)\circ\frac{iI}{2}\bullet\frac{I}{2}+A\circ\Phi(\frac{iI}{2})\bullet\frac{I}{2}+A\circ\frac{iI}{2}\bullet\Phi(\frac{I}{2})\\ &= i\Phi(A)+A\Phi(\frac{iI}{2})+\Phi(\frac{iI}{2})A.\end{aligned}\tag{2.5}$$

By choosing $A = \frac{iI}{2}$ we obtain $\Phi(\dfrac{iI}{2}) = 0$ and so from (2.5) we get $\Phi(iA) = i\Phi(A)$.

Step 4. Let $A, B, C \in \mathcal{A}$ and $T = \Phi(A+B) - \Phi(A) - \Phi(B)$. If the triplets (A, X, C), (B, X, C) and $(A+B, X, C)$ satisfy in the condition (1.1) and $B \circ X \bullet C = 0$ for some $X \in \mathcal{A}$, then $T \circ X \bullet C = 0$.

By applying Step 1 we can write

$$\begin{aligned}\Phi(((A+B)\circ X)\bullet C) &= \Phi((A\circ X)\bullet C) + \Phi((B\circ X)\bullet C)\\ &= \Phi((A\circ X)\bullet C + (A\circ\Phi(X))\bullet C + (A\circ X)\bullet\Phi(C)\\ &+ (\Phi(B)\circ X)\bullet C + (B\circ\Phi(X))\bullet C + (B\circ X)\bullet\Phi(C)\\ &= (\Phi((A+B))\circ X)\bullet C + ((\Phi(A)+\Phi(B))\circ X)\bullet C\\ &+ ((A+B)\circ X)\bullet\Phi(C).\end{aligned}$$

On the other hand,

$$\begin{aligned}\Phi(((A+B)\circ X)\bullet C) &= (\Phi(A+B)\circ X)\bullet C + ((A+B)\circ\Phi(X)\bullet C\\ &+ ((A+B)\circ X)\bullet\Phi(C).\end{aligned}$$

It follows from the last two relations that $T \circ X \bullet C = 0$.

Step 5. For each $A \in \mathcal{A}_{sa}$ and $B \in \mathcal{A}$, we have

$$\Phi(\ \mathbf{B}_{12} + A_{11}) = \Phi(\ \mathbf{B}_{12}) + \Phi(A_{11})$$

and

$$\Phi(\ \mathbf{B}_{12} + A_{22}) = \Phi(\ \mathbf{B}_{12}) + \Phi(A_{22})$$

.

Let $T = \Phi(A_{11} +\ \mathbf{B}_{12}) - \Phi(A_{11}) - \Phi(\ \mathbf{B}_{12})$. Then From Step 2 we have $T^* = T$. Since $A_{11} \circ \frac{iI}{2} \bullet \frac{P_2}{2} = 0$, from Step 4 we get that $T \circ \frac{iI}{2} \bullet \frac{P_2}{2} = 0$. We obtain from the last relation $T_{12} = T_{21} = T_{22} = 0$. Similarly, from $\mathbf{B}_{12} \circ \frac{iI}{2} \bullet \frac{P_1 - P_2}{2} = 0$, we have $T \circ \frac{iI}{2} \bullet \frac{P_1 - P_2}{2} = 0$, which implies that $T_{11} = 0$. Therefore $T = 0$, i.e.

$$\Phi(A_{11} +\ \mathbf{B}_{12}) = \Phi(A_{11}) + \Phi(\ \mathbf{B}_{12}).$$

By similar arguments to those employed above one can show that

$$\Phi(A_{22} +\ \mathbf{B}_{12}) = \Phi(A_{22}) + \Phi(\ \mathbf{B}_{12}).$$

Step 6. For each $A, C \in \mathcal{A}_{sa}$ and $B \in \mathcal{A}$, we have

$$\Phi(A_{11} +\ \mathbf{B}_{12} + C_{22}) = \Phi(A_{11}) + \Phi(\ \mathbf{B}_{12}) + \Phi(C_{22}).$$

Let $T = \Phi(A_{11} +\ \mathbf{B}_{12} + C_{22}) - \Phi(A_{11} +\ \mathbf{B}_{12}) - \Phi(C_{22})$. From Step 2 we have $T^* = T$. Since $A_{22} \circ \frac{iI}{2} \bullet \frac{P_1}{2} = 0$, from the Step 4 we get $T \circ \frac{iI}{2} \bullet \frac{P_1}{2} = 0$. We obtain from the last relation $T_{12} = T_{21} = T_{11} = 0$. Similarly, from $\mathbf{B}_{12} \circ \frac{iI}{2} \bullet \frac{P_1 - P_2}{2} = 0$, we have $T \circ \frac{iI}{2} \bullet \frac{P_1 - P_2}{2} = 0$, which implies that $T_{22} = 0$. Therefore $T = 0$, i.e.

$$\Phi(A_{11} + \mathbf{B}_{12} + C_{22}) = \Phi(A_{11}) + \Phi(\mathbf{B}_{12}) + \Phi(C_{22}).$$

Step 7. For each $A, B \in \mathcal{A}$, we have

$$\Phi(\mathbf{A}_{jk} + \mathbf{B}_{jk}) = \Phi(\mathbf{A}_{jk}) + \Phi(\mathbf{B}_{jk}), \quad 1 \leqslant j \neq k \leqslant 2.$$

Since

$$\begin{aligned}(P_j + \mathbf{A}_{jk}) \circ i(P_k + \mathbf{B}_{jk}) \bullet \frac{I}{2} &= i\mathbf{B}_{jk} + i\mathbf{A}_{jk} + i\mathbf{A}_{jk}\mathbf{B}_{jk} + i\mathbf{B}^*_{jk} + i\mathbf{B}^*_{jk}\mathbf{A}_{jk} \\ &= i\mathbf{A}_{jk} + i\mathbf{B}_{jk} + i\mathbf{A}_{jk}(\mathbf{B}_{jk} + \mathbf{B}^*_{jk}) \\ &= i\mathbf{A}_{jk} + i\mathbf{B}_{jk} + i\mathbf{A}_{jk}\mathbf{B}_{jk}\end{aligned}$$

We get from step 5 and 6

$$\begin{aligned}&\Phi(i(\mathbf{A}_{jk} + \mathbf{B}_{jk})) + \Phi(i\mathbf{A}_{jk}\mathbf{B}_{jk}) = \Phi((P_j + \mathbf{A}_{jk}) \circ i(P_k + \mathbf{B}_{jk}) \bullet \frac{I}{2}) \\ &= (\Phi(P_j) + \Phi(\mathbf{A}_{jk})) \circ i(P_k + \mathbf{B}_{jk}) \bullet \frac{I}{2} + (P_j + \mathbf{A}_{jk}) \circ (\Phi(iP_k) + \Phi(i\mathbf{B}_{jk})) \bullet \frac{I}{2} \\ &= (\Phi(P_j) + \Phi(\mathbf{A}_{jk})) \circ i(P_k + \mathbf{B}_{jk}) \bullet \frac{I}{2} + (P_j + \mathbf{A}_{jk}) \circ (\Phi(iP_k) + \Phi(i\mathbf{B}_{jk})) \bullet \frac{I}{2} \\ &+ (P_j + \mathbf{A}_{jk}) \circ iI(P_k + \mathbf{B}_{jk}) \bullet \Phi(\frac{I}{2}) \\ &= \Phi(P_j) \circ iP_k \bullet \frac{I}{2} + \Phi(P_j) \circ iB_{jk} \bullet \frac{I}{2} + \Phi(\mathbf{A}_{jk}) \circ iP_k \bullet \frac{I}{2} + \Phi(\mathbf{A}_{jk}) \circ iB_{jk} \bullet \frac{I}{2} \\ &+ P_j \circ \Phi(iP_k) \bullet \frac{I}{2} + P_j \circ \Phi(i\mathbf{B}_{jk}) \bullet \frac{I}{2} + \mathbf{A}_{jk} \circ \Phi(iP_k) \bullet \frac{I}{2} + \mathbf{A}_{jk} \circ \Phi(i\mathbf{B}_{jk}) \bullet \frac{I}{2} \\ &+ P_j \circ iP_k \bullet \Phi(\frac{I}{2}) + P_j \circ i\mathbf{B}_{jk} \bullet \Phi(\frac{I}{2}) + \mathbf{A}_{jk} \circ iP_k \bullet \Phi(\frac{I}{2}) + \mathbf{A}_{jk} \circ iB_{jk} \bullet \Phi(\frac{I}{2}) \\ &= \Phi(i\mathbf{B}_{jk}) + \Phi(i\mathbf{A}_{jk}) + \Phi(i\mathbf{B}_{jk}A_{jk}).\end{aligned}$$

This implies $\Phi(i(\mathbf{A}_{jk} + \mathbf{B}_{jk})) + \Phi(i\mathbf{A}_{jk}\mathbf{B}_{jk}) = \Phi(i\mathbf{B}_{jk}) + \Phi(i\mathbf{A}_{jk}) + \Phi(i\mathbf{A}_{jk}\mathbf{B}_{jk})$, and then $\Phi(i(\mathbf{A}_{jk} + \mathbf{B}_{jk})) = \Phi(i\mathbf{B}_{jk}) + \Phi(i\mathbf{A}_{jk})$.

From Step 3 we obtain

$$\Phi(\mathbf{A}_{jk} + \mathbf{B}_{jk}) = \Phi(\mathbf{A}_{jk}) + \Phi(\mathbf{B}_{jk}).$$

Step 8. For every $A, B \in \mathcal{A}_{sa}$, we have

$$\Phi(A_{jj} + B_{jj}) = \Phi(A_{jj}) + \Phi B_{jj}) \quad 1 \leqslant j \neq k \leqslant 2.$$

Let $T = \Phi(A_{jj} + B_{jj}) - \Phi(A_{jj}) - \Phi(B_{jj})$. From Step 2 we have $T^* = T$. Since $A_{11} \circ iP_2 \bullet \frac{I}{2} = 0$, from the Step 4 we have $T \circ iP_2 \bullet \frac{I}{2} = 0$. We obtain from the last relation $T_{12} = T_{21} = T_{22} = 0$. Similarly, if $S \in \mathcal{A}_{sa}$, then from $A_{11} \circ iS_{21} \bullet \frac{I}{2} = 0$, we have $T \circ iS_{21} \bullet \frac{I}{2} = 0$, which implies that $T_{11} = 0$. Therefore $T = 0$. i.e.

$$\Phi(A_{jj} + B_{jj}) = \Phi(A_{jj}) + \Phi B_{jj}) \quad 1 \leqslant j \neq k \leqslant 2.$$

Step 9. Φ is additive on $\mathcal{A}_{sa}$.

Let $A=\sum_{i,j=1}^{2}A_{ij}\in\mathcal{A}_{sa}$, $B=\sum_{i,j=1}^{2}B_{ij}\in\mathcal{A}_{sa}$. Now it follows from Steps 5, 6, 7 and 8, that

$$\begin{aligned}\Phi(A+B) &= \Phi(\sum_{i,j=1}^{2}A_{ij}+\sum_{i,j=1}^{2}B_{ij})=\Phi(\sum_{i,j}^{2}(A_{ij}+B_{ij}))\\ &= \sum_{i,j=1}^{2}\Phi(A_{ij}+B_{ij})=\sum_{i,j=1}^{2}\Phi(A_{ij})+\sum_{i,j=1}^{2}\Phi(B_{ij})\\ &= \Phi(\sum_{i,j=1}^{2}A_{ij})+\Phi(\sum_{i,j=1}^{2}B_{ij})\\ &= \Phi(A)+\Phi(B).\end{aligned}$$

Step 10. Φ is additive on $\mathcal{A}_{sk}$.

This follows from Steps 3 and 9.

From now on we assume that condition (1.1) is valid for every $A,B\in\mathcal{A}$ and $C\in\mathcal{A}_{p_1}$

Step 11. Φ is $*$-additive on $\mathcal{A}$.

Let $A\in\mathcal{A}$. By choosing $B=\frac{iI}{2}$ from (2.2) we obtain $\Phi(iA)=i\Phi(A)$. According to Step 2 we can conclude $\Phi(A+A^*)=-i\Phi(i(A+A^*))=-i(\Phi(iA)-\Phi(iA)^*)=\Phi(A)+\Phi(A)^*$. This together with Step 2 follows that we have $\Phi(A)=\frac{\Phi(A)+\Phi(A)^*}{2}+\frac{\Phi(A)-\Phi(A)^*}{2}=\Phi(\frac{A+A^*}{2})+\Phi(\frac{A-A^*}{2})$. Since Φ is the preserver of self-adjoint and skew self-adjoint elements, it follows from this relationship that Φ will be the preserver of involution. Let $A,B\in\mathcal{A}$. By applying Steps 9 and 10 we have

$$\begin{aligned}\Phi(A+B) &= \Phi(\frac{(A+B)+(A+B)^*}{2})+\Phi(\frac{(A+B)-(A+B)^*}{2})\\ &= \Phi(\frac{A+A^*}{2})+\Phi(\frac{A-A^*}{2})\\ &= \Phi(\frac{B+B^*}{2})+\Phi(\frac{B-B^*}{2}),\end{aligned}$$

which implies that Φ is $*$-additive.

Step 12. Φ is additive $*$-derivation.

In view of the proof of Step 2, we have

$$\Phi(AB-B^*A)=\Phi(A)B-B^*\Phi(A)+A\Phi(B)-\Phi(B^*)A.$$

Replacing B by iB in the above Equation and by using $\Phi(iB)=i\Phi(B)$, we get

$$\Phi(AB+B^*A)=\Phi(A)B+B^*\Phi(A)+A\Phi(B)+\Phi(B^*)A.$$

From two above Equations we obtain

$$\Phi(AB)=\Phi(A)B+A\Phi(B),$$

i.e. Φ is an additive $*$-derivation. This completes the proof of Theorem 2.1.

$\square$

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