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Research Paper

STRONG SUM d^d DISTANCE - d^d_{ss} IN FUZZY GRAPHS

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ABSTRACT

For any connected fuzzy graphs $G : (V, \sigma, \mu)$,the strong sum distance between x and y is defined as $d^d_{ss}(x, y) = min\{L(P_s)\}$ where P_s is the strong $x - y$ path. For any two vertices x and y of a graph G, d^d-distance is defined by considering the degrees of x and y along with the length of the path. In this paper we introduced strong sum d^d distance in fuzzy graphs by considering distance as strong sum distance and degree of vertex as the sum of membership values of all edges which are incident at that vertex. We study some properties with this new distance. First we prove that the new distance is a metric on the set of vertices of G. Some properties of strong neighbors and eccentric nodes are obtained.

1. Introduction

Graph theory was first introduced in the 18th century by the Swiss mathematician Leonhard Euler. His work on the famous "Seven Bridges of Konigsberg problem" is considered as the origin of graph theory. Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra,

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number theory, topology, optimization and computer science. When we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. Fuzzy graphs have emerged as a powerful tool for modeling complex systems with uncertain relationships. Zadeh introduced his landmark paper "Fuzzy Sets" in the year 1965 [?]. Ten years after this remarkable paper, theory of fuzzy graphs was independently developed by Rosenfeld [?], Yeh and Bang [?] in 1975. One of the fundamental concepts in fuzzy graph theory is distance, which measures the degree of separation between vertices. Distance play important roles in applications related with graphs and fuzzy graphs. Distance measures in fuzzy graphs have numerous applications in network analysis, pattern recognition, and decision making under uncertainity. The concept of distance in fuzzy graphs was first introduced by Azriel Rosenfeld [?] in 1975. Rosenfled [?]introduced the concept of μ-distance in fuzzy graphs. Despite the importance of distance measures in fuzzy graphs, there is a need for further research on developing new distance measures and exploring their properties. Sum distance in fuzzy graphs was introduced by Mini Tom and M. S. Sunitha [?] in 2014. Strong sum distance in fuzzy graphs was introduced by Mini Tom and Muraleedharan Shetty [?] in 2015. d-distance in graphs was introduced by T. Jackuline, Dr. J. Golden Ebenzer Jebamani and Dr. D. Premalatha [?]in 2022. We introduce the concept of strong sum d^d-distance in Fuzzy graphs. The objective of this research paper is to introduce a new distance measure for fuzzy graphs and explore its properties and applications. This research paper is expected to contribute to the field of fuzzy graph theory by introducing a new distance measure and demonstrating its applications in real world problems.

Section 2 contains preliminaries and in section 3, a strong sum d^ddistance in fuzzy graphs is defined and proved that it is a metric. Based on this metric strong sum d^d eccentricity, radius, diameter, eccentric vertex in fuzzy graphs are defined. In section 4, a real life application on this distance is given. Section 5 presents the conclusion.

2. Preliminaries

Definition 2.1. [?] A fuzzy graph is a triplet $G : (V, \sigma, \mu)$ where V the vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ such that $\sigma(u, v) \leq \sigma(u) \bigwedge \sigma(v)$ for all $u, v \in V$.

Definition 2.2. [?] A path P of length n is a sequence of nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1, 2, 3, \ldots, n$ and the degree of membership of a weakest arc in the path is defined as its strength.

Definition 2.3. [?] The strength of connectedness between two nodes u and v is defined as the maximum of the strengths of all paths between u and v and is denoted by $CONN_G(u, v)$.

Definition 2.4. [?] A weakest arc of $G : (V, \sigma, \mu)$ is an arc with least membership value.

Definition 2.5. [?] An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted and a $u-v$ path is called a strong path if it contains only strong arcs.

Definition 2.6. [?] If $\mu(u, v) > 0$, then u and v are called neighbors. Also v is called a strong neighbor if arc (u, v) is strong. The set of all neighbors is denoted by $N(u)$ and the set of all strong neighbors is denoted by $N_s(u)$.

Definition 2.7. [?] The center of a fuzzy graph is the fuzzy subgraph induced by the central nodes, which are the vertices with the smallest eccentricity. The center of every connected fuzzy graph $G : (V, \sigma, \mu)$ lies in a block of G^*.

Definition 2.8. [?] Let $G : (V, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u) = d(u) = \sum_{u \neq v} \mu(u, v)$.

Definition 2.9. [?]

A node is a fuzzy cut node of G if removal of it reduces the strength of connectedness between some other pair of nodes. Equivalently w is a fuzzy cut node if and only if there exist u, v distinct from w such that w is on every strongest $u - v$ path.

3. Fuzzy Strong Sum d^d distance and its properties

Definition 3.1. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph. For any path $P : x_0, x_1, x_2, x_3, \ldots, x_n$, length of P is defined as the sum of the weights of the arcs in P. ie.) $L(P) = \sum_{i=1}^{n} \mu(x_{i-1}, x_i)$. If $n = 0$ then $L(P) = 0$ and for $n \geq 1$ then $L(P) > 0$. Also if G is disconnected then $L(P)$ may be zero. For any two vertex x and y in G, let$P = \{P_s : P_s$ is a strong $x - y$ path, $s = 1, 2, 3, \ldots\}$. For any vertex $x_i, i = 1, 2, 3, \ldots$ degree of vertex x is defined as the sum of membership values of all the edges which are incident at x. ie.) $deg(x) = \sum_{i=1}^{n} x_i$. The fuzzy strong sum distance between x and y is defined as $d_{ss}(x, y) = min\{L(P_s)\}$. The fuzzy strong sum d^d Distance between x and y is defined as, $d^d_{ss}(x, y) = d_{ss}(x, y)+deg(x)+deg(y)+deg(x).deg(y)$

Example 3.2.

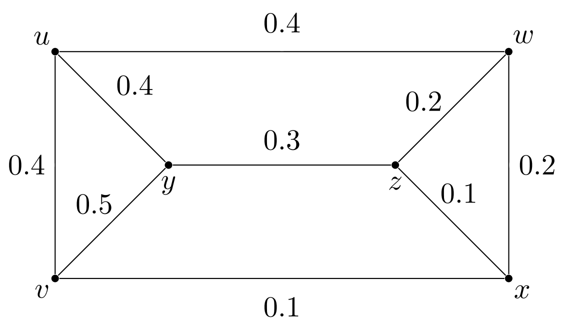

fig. 1

The following are the strong sum distances of this graph,

$d_{ss}(u, v) = 0.4, d_{ss}(u, w) = 0.4, d_{ss}(u, x) = 0.6, d_{ss}(u, y) = 0.4, d_{ss}(u, z) = 0.7,$
$d_{ss}(v, u) = 0.4, d_{ss}(v, w) = 0.8, d_{ss}(v, x) = 1, d_{ss}(v, y) = 0.5, d_{ss}(v, z) = 0.8,$
$d_{ss}(w, u) = 0.4, d_{ss}(w, v) = 0.8, d_{ss}(w, x) = 0.2, d_{ss}(w, y) = 0.8, d_{ss}(w, z) = 1.1,$
$d_{ss}(x, u) = 0.6, d_{ss}(x, v) = 0.8, d_{ss}(x, w) = 0.2, d_{ss}(x, y) = 0.7, d_{ss}(x, z) = 0.4$
$d_{ss}(y, u) = 0.4, d_{ss}(y, v) = 0.5, d_{ss}(y, w) = 0.8, d_{ss}(y, x) = 0.7, d_{ss}(y, z) = 0.3$
$d_{ss}(z, u) = 0.7, d_{ss}(z, v) = 0.8, d_{ss}(z, w) = 1.1, d_{ss}(z, x) = 0.4, d_{ss}(z, y) = 0.3.$

The degrees of each vertices are,

$deg(u) = 1.2, deg(v) = 1, deg(w) = 0.8, deg(x) = 0.4, deg(y) = 1.2, deg(z) = 0.6.$

The following are the strong sum d^d distances of this graph,

$d^d_{ss}(u,v) = 3.8, d^d_{ss}(u,w) = 3.36, d^d_{ss}(u,x) = 2.68, d^d_{ss}(u,y) = 4.24, d^d_{ss}(u,z) = 3.22$
$d^d_{ss}(v,u) = 3.8, d^d_{ss}(v,w) = 3.4, d^d_{ss}(v,x) = 2.8, d^d_{ss}(v,y) = 3.9, d^d_{ss}(v,z) = 3$
$d^d_{ss}(w,u) = 3.36, d^d_{ss}(w,v) = 3.4, d^d_{ss}(w,x) = 1.72, d^d_{ss}(w,y) = 3.76, d^d_{ss}(w,z) = 2.98,$
$d^d_{ss}(x,u) = 2.68, d^d_{ss}(x,v) = 2.8, d^d_{ss}(x,w) = 1.72, d^d_{ss}(x,y) = 2.78, d^d_{ss}(x,z) = 1.64$
$d^d_{ss}(y,u) = 4.24, d^d_{ss}(y,v) = 3.9, d^d_{ss}(y,w) = 3.76, d^d_{ss}(y,x) = 2.78, d^d_{ss}(y,z) = 2.82$
$d^d_{ss}(z,u) = 3.22, d^d_{ss}(z,v) = 3, d^d_{ss}(z,w) = 2.98, d^d_{ss}(z,x) = 1.64, d^d_{ss}(z,y) = 2.82$

Theorem 3.3. *In a fuzzy graph $G : (V, \sigma, \mu)$, $d^d_{ss} : V \times V \to [0,1]$ is a metric on V. i.e. $\forall x, y, z \in V$*

(1) $d^d_{ss}(x,y) \geq 0 \ \forall\ x, y \in V$
(2) $d^d_{ss}(x,y) = 0 \ iff x = y$
(3) $d^d_{ss}(x,y) = d^d_{ss}(y,x)$
(4) $d^d_{ss}(x,z) \leq d^d_{ss}(x,y) + d^d_{ss}(y,z)$.

Proof. Let G be a connected fuzzy graph. Then it is clear by definition that $d^d_{ss}(x,y) \geq 0$ and $d^d_{ss}(x,y) = 0$ which implies $x = y$.
Since the reversal of a strong path from x to y is a strong path from y to x and vice versa $d^d_{ss}(x,y) = d^d_{ss}(y,x)$.
It can be proved that the distance d^d_{ss} satisfies the triangle inequality.
Let P_1 be a strong $x - y$ path, $deg(x) > 0$ and $deg(y) > 0$ such that $d^d_{ss}(x,y) = d_{ss}(x,y) + deg(x) + deg(y) + deg(x).deg(y)$
Let P_2 be a strong $y - z$ path, $deg(y) > 0$ and $deg(z) > 0$ such that $d^d_{ss}(y,z) = d_{ss}(y,z) + deg(y) + deg(z) + deg(y).deg(z)$
The strong path P_1 followed by strong path P_2 is a $x - z$ walk and since every walk contains one path, there exists a strong $x - z$ path in G whose length along with degree of the vertices is atmost $d^d_{ss}(x,y) + d^d_{ss}(y,z)$.
Therefore $d^d_{ss}(x,z) \leq d^d_{ss}(x,y) + d^d_{ss}(y,z)$.

Definition 3.4. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and let x be a vertex of G. The **strong sum d^d eccentricity** $e_{d^d_{ss}}(x)$ of x is the strong sum d^d distance to a node farthest from x. Thus $e_{d^d_{ss}}(x) = max\{d^d_{ss}(x,y) : y \in V\}$. The **strong sum d^d radius** $r_{d^d_{ss}}(G)$ is the minimum eccentricity of the nodes. The **strong sum d^d diameter** $d_{d^d_{ss}}(G)$ is the maximum eccentricity of the nodes. A node x is a **central node** if $e_{d^d_{ss}}(x) = r_{d^d_{ss}}(G)$. It is denoted by $C(G)$. $C(G)$ is the set of all central nodes. A node x is **peripheral node** if $e_{d^d_{ss}}(x) = d_{d^d_{ss}}(G)$.

Example 3.5.

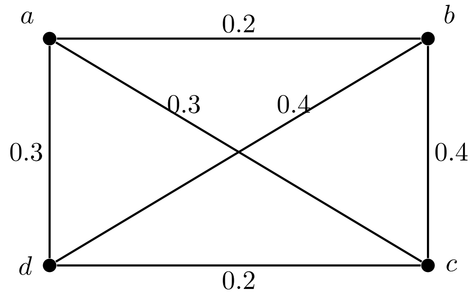

fig.2

The following are the strong sum distances of this graph,
$d_{ss}(a,b) = 0.7, d_{ss}(a,c) = 0.3, d_{ss}(a,d) = 0.3,$
$d_{ss}(b,a) = 0.7, d_{ss}(b,c) = 0.4, d_{ss}(b,d) = 0.4,$
$d_{ss}(c,a) = 0.3, d_{ss}(c,b) = 0.4, d_{ss}(c,d) = 0.4,$
$d_{ss}(d,a) = 0.3, d_{ss}(d,b) = 0.4, d_{ss}(d,c) = 0.4.$

The degrees of each vertices are,
$deg(a) = 0.8, deg(b) = 1.0, deg(c) = 0.9, deg(d) = 0.9.$

The following are the strong sum d^d distances of this graph,
$d^d_{ss}(a,b) = 3.3, d^d_{ss}(a,c) = 2.72, d^d_{ss}(a,d) = 2.72,$
$d^d_{ss}(b,a) = 3.3, d^d_{ss}(b,c) = 3.2, d^d_{ss}(b,d) = 3.2,$
$d^d_{ss}(c,a) = 2.72, d^d_{ss}(c,b) = 3.2, d^d_{ss}(c,d) = 3.41,$
$d^d_{ss}(d,a) = 2.72, d^d_{ss}(d,b) = 3.2, d^d_{ss}(d,c) = 3.41.$

The eccentricities are,
$e_{d^d_{ss}}(a) = 3.3, e_{d^d_{ss}}(b) = 3.3, e_{d^d_{ss}}(c) = 3.41, e_{d^d_{ss}}(d) = 3.41.$

Here, $r_{d^d_{ss}}(G) = 3.3$. Therefore a and b are central nodes.
$d_{d^d_{ss}}(G) = 3.41$. Therefore c and d are peripheral nodes.

Theorem 3.6. *For any connected fuzzy graph $G : (V, \sigma, \mu)$ the radius and diameter satisfy $r_{d^d_{ss}}(G) \leq d_{d^d_{ss}}(G) \leq 2r_{d^d_{ss}}(G)$.*

Proof. From the definition of radius and diameter, it is obvious that $r_{d^d_{ss}}(G) \leq d_{d^d_{ss}}(G) \ldots (1)$.
Let z be a central node of G.
Therefore, $e_{d^d_{ss}}(z) = r_{d^d_{ss}}(G)$.
let x and y be the peripheral nodes of G. Therefore $e_{d^d_{ss}}(x) = e_{d^d_{ss}}(y) = d_{d^d_{ss}}(G)$.
By triangle inequality, $d^d_{ss}(x,y) \leq d^d_{ss}(x,y) + d^d_{ss}(y,z)$.
ie.) $d_{d^d_{ss}}(G) \leq r_{d^d_{ss}}(G) + r_{d^d_{ss}}(G)$.
$d_{d^d_{ss}}(G) \leq 2r_{d^d_{ss}}(G). \ldots (2)$.
Therefore from (1) and (2), $r_{d^d_{ss}}(G) \leq d_{d^d_{ss}}(G) \leq 2r_{d^d_{ss}}(G)$.

Result 3.7. For any two strong neighbors x and y in a connected graph $G : (V, \sigma, \mu)$, $|e_{d^d_{ss}}(x) – e_{d^d_{ss}}(y)| \leq 1$.
From Fig.2, ac is a strong arc. $e_{d^d_{ss}}(a) = 3.3, e_{d^d_{ss}}(c) = 3.41$. $|3.3–3.41| = 0.11 \leq 1$. Therefore a and c are strong neighbors.
Also bc is a strong arc, $e_{d^d_{ss}}(b) = 3.3, e_{d^d_{ss}}(c) = 3.41$. $|3.3–3.41| = 0.11 \leq 1$. Therefore b and c are strong neighbors.

Remark 3.8. It is obvious that $e_{d^d_{ss}}(x) \geq d^d_{ss}(x,y)$ for any vertex x of G.
Every vertex of a fuzzy graph has fuzzy strong sum d^d eccentricity. The fuzzy strong sum d^d eccentricities of any two vertices of a fuzzy graph can be related with the fuzzy strong sum d^d distance between these two vertices. This can be seen from the following theorem 3.9

Theorem 3.9. *For any two nodes x and y in a connected fuzzy graph $G : (V, \sigma, \mu)$, $|e_{d^d_{ss}}(x) – e_{d^d_{ss}}(y)| \leq d^d_{ss}(x,y)$.*

Proof. Assume without loss of generality $e_{d^d_{ss}}(x) \geq e_{d^d_{ss}}(y)$.
Let z be a node farthest from x.
ie.) $e_{d^d_{ss}}(x) = d^d_{ss}(x, z) \leq d^d_{ss}(x, y) + d^d_{ss}(y, z) \leq d^d_{ss}(x, y) + e_{d^d_{ss}}(y)$.
Since $e_{d^d_{ss}}(y) \geq d^d_{ss}(y, z)$.
ie.) $0 \leq e_{d^d_{ss}}(x) – e_{d^d_{ss}}(y) \leq d^d_{ss}(x, y)$.
Therefore $|e_{d^d_{ss}}(x) – e_{d^d_{ss}}(y)| \leq d^d_{ss}(x, y)$.

Definition 3.10. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and $x, y \in V$ be any two vertices of G. Then y is said to be fuzzy strong sum d^d eccentric vertex of x if $e_{d^d_{ss}}(x) = d^d_{ss}(x, y)$. $x^*_{d^d_{ss}}$ denotes the set of fuzzy strong sum d^d eccentric vertices of x. That is, $x^*_{d^d_{ss}} = \{y \in V \mid d^d_{ss}(x, y) = e_{d^d_{ss}}(x)\}$.

Theorem 3.11. *Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and x and y be any two vertices of G. If x is an eccentric vertex of y, and y is an eccentric vertex of x, then $e_{d^d_{ss}}(x) = e_{d^d_{ss}}(y)$.*

Proof. Let x and y be any two vertices of a connected fuzzy graph G, such that x is an fuzzy strong sum d^d eccentric vertex of y and y is an fuzzy strong sum eccentric vertex of x.
That is, $x \in y^*_{d^d_{ss}}$ and $y \in x^*_{d^d_{ss}}$.
$x \in y^*_{d^d_{ss}}$ implies that $e_{d^d_{ss}}(y) = d^d_{ss}(x, y)$. Also, $y \in x^*_{d^d_{ss}}$ implies that $e_{d^d_{ss}}(x) = d^d_{ss}(x, y)$.
Therefore, $e_{d^d_{ss}}(y) = d^d_{ss}(x, y) = e_{d^d_{ss}}(x)$.
Therefore, $e_{d^d_{ss}}(y) = e_{d^d_{ss}}(x)$.
Hence the theorem is proved.

Remark 3.12. Converse of the above theorem need not be true. That is, if $e_{d^d_{ss}}(x) = e_{d^d_{ss}}(y)$, then it need not be true that x is an fuzzy strong sum d^d eccentric vertex of y and y is an fuzzy strong sum d^d eccentric vertex of x. $Fig.2$ is an example for this graph.

Theorem 3.13. *For every two strong neighbors x and y in a connected fuzzy graph $G : (V, \sigma, \mu)$, $|d^d_{ss}(x, z) - d^d_{ss}(y, z)| \leq 1$ for every node z of G.*

Proof. Let x and y be strong neighbors in G and let z be any node of G.
Assume $d^d_{ss}(x, z) \geq d^d_{ss}(y, z)$.
Then by triangle inequality we have, $d^d_{ss}(x, z) \leq d^d_{ss}(x, y) + d^d_{ss}(y, z)$.
since x and y are strong neighbors, $d^d_{ss}(x, z) \leq 1 + d^d_{ss}(y, z)$.
$0 \leq d^d_{ss}(x, y) - d^d_{ss}(y, z) \leq 1$.
Therefore $|d^d_{ss}(x, z) - d^d_{ss}(y, z)| \leq 1$.

Theorem 3.14. *If x and y are any two vertices in a connected fuzzy graph $G : (V, \sigma, \mu)$. Then $0 \leq d_{ss}(x, y) \leq d^d_{ss}(x, y) < \infty$.*

Proof. It is clear that the strong sum distance $d_{ss}(x, y) \geq 0$.
The fuzzy strong sum d^d distance (d^d_{ss}) is the minimum strong sum length of any $x - y$ path along with their degree sum and product of their degrees.
Therefore, $d^d_{ss}(x, y) \geq d_{ss}(x, y)$.
Also, $d^d_{ss}(x, y) < \infty$.
Hence $0 \leq d_{ss}(x, y) \leq d^d_{ss}(x, y) < \infty$.

Result 3.15.

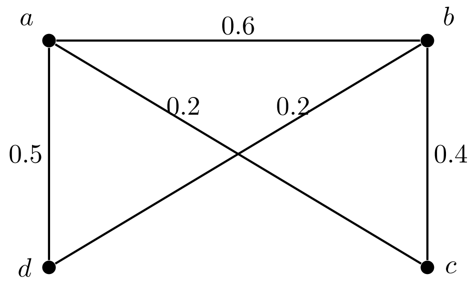

fig.3
Fuzzy graph $G : (V, \sigma, \mu)$ with disconnected center

Theorem 3.16. *The center of every connected fuzzy graph $G : (V, \sigma, \mu)$ lies in a block of G^*.*

Proof. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph. Let us assume that center of G does not lie in a block of G^*.
Then there exist a node y such that, y is a cut node of G.
Let $G_1$and G_2 be any two components of $G^* - y$. Therefore each of G_1 and G_2 contain atleast one central node of G.
Let x be a node of G such that $d^d_{ss}(x, y) = e_{d^d_{ss}}(y)$.
Let P_1 be a strong $x - y$ path such that $d^d_{ss}(x, y) = L(P_1)$, length of P_1. Then one of G_1 and G_2 contains no node in the path P_1, say G_2 contains no node of P_1.
Let z be a central node of G that belongs to G_2 and let P_2 be a strong $y - z$ path such that $d^d_{ss}(y, z) = L(P_2)$, length of P_2.
Therefore, $d^d_{ss}(x, z) = L(P_1) + L(P_2)$.
Hence we know that $e_{d^d_{ss}}(z) > e_{d^d_{ss}}(y)$, which contradicts z is a central node of G.
Hence center of every connected fuzzy graph $G : (V, \sigma, \mu)$ lies in a block of G^*

Theorem 3.17. *For every vertex x of a connected fuzzy graph G, $d_{d^d_{ss}}(G) - e_{d^d_{ss}}(x) \geq k$, where k is some non-negative real number.*

Proof. Let $G : (V, \sigma, \mu)$ be a connected fuzzy graph and x is any vertex of G.
By definition of fuzzy strong sum d^d radius and fuzzy strong sum d^d diameter,
$r_{d^d_{ss}}(G) \leq e_{d^d_{ss}}(x) \leq d_{d^d_{ss}}(G)$, which is true for any vertex x of G.
Considering the right hand side inequality, we get,
$d_{d^d_{ss}}(G) - e_{d^d_{ss}}(x) \geq k$, Where $k \geq 0$.

Theorem 3.18. *Let x and y be any two vertices in a connected fuzzy graph G. Then, $|d^d_{ss}(x, z) - d^d_{ss}(y, z)| \leq d^d_{ss}(x, y)$ for all z of G.*

Proof. We know that, $d^d_{ss}(z, x)$ is a metric.
We have $d^d_{ss}(z, x) \leq d^d_{ss}(z, y) + d^d_{ss}(y, x)$
$d^d_{ss}(z, x) - d^d_{ss}(z, y) \leq d^d_{ss}(y, x)$
Also, $d^d_{ss}(z, y) \leq d^d_{ss}(z, x) + d^d_{ss}(x, y)$
$-d^d_{ss}(x, y) \leq d^d_{ss}(z, x) - d^d_{ss}(z, y)$
Combining the above results,
$-d^d_{ss}(x, y) \leq d^d_{ss}(z, x) - d^d_{ss}(z, y) \leq d^d_{ss}(x, y)$
$|d^d_{ss}(z, x) - d^d_{ss}(z, y)| \leq d^d_{ss}(x, y)$.

4. Application

In real life, strong sum distance has its applications in logistics and transportation, communication network, social network analysis etc. Diagnosing cancer can be challenging due to several factors, including vague or absent of early symptoms, cancer will develop deep within the body.

We are using strong sum d^d distance to predict the type of cancer according to the symptoms, a patient have. The relation between disease and symptoms and patients and disease are drawn as a fuzzy graph. By considering the strong paths, fuzzy strong sum d^d distance is calculated. Using this we can predict the type of cancer affected for the patient.

Symptoms for Breast Cancer (S_1)**:** lump in the breast or under arm, changes in breast size or shape, skin changes, nipple discharge or pain.
Symptoms of Lung Cancer (S_2)**:** persistant cough, coughing up blood, chest pain, shortness of breath and unexplained weight loss.
Symptoms of Skin Cancer (S_3)**:** changes in moles, new growths or sores that don't heal and changes in skin.
Symptoms of Colorectal Cancer (S_4)**:** changes in bowel habits, rectal bleeding, persistent abdominal pain and unexplained weight loss.
Let S denote the universal set of all symptoms $S = \{S_1, S_2, S_3, S_4\}$
Let D denote the universal set of all diseases $D = \{D_1, D_2, D_3, D_4\}$
Let P denotes the universal set of all patients $P = \{P_1, P_2, P_3, P_4\}$.

- D_1 = Breast Cancer
- D_2 = Lung Cancer
- D_3 = Skin Cancer
- D_4 = Colorectal Cancer

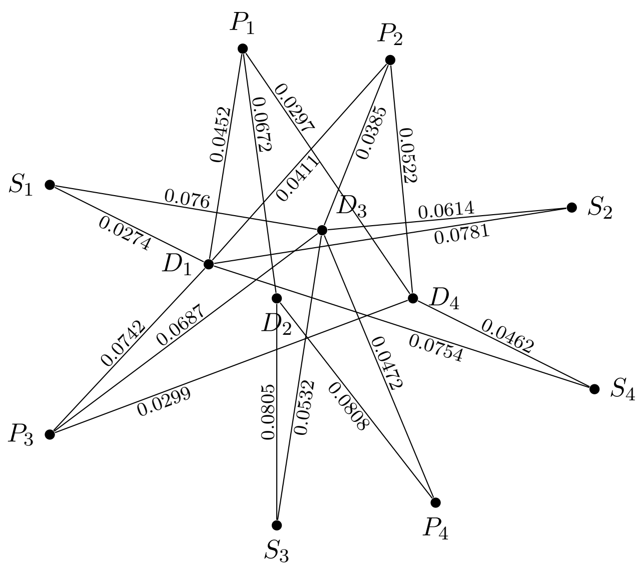

Fig.4 Fuzzy Graph between patients, symptoms and diseases

Strong Path	**Strong sum d^d Distance**
$S_1 \to D_1 \to P_1$	0.33279314
$S_1 \to D_3 \to P_2$	0.36332812
$S_1 \to D_3 \to P_3$	0.4314644
$S_1 \to D_3 \to P_4$	0.3678352652
$S_2 \to D_1 \to P_1$	0.42472295
$S_2 \to D_1 \to P_2$	0.4088861
$S_2 \to D_1 \to P_3$	0.480957
$S_2 \to D_3 \to P_4$	0.393956
$S_3 \to D_2 \to P_1$	0.44249877
$S_3 \to D_3 \to P_2$	0.37482166
$S_3 \to D_3 \to P_3$	0.4439942
$S_3 \to D_2 \to P_4$	0.4401136
$S_4 \to D_1 \to P_1$	0.40157936
$S_4 \to D_4 \to P_2$	0.36782688
$S_4 \to D_1 \to P_3$	0.4573856

From the above data, the maximum distance for P_1 is 0.44249877 and the corresponding path is $S_3 \to D_2 \to P_1$. The maximum distance for P_2 is 0.4088861 and the corresponding path is $S_2 \to D_1 \to P_2$. The maximum distance for P_3 is 0.480957 and the corresponding path is $S_2 \to D_1 \to P_3$. The maximum distance for P_4 is 0.4401136 and the corresponding path is $S_3 \to D_2 \to P_4$.

Therefore, P_1 and P_4 are confirmed with D_2 and P_2 and P_3 are confirmed with D_1.

5. CONCLUTIONS

Fuzzy distance has many real life application like decision making and optimization, pattern recognition, biometric etc. In this paper we defined strong sum d^d distance in fuzzy graphs and its properties are discussed. We applied our proposed work in a real-life situation to predict the type of cancer affected for a patient based on the symptoms.

References

[1] K. R. Bhutani and A Rosenfeld, *Strong arcs in Fuzzy Graphs*, Information Sciences, 152(2003), 319-322.

[2] F. Buckley and F. Harary, *Distance in Graphs*, Addison-Wesley, Longman (1990).

[3] T. Jackuline, J. Golden Ebenzer Jebamani, and D. Premalatha, *d-distance in graphs*, Journal of Xi'an Shiyou University, Natural Science Edition, 18(8)(2022), 132-138.

[4] Jill K. Mathew and Sunil Mathew, *A characterization of fuzzy self centered graphs*, Annals of Fuzzy Mathematics and Informatics, 10(2015), 249-260.

[5] A. Nagoorgani and J. Umamaheshwari, *Fuzzy Detour μ center in fuzzy graphs*, Int J Algorithms Comput Math, 3(2)(2010), 57-63.

[6] D. Reddy Babu and L. N. Varma, *D- Distance in graphs*, Golden Research Thoughts, 9 (2013), 2231-5063.

[7] A. Rosenfeld, *Fuzzy Sets and applications to Cognitive and Decision Process*, Academic Press, New York, (1975), 77-95.

[8] S. Sangeetha, *A study on fuzzy graph theory and its applications*, Journal of algebraic statistics, 13(3)(2022), 3877-3882.

[9] T. Sudha and G. Jayalalitha, *Application of fuzzy network graph in hospital*, Advances and Applications in Mathematical Sciences, 23(9)(2021), 1823-1829.

[10] M. P. Sunil and J. Suresh Kumar, *On fuzzy distance in fuzzy graphs*, Int. J. Math. and Appl., 8(1)(2020), 89-93.

[11] Mini Tom and M. S. Sunitha, *Boundary and interior nodes in a fuzzy graph using sum distance* , Fuzzy Inf. Eng., 8(2016), 75-85.

[12] Mini Tom and M. S. Sunitha, *Strong sum distance in fuzzy graphs*, SpringerPlus,(2015), 4:214.

[13] Mini Tom and M. S. Sunitha, *Sum distance in fuzzy graphs*, Annals of pure and applied mathematics, 7(2)(2014), 73-89.

[14] R. T. Yeh and S. Y. Bang, *Fuzzy relations, Fuzzy graphs and their application to clustering analysis*, Fuzzy Sets and their applications to Cognitive and Decision Process (1975), 125-149.

[15] L. S. Zadeh,, *Fuzzy Sets* , Information and Control, 8(1965), 338-353.