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Research Paper

THE EDGE SEIDEL AND MINIMUM EDGE COVERING SEIDEL ENERGY OF THE $K_{1,n}$ AND $K_{2,n}$ GRAPHS

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ABSTRACT

The Seidel energy of a graph is the sum of the absolute values of the eigenvalues of its Seidel matrix. In this paper, we introduce the concept of edge Seidel energy $E(L_s(G))$ and edge covering Seidel energy $E(L_{s_{ec}}(G))$ for the $K_{1,n}$ and $K_{2,n}$ Graphs, and we have obtained some results.

1. Introduction

Another well-known matrix corresponding to a graph is the Seidel matrix S(G) introduced by van Lint and Seidel in 1966 [15]. It is defined as $S(G) = J_n - I - 2A(G)$, where J_n is the matrix with all its entries equal to 1 and I is an identity matrix both of the same order $n \times n$.

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Seidel energy in graphs has various applications, especially in network analysis, clustering, and studying the resilience features of networks. Seidel energy serves as a measure to assess the resilience of a graph against removal or failure of edges and vertices. This metric helps identify critical points in communication, power, or transportation networks. Seidel energy can be used for detecting clusters and independent components within a graph through spectral analysis. It plays an effective role in identifying important regions and central data points in clustering algorithms. In designing and improving distribution networks, Seidel energy functions as an indicator of network robustness and resistance to interference or failures. It helps evaluate efficiency and security structures in wireless and IoT networks [6, 9, 11, 12, 13, 14]. The one of important spectral properties of the Seidel matrix is that the multiplicity of the least Seidel eigenvalue has a connection with equiangular lines in Euclidean space [8]. The energy of a graph G is the sum of absolute values of the eigenvalues of G. Haemers introduced the Seidel energy of a graph G, defined as the sum of absolute values of the Seidel eigenvalues of G and showed a connection with the energy of G [5]. The study on Seidel energy of a graph can be found in [1, 2, 3, 10, 11, 14]. The concept of energy in a graph was introduced by I. Gutman in the year 1978 [7]. Let G be a simple graph and let its vertex set be $V(G) = \{v_1, v_2, \ldots, v_n\}$. The adjacency matrix $A(G)$ of the graph G is a square matrix of order n, $A = A(G)$, where entries a_{ij} are given by $a_{ij} = 1$ if v_i and v_j are adjacent, $a_{ij} = 0$ otherwise. The eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ of $A(G)$, assumed in non-increasing order, are the eigenvalues of the graph G. The energy $E(G)$ of a graph G is the sum of the absolute values of $A(G)$ eigenvalues. In this paper, all graphs are assumed to be simple, finite, and connected. A graph $G = (V, E)$ is a simple graph with no loops, no multiple and directed edges. As usual, we denote by $n = |V|$ and $m = |E|$ to the number of vertices and edges in a graph G, respectively.

2. SEIDEL AND EDGE SEIDEL ENERGY

In this section, we first define Seidel energy and then introduce an edge Seidel energy of a graph. Subsequently, we examine an edge Seidel energy of $K_{1,n}$ and $K_{2,n}$ graphs and prove some of properties.

Definition 2.1. Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set E. The Seidel matrix of G is the $n \times n$ matrix defined by $S(G) = s_{ij}$, where

$$s_{ij} = \begin{cases} -1 & \text{if } v_iv_j \in E, \\ 1 & \text{if } v_iv_j \notin E, \\ 0 & \text{if } v_i = v_j. \end{cases}$$

The characteristic polynomial of $S(G)$ is denoted by $f_n(G, \lambda) = \det(\lambda I - S(G))$. The Seidel eigenvalues of a graph G are the eigenvalues of $S(G)$. The Seidel energy of G is defined as $SE(G) = \sum_{i=1}^{n} |\lambda_i|$ [8].

Definition 2.2. Let G be a simple graph and let its vertex set be $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. The edge Seidel matrix of G is defined as $S_e(G) = s_{e_{ij}}$, where

$$s_{e_{ij}} = \begin{cases} -1 & \text{if } e_i \text{ and } e_j \text{ are adjacent,} \\ 1 & \text{if } e_i \text{ and } e_j \text{ are not adjacent,} \\ 0 & \text{if } e_i = e_j. \end{cases}$$

The characteristic polynomial of $S_e(G)$ is denoted by $f_m(G,\lambda) = \det(\lambda I - S_e(G))$. The edge Seidel eigenvalues of a graph G are the eigenvalues of $S_e(G)$. The edge Seidel energy of G is defined as $S_e(E(G)) = \sum_{i=1}^{m} |\lambda_i|$.

2.1. **Edge Seidel Energy of $K_{1,n}$.**

Theorem 2.3. *If G be $K_{1,n}$ graph of order m, then*

(i) $f_m(G,\lambda) = (\lambda-1)^{m-1}(\lambda+(m-1)); \qquad m = 2,3,...$

(ii) $\det S_e(G) = 1 - m$

Proof. Let $S_e(G)$ be an edge Seidel matrix of G. Then

(i)

$$S_e = \begin{pmatrix} 0 & -1 & -1 & \cdots & -1 \\ -1 & 0 & -1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \cdots & 0 \end{pmatrix}.$$

The characteristic polynomial of $S_e(G)$ is $f_m(G,\lambda) = det(\lambda I - S_e)$. Using the elementary row operations, we convert $f_m(G,\lambda)$ into an upper triangular matrix. Then the characteristic polynomial is as follows:

$$f_m(G,\lambda) = (\lambda-1)^{m-1}(\lambda+(m-1)); \qquad m = 2,3,...$$

(ii) The eigenvalues of the matrix J, which is an $n \times n$ matrix of all ones, are as follows: there is one eigenvalue equal to n, and the remaining $n-1$ eigenvalues are equal to zero. Consequently, the eigenvalues of the matrix $-J+I$, where I is the identity matrix, are $1-n$(corresponding to the eigenvector associated with the eigenvalue n of J and 1(with multiplicity $n-1$)). Therefore, the determinant of the matrix $-J+I$ is the product of its eigenvalues. So $\det(-J+I) = (1-n) \times 1^{n-1} = 1-n$. Since the matrix $S_e(G)$ for the star graph $K_{1,n}$ has this structure, it follows that $\det S_e(G) = 1-m$ where $m = n$. ☐

Theorem 2.4. *Let G be $K_{1,n}$ graph of order m. The edge Seidel energy of $K_{1,n}$ is*

$$E(L_s(G)) = 2(m-1); \qquad m = 2,3,...$$

Proof. Using Theorem 2.3, $f_m(G,\lambda) = (\lambda-1)^{m-1}(\lambda+(m-1))$. So

$$(1 \times (m-1)) + ((m-1) \times 1) = m-1+m-1 = 2m-2 = 2(m-1).$$ ☐

Proposition 2.5. *If G be $K_{1,n}$ graph of order m, then*

$$\sum_{i=1}^{m} \lambda_i^2 = m(m-1); \qquad m = 2,3,...$$

Proof. Using Theorem 2.3, $f_m(G,\lambda) = (\lambda-1)^{m-1}(\lambda+(m-1))$. So

$$(1^2 \times (m-1)) + ((m-1)^2 \times 1) = m - 1 + m^2 - 2m + 1 = m^2 - m = m(m-1).$$

□

2.2. **Edge Seidel Energy of $K_{2,n}$.**

Theorem 2.6. *If G be $K_{2,n}$ graph of order m, then*

$$f_m(G,\lambda) = (\lambda-3)^{\frac{m}{2}-1}(\lambda+1)^{\frac{m}{2}}(\lambda+(m-3)); \quad m = 2n, \quad n = 1, 2, \ldots$$

Proof. Let $S_e(G)$ be an edge Seidel matrix of G. Then

$$\begin{array}{cc|c}
 & & \overbrace{e_1 \quad e_2 \quad \cdots \quad e_n}^{n} \quad \overbrace{e_{n+1} \quad \cdots \quad e_{n\times m}}^{n-1} \\
n\left\{\begin{array}{c} e_1 \\ \vdots \\ e_n \end{array}\right. & & \\
n-1\left\{\begin{array}{c} e_{n+1} \\ \vdots \\ e_{n\times m} \end{array}\right. & &
\end{array}
\begin{bmatrix}
0 & -1 & \cdots & -1 & 1 & \cdots & 1 \\
-1 & \ddots & & & & & \vdots \\
\vdots & & \ddots & & & & \vdots \\
-1 & & & \ddots & & & -1 \\
1 & & \cdots & & \ddots & & -1 \\
\vdots & & & & & \ddots & \vdots \\
1 & & \cdots & -1 & & \cdots & 0
\end{bmatrix}$$

The characteristic polynomial of $S_e(G)$ is $f_m(G,\lambda) = \det(\lambda I - S_e)$. Using the elementary row operations, we convert $f_m(G,\lambda)$ into an upper triangular matrix. Then the characteristic polynomial is as follows:

$$f_m(G,\lambda) = (\lambda-3)^{\frac{m}{2}-1}(\lambda+1)^{\frac{m}{2}}(\lambda+(m-3)); \quad m = 2n, \quad n = 1, 2, \ldots$$

□

Theorem 2.7. *The edge Seidel energy of $K_{2,n}$ is*

(i) $E(L_s(G)) = 3(m-2); \quad m = 2n, \quad n = 2, 3, \ldots$

(ii) $E(L_s(G)) = 2 \quad for \quad n = 1$

Proof. Using Theorem 2.6,

(i) $f_m(G,\lambda) = (\lambda-3)^{\frac{m}{2}-1}(\lambda+1)^{\frac{m}{2}}(\lambda+(m-3))$. So

$$(\frac{m}{2}-1) \times 3 + (\frac{m}{2} \times 1) + (m-3) = \frac{3m}{2} - 3 + \frac{m}{2} - 3 = \frac{4m}{2} + m - 6 = 3m - 6 = 3(m-2).$$

(ii) Let G be $K_{2,1}$ graph of order 2. Then $f_2(G,\lambda) = (\lambda+1)(\lambda-1)$. So $\lambda_1 = 1$ and $\lambda_2 = -1$ and $E(L_s(G)) = \sum_{i=1}^{2} \lambda_i^2 = 2$. □

Proposition 2.8. *If G be $K_{2,n}$ graph of order m, then*

$$\sum_{i=1}^{m} \lambda_i^2 = m(m-1); \quad m = 2n, \quad n = 1,2,...$$

Proof. Using Theorem 2.6, $f_m(G,\lambda) = (\lambda-3)^{\frac{m}{2}-1}(\lambda+1)^{\frac{m}{2}}(\lambda+(m-3))$. So

$$(\frac{m}{2}-1)\times(3^2) + (\frac{m}{2}\times(-1^2) + (m-3)^2) = \frac{9m}{2} - 9 + \frac{m}{2} + m^2 - 6m + 9$$
$$= \frac{10m}{2} + m^2 - 6m = 5m + m^2 - 6m = m^2 - m = m(m-1).$$

□

3. Minimum Edge Covering Seidel Energy

Let G be a simple graph and let its vertex set be $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$. A subset C of V is called an edge covering set of G if every vertex of G is incident to at least one edge in C. Any edge covering set with minimum cardinality is called a minimum edge covering set [4]. Let C_e be the minimum edge covering set of a graph G. The minimum edge covering Seidel matrix of G is the $n\times n$ matrix defined by $S_{C_e}(G) = {S'}_{e_{ij}}$, where

$${s'}_{e_{ij}} = \begin{cases} -1 & \text{if } e_ie_j \in E, \\ 1 & \text{if } e_ie_j \notin E, \\ 1 & \text{if } i = j \text{ and } e_i \in C_e, \\ 0 & \text{if } i = j \text{ and } e_i \notin C_e. \end{cases}$$

The characteristic polynomial of $S_{C_e}(G)$ is denoted by $f_m(G,\lambda) = \det(\lambda I - S_{C_e}(G))$. Since $S_{C_e}(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. The edge covering Seidel energy of G is defined as $S_{C_e}(G) = \sum_{i=1}^{m} |\lambda_i|$.

3.1. Minimum Edge Covering Seidel Energy of $K_{1,n}$.

Theorem 3.1. *If G be $K_{1,n}$ graph of order m, then*

$$f_m(G,\lambda) = (\lambda+(m-2))(\lambda-2)^{m-1}; \qquad m = 2,3,...$$

Proof. Let ${S'}_e(G)$ be an edge covering Seidel matrix of G. Then

$${S'}_e = \begin{pmatrix} 1 & -1 & -1 & \cdots & -1 \\ -1 & 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & \cdots & 1 & -1 \\ -1 & -1 & -1 & \cdots & 1 \end{pmatrix}.$$

The characteristic polynomial of ${S'}_e(G)$ is $f_m(G,\lambda) = \det(\lambda I - {S'}_e)$. Using the elementary row operations, we convert $f_m(G,\lambda)$ into an upper triangular matrix. Then the characteristic polynomial is as follows:

$$f_m(G,\lambda) = (\lambda + (m-2))(\lambda-2)^{m-1}; \qquad m = 2,3,...$$

$\square$

Theorem 3.2. *The edge covering Seidel energy of $K_{1,n}$ is*

$$E(L_{s_{ec}}(G)) = 3m - 4; \qquad m = 2,3,...$$

Proof. Using Theorem 3.1, $f_m(G,\lambda) = (\lambda + (m-2))(\lambda-2)^{m-1}$. So

$$((m-2)\times 1) + (2\times(m-1)) = m - 2 + 2m - 2 = 3m - 4.$$ $\square$

Proposition 3.3. *If G be $K_{1,n}$ graph of order m, then*

$$\sum_{i=1}^{m} \lambda_i^2 = m^2; \qquad m = 2,3,...$$

Proof. Using Theorem 3.1, $f_m(G,\lambda) = (\lambda + (m-2))(\lambda-2)^{m-1}$. So

$$(m-2)^2 + (m-1)\times(2^2) = m^2 - 4m + 4 + 4m - 4 = m^2.$$

$\square$

3.2. **Edge Covering Seidel Energy of $K_{2,n}$.**

Theorem 3.4. *If G be $K_{2,n}$ graph of order m, then*

$$f_m(G,\lambda) = (\lambda^2 + (m-3)\lambda + \frac{m}{2} - 2)(\lambda^2 - 3\lambda - 2)^{\frac{m}{2}-1}; \qquad m>2$$

Proof. Let ${S'}_e(G)$ be an edge covering Seidel matrix of G. Then

$${S'}_e = \begin{pmatrix} 1 & -1 & -1 & 1 & -1 & \cdots & -1 & 1 \\ -1 & 0 & 1 & -1 & 1 & \cdots & 1 & -1 \\ -1 & 1 & 0 & -1 & -1 & \cdots & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & \cdots & 1 & -1 \\ \vdots & & & & \ddots & & & \vdots \\ -1 & 1 & -1 & 1 & -1 & \cdots & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & \cdots & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & \cdots & 0 & -1 \\ 1 & -1 & 1 & -1 & 1\cdots & -1 & -1 & 1 \end{pmatrix}$$

The characteristic polynomial of $S_e(G)$ is $f_m(G,\lambda) = \det(\lambda I - {S'}_e)$. Using the elementary row operations, we convert $f_m(G,\lambda)$ into an upper triangular matrix. The characteristic polynomial is as follows:

$$f_m(G,\lambda) = (\lambda^2 + (m-3)\lambda + \frac{m}{2} - 2)(\lambda^2 - 3\lambda - 2)^{\frac{m}{2}-1}; \qquad m>2$$

□

Theorem 3.5. *The edge covering Seidel energy of $K_{2,n}$ is*

$$E(L_{s_{ec}}(G)) = (\frac{2+\sqrt{17}}{2})m - (3+\sqrt{17}); \quad m>2$$

Proof. Using Theorem 3.4 and Δ method,

$$\begin{aligned} E(L_{s_{ec}}(G)) &= ((m-3)^2 - 4(\frac{m}{2}-2)) + (9 - 4(-2\times 1))(\frac{m}{2}-1) \\ &= (m^2 - 6m + 9 - 2m + 8) + 17(\frac{m}{2}-1) \\ &= (m^2 - 8m + 17) + 17(\frac{m}{2}-1) \end{aligned}$$

We form the Δ for the $(m^2 - 8m + 17)$. Then

$m_1 = 4 - i$, $m_2 = 4 + i$ and $\Delta = (m-4+i)(m-4-i)$. So

$\lambda_1 = \dfrac{3 - m - \sqrt{(m-4+i)(m-4-i)}}{2}$ and $\lambda_2 = \dfrac{3 - m + \sqrt{(m-4+i)(m-4-i)}}{2}$.

For $(\lambda^2 - 3\lambda - 2)$ we have $\Delta = 17$. So $\lambda_3 = \dfrac{3+\sqrt{17}}{2}$ and $\lambda_4 = \dfrac{3-\sqrt{17}}{2}$.

We put $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ in the $(m^2 - 8m + 17) + 17(\frac{m}{2} - 1)$ and $E(L(G))$ is obtained as follows:

$$\begin{aligned} E(L(G)) =& \frac{m-3+\sqrt{(m-4+i)(m-4-i)}}{2} + \frac{m-3-\sqrt{(m-9+i)(m-4-i)}}{2} \\ &+ \frac{3+\sqrt{17}}{12}\left(\frac{m}{2}-1\right). \end{aligned}$$

And $E(L_{s_{ec}}(G))$ is calculated as follows:

$$\begin{aligned} E(L_{s_{ec}}(G)) &= \frac{2m-6}{2} + \left(\frac{3+\sqrt{17}}{4}\right)m - \left(\frac{3+\sqrt{17}}{2}\right) \\ &+ \left(\frac{3+\sqrt{17}}{4}\right)m - \left(\frac{3+\sqrt{17}}{4}\right)m - \left(\frac{3+\sqrt{17}}{2}\right) \\ &= m - 3 + \frac{3m}{4} + \frac{\sqrt{17}}{4}m - \frac{3}{2} - \frac{\sqrt{17}}{2} - \frac{3m}{4} - \frac{\sqrt{17}}{4}m + \frac{3}{2} - \frac{\sqrt{17}}{2} \\ &= m - 2 + \frac{2\sqrt{17}}{4}m - \sqrt{17} = m - 3 + \frac{\sqrt{17}}{2}m - \sqrt{17} \\ &= \left(1 + \frac{\sqrt{17}}{2}\right)m - (3+\sqrt{17}). \end{aligned}$$

□

Proposition 3.6. *If G be $K_{2,n}$ graph of order m, then*

$$\sum_{i=1}^{m} \lambda_i^2 = m^2 - \frac{m}{2}, \quad m>2$$

Proof. Using Theorems 3.4 and 3.5,

$$f_m(G,\lambda) = (\lambda^2 + (m-3)\lambda + \frac{m}{2} - 2)(\lambda^2 - 3\lambda - 2)^{\frac{m}{2}-1}; \quad m>2. \text{ So}$$

$$\begin{aligned}
\sum_{i=1}^{m} \lambda_i^2 &= \frac{2(3-m)^2 + 2(m-4+i)(m-4-i)}{4} \\
&+ \left(\frac{26+6\sqrt{17}}{4}\right)(\frac{m}{2}-1) + \left(\frac{26-6\sqrt{17}}{4}\right)(\frac{m}{2}-1) \\
&= \frac{(3-m)^2 + (m-4+i)(m-4-i)}{2} \\
&+ \left(\frac{13+3\sqrt{17}}{2}\right)(\frac{m}{2}-1) + \left(\frac{13-3\sqrt{17}}{2}\right)(\frac{m}{2}-1) \\
&= \frac{9-6m+m^2+m^2-8m+17}{2} + \frac{13m}{4} - \frac{13}{2} \\
&+ \frac{3\sqrt{17}m}{4} - \frac{3\sqrt{17}}{2} + \frac{13m}{4} - \frac{13}{2} - \frac{3\sqrt{17}m}{4} + \frac{3\sqrt{17}}{2} \\
&= \frac{2m^2-14m+26}{2} + \frac{26m}{4} - \frac{26}{2} = m^2 - 7m + 13 + \frac{13}{2}m - 13 \\
&= m^2 - \frac{m}{2}
\end{aligned}$$

Consequently, $\sum_{i=1}^{m} \lambda_i^2 = m^2 - \frac{m}{2}$ for $m > 2$

□

4. Conclusions

The results of this study demonstrate that the edge Seidel energy can serve as an effective and valuable tool in analyzing the structural characteristics and internal relationships of graphs. This tool enables the identification and differentiation of critical points and key elements within various networks and, through spectral analysis, provides a deeper understanding of the structure and behavior of graphs. Furthermore, the findings of this research offer new perspectives on the spectral behaviors of the graphs $K_{1,n}$ and $K_{2,n}$ as well as their interactions, which can serve as a foundation for more advanced studies. These achievements contribute to a better comprehension of the applications of Seidel energy within the fields of graph theory and applied mathematics. For instance, they can be utilized in designing and optimizing communication networks, transportation systems, power grids, and clustering algorithms. Additionally, investigating and analyzing the relationships among different types of graph energies such as Seidel energy, spectral energy, and other metrics can open pathways for developing innovative methods for analyzing complex systems. Finally, it is recommended that future research focus on more complex and diverse graphs and explore the relationships and comparisons between various energies in graphs. This is of great importance because real-world networks are often highly intricate and multidimensional, requiring deeper and broader studies to fully understand their interrelations. Developing analytical models and robust quantitative tools in this area could not only enrich theoretical graph analysis but also expand practical applications across various industries and technological fields.

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