Journal of Hyperstructures 13 *(1) (2024), 33-45.*
ISSN: 2322-1666 print/2251-8436 online
https://doi.org/10.22098/jhs.2024.14780.1009

SOME PROPERTIES OF α-PRODUCT ON SPHERICAL FUZZY GRAPHS

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Abstract. In a network model involves incomplete and imperfect network states for which sophisticated analysis is needed. Spherical fuzzy sets could be more helpful for the analysis of network state more accurately. The following evaluation information may be provided by the decision maker: Yes, Abstain; No, Reject. To solve such problems, we can use a mathematical model based on spherical fuzzy sets. The mathematical models are something like a theory which should be able to explain observations of making useful predictions. In this paper, some properties of α-product on spherical fuzzy graphs are studied. Also, the regularity properties of α-product of two spherical fuzzy graphs are discussed. Theorems related these concepts are stated and proved.

Key Words: Spherical fuzzy graphs, $\alpha-$ product, Regular spherical fuzzy graphs.
2010 Mathematics Subject Classification: 05C22, 05C90.

1. Introduction

Fuzzy set theory introduced by Zadeh [16] is a generalization of crisp set theory. Fuzzy set theory has got many applications in several fields, including chemical industry, telecommunication, decision theory, networking, computer science, and management science. In 1986, Atanassov [2] proposed the intuitionistic fuzzy sets (IFSs) as an extension of fuzzy

Research Paper
Received: 07 March 2024, Accepted: 23 April 2024. Communicated by Ahmad Yousefian Darani;
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set (FS) theory. In 2013, Cuong [3, 4] initiated the concept of the picture fuzzy set (PFS) as a direct extension of intuitonistic fuzzy sets, which may be adequate in cases when human opinions are of types: yes, abstain, no, and refusal. Mahmood et al. [12] introduced the concept of spherical fuzzy set which gives an additional strength to the concept of picture fuzzy set by enlarging the space for the grades for all the four parameters. Yager [14] introduced Pythagorean fuzzy subsets. Kifayat et al. [7] studied the geometrical comparison of fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets with spherical fuzzy sets. Cen Zuo, et al. [15] introduced the some new concepts of picture fuzzy graph. Akram et.al [1] introduced the notion of spherical fuzzy graphs and Abhishek Guleria [6] also introduced generalized version of spherical fuzzy graphs using T-spherical fuzzy sets. Muhammad Shoaib et. al [11] discussed the concept of complex spherical fuzzy graphs. Mordeson and Peng [13] introduced some operations on fuzzy graphs. Gani et al.[5] studied alpha product on fuzzy graphs. B. Mohamed Harif and A. Nazeera Begam [8, 9, 10] defined regular spherical fuzzy graphs and alpha product on spherical fuzzy graphs. The research paper is organized as follows: Introduction presents the various works done by several researchers in fuzzy graphs, intuitionistic fuzzy graphs, picture fuzzy graphs and spherical fuzzy graphs. In Preliminaries, we have provided some basic definitions of spherical fuzzy graphs. In Main Result some theorems on regularity properties of alpha product of two spherical fuzzy graphs are given. In conclusion, we conclude present studies and recommendation for future work.

2. Preliminaries

Definition 2.1. [12] A spherical fuzzy set S in U (universe of discourse) is given by $S = \{< \alpha, \mu_S(\alpha), \eta_S(\alpha), \nu_S(\alpha) >: \alpha \in U\}$ where $\mu_S : U \to [0,1]$, $\eta_S : U \to [0,1]$ and $\nu_S : U \to [0,1]$ denote degree of membership, degree of neutral membership and degree of nonmembership respectively, and for each $\alpha \in U$ satisfying the condition $0 \leq \mu_S^2(\alpha) + \eta_S^2(\alpha) + \nu_S^2(\alpha) \leq 1, \forall \alpha \in U$. The degree of refusal for any spherical fuzzy set S and $\alpha \in U$ is given by $r_S(\alpha) = \sqrt{1 - (\mu_S^2(\alpha) + \eta_S^2(\alpha) + \nu_S^2(\alpha))}$.

Definition 2.2. [1] A spherical fuzzy graph(SFG) $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ where

- $\mathcal{N} = \{v_1, v_2, ..., v_n\}$ such that $\sigma_1 : \mathcal{N} \to [0,1], \sigma_2 : \mathcal{N} \to [0,1]$ and $\sigma_3 : \mathcal{N} \to [0,1]$ denote the degree of membership, degree of neutral membership and degree of non-membership of each element $v_i \in \mathcal{N}$ respectively, and

$$0 \leq \sigma_1^2(v_i) + \sigma_2^2(v_i) + \sigma_3^2(v_i) \leq 1, \tag{2.1}$$

for every $v_i \in \mathcal{N}, (i = 1, 2, 3..., n)$

- $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ where $\mu_1 : \mathcal{L} \to [0, 1], \mu_2 : \mathcal{L} \to [0, 1]$ and $\mu_3 : \mathcal{L} \to [0, 1]$ are such that

$$\begin{aligned} &\mu_1(u_i, u_j) \leq \min\{\sigma_1(u_i), \sigma_1(u_j)\}, \\ &\mu_2(u_i, u_j) \leq \min\{\sigma_2(u_i), \sigma_2(u_j)\}, \\ &\mu_3(u_i, u_j) \leq \max\{\sigma_3(u_i), \sigma_3(u_j)\}, \text{ and} \\ &0 \leq \mu_1^2(u_i, u_j) + \mu_2^2(u_i, u_j) + \mu_3^2(u_i, u_j) \leq 1 \end{aligned} \tag{2.2}$$

for every $(u_i, u_j) \in \mathcal{L}, (i, j = 1, 2, 3, ..., n)$.

Definition 2.3. [1] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. The degree of a vertex u of a SFG is

$$d_{\mathcal{G}}(u) = \left(\sum_{u \neq v} \mu_1(u, v), \sum_{u \neq v} \mu_2(u, v), \sum_{u \neq v} \mu_3(u, v) \right)$$

.

Definition 2.4. [8] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. If $d_{\mathcal{G}}(u) = (c_1, c_2, c_3), \forall u \in \mathcal{N}$. (i.e) each vertex has same degree (c_1, c_2, c_3), then $\mathcal{G}$ is said to be regular spherical fuzzy graph(RSFG) of degree (c_1, c_2, c_3) or a $(c_1, c_2, c_3)-$ regular spherical fuzzy graph((c_1, c_2, c_3)-RSFG).

Definition 2.5. [8] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG, then the order of $\mathcal{G}$ is denoted by $O(\mathcal{G})$ and defined by

$$O(\mathcal{G}) = \left(\sum_{v \in \mathcal{N}} \sigma_1(v), \sum_{v \in \mathcal{N}} \sigma_2(v), \sum_{v \in \mathcal{N}} \sigma_3(v) \right).$$

Definition 2.6. [8] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG, then the size of $\mathcal{G}$ is denoted by $S(\mathcal{G})$ and defined by

$$S(\mathcal{G}) = \left(\sum_{uv \in \mathcal{L}} \mu_1(uv), \sum_{uv \in \mathcal{L}} \mu_2(uv), \sum_{uv \in \mathcal{L}} \mu_3(uv) \right).$$

Definition 2.7. [1] A spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is said to be a complete spherical fuzzy graph if

$$\mu_1(u,v) = \min\{\sigma_1(u), \sigma_1(v)\},$$
$$\mu_2(u,v) = \min\{\sigma_2(u), \sigma_2(v)\},$$
$$\mu_3(u,v) = \max\{\sigma_3(u), \sigma_3(v)\}$$

for all $u, v \in \mathcal{N}$ and is denoted by $K_{\mathcal{N}}$.

Definition 2.8. [1] The complement of a spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is a spherical fuzzy graph $\overline{\mathcal{G}} = (\overline{\mathcal{N}}, \overline{\mathcal{L}})$ where $\overline{\mathcal{N}} = \mathcal{N}$ and $\overline{\mathcal{L}} = (\overline{\mu_1}, \overline{\mu_2}, \overline{\mu_3})$ defined by

$$\overline{\mu_1}(u,v) = \min\{\sigma_1(u), \sigma_1(v)\} - \mu_1(u,v),$$
$$\overline{\mu_2}(u,v) = \min\{\sigma_2(u), \sigma_2(v)\} - \mu_2(u,v),$$
$$\overline{\mu_3}(u,v) = \max\{\sigma_3(u), \sigma_3(v)\} - \mu_3(u,v).$$

Definition 2.9. [10] Let $\mathcal{G}_1 = (\mathcal{N}_1, \mathcal{L}_1) = \left(\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_1}, \mu_3^{\mathcal{G}_1}\right)\right)$ and $\mathcal{G}_2 = (\mathcal{N}_2, \mathcal{L}_2) = \left(\left(\sigma_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right), \left(\mu_1^{\mathcal{G}_2}, \mu_2^{\mathcal{G}_2}, \mu_3^{\mathcal{G}_2}\right)\right)$ be two spherical fuzzy graphs. The α-product of two spherical fuzzy graph $\mathcal{G}_1$ and $\mathcal{G}_2$ is defined as a spherical fuzzy graph $\mathcal{G} = \mathcal{G}_1 \times_\alpha \mathcal{G}_2$ and
$E = \{((u_1,u_2),(v_1,v_2)/u_1 = v_1, u_2v_2 \in \mathcal{L}_2$ (or) $u_2 = v_2, u_1v_1 \in \mathcal{L}_1$ (or) $u_1v_1 \in \mathcal{L}_1,\ u_2v_2 \notin \mathcal{L}_2$ (or) $u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2\}$ with

$$\sigma_1^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(u_1,u_2) = \sigma_1^{\mathcal{G}_1}(u_1) \wedge \sigma_1^{\mathcal{G}_2}(u_2),$$
$$\sigma_2^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(u_1,u_2) = \sigma_2^{\mathcal{G}_1}(u_1) \wedge \sigma_2^{\mathcal{G}_2}(u_2),$$
$$\sigma_3^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(u_1,u_2) = \sigma_3^{\mathcal{G}_1}(u_1) \vee \sigma_3^{\mathcal{G}_2}(u_2)$$

and

$$\mu_1^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}((u_1,u_2),(v_1,v_2)) = \begin{cases} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in \mathcal{L}_2 \\ \sigma_1^{\mathcal{G}_2}(u_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in \mathcal{L}_1 \\ \sigma_1^{\mathcal{G}_2}(u_2) \wedge \sigma_1^{\mathcal{G}_2}(v_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1) & \text{if } u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2 \\ \sigma_1^{\mathcal{G}_1}(u_1) \wedge \sigma_1^{\mathcal{G}_1}(v_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2) & \text{if } u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2 \end{cases}$$

$$\mu_2^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}((u_1,u_2),(v_1,v_2)) = \begin{cases} \sigma_2^{\mathcal{G}_1}(u_1) \wedge \mu_2^{\mathcal{G}_2}(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in \mathcal{L}_2 \\ \sigma_2^{\mathcal{G}_2}(u_2) \wedge \mu_2^{\mathcal{G}_1}(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in \mathcal{L}_1 \\ \sigma_2^{\mathcal{G}_2}(u_2) \wedge \sigma_2^{\mathcal{G}_2}(v_2) \wedge \mu_2^{\mathcal{G}_1}(u_1v_1) & \text{if } u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2 \\ \sigma_2^{\mathcal{G}_1}(u_1) \wedge \sigma_2^{\mathcal{G}_1}(v_1) \wedge \mu_2^{\mathcal{G}_2}(u_2v_2) & \text{if } u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2 \end{cases}$$

$$\mu_3^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}((u_1,u_2),(v_1,v_2)) = \begin{cases} \sigma_3^{\mathcal{G}_1}(u_1) \vee \mu_3^{\mathcal{G}_2}(u_2v_2), & \text{if } u_1 = v_1, u_2v_2 \in \mathcal{L}_2 \\ \sigma_3^{\mathcal{G}_2}(u_2) \vee \mu_3^{\mathcal{G}_1}(u_1v_1), & \text{if } u_2 = v_2, u_1v_1 \in \mathcal{L}_1 \\ \sigma_3^{\mathcal{G}_2}(u_2) \vee \sigma_3^{\mathcal{G}_2}(v_2) \vee \mu_3^{\mathcal{G}_1}(u_1v_1) & \text{if } u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2 \\ \sigma_3^{\mathcal{G}_1}(u_1) \vee \sigma_3^{\mathcal{G}_1}(v_1) \vee \mu_3^{\mathcal{G}_2}(u_2v_2) & \text{if } u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2 \end{cases}$$

Example 2.10. Consider the spherical fuzzy graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ are given in Figure 1. The α-product $\mathcal{G}_1 \times_\alpha \mathcal{G}_2$ is given in Figure 2.

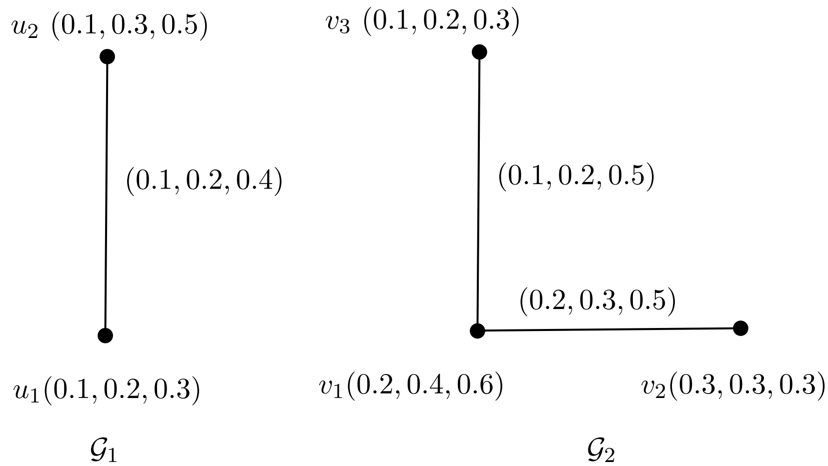

FIGURE 1. Spherical Fuzzy Graphs

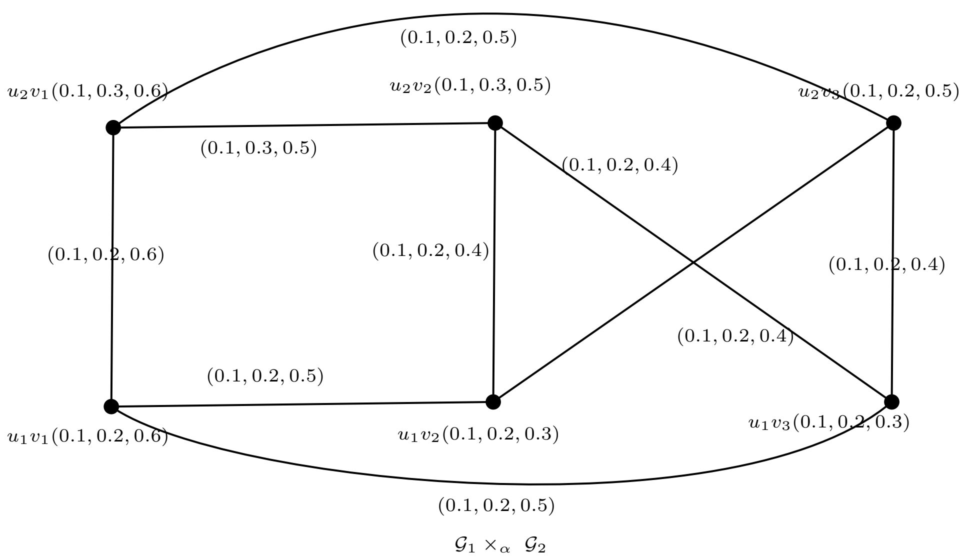

FIGURE 2. $\alpha-$Product of $\mathcal{G}_1$ and $\mathcal{G}_2$

3. Main Results

Theorem 3.1. *Let* $\mathcal{G}_1 = (\mathcal{N}_1, \mathcal{L}_1) = \left(\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_1}, \mu_3^{\mathcal{G}_1}\right)\right)$ *and* $\mathcal{G}_2 = (\mathcal{N}_2, \mathcal{L}_2) = \left(\left(\sigma_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right), \left(\mu_1^{\mathcal{G}_2}, \mu_2^{\mathcal{G}_2}, \mu_3^{\mathcal{G}_2}\right)\right)$ *be two spherical fuzzy graphs such that* $\sigma_1^{\mathcal{G}_1} \leq \mu_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_1} \leq \mu_2^{\mathcal{G}_2}$ *and* $\sigma_3^{\mathcal{G}_1} \geq \mu_3^{\mathcal{G}_2}$. *If* $\mathcal{G}_1 \times_\alpha \mathcal{G}_2$ *and* $\mathcal{G}_1$ *are regular spherical fuzzy graphs and if* G_2^* *is a regular graph, then* $\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right)$ *is a constant function.*

Proof. Suppose that $\mathcal{G}_1$ is a regular spherical fuzzy graph of degree (r_1, r_2, r_3) and G_2^* is a regular graphs of degree n. Since $\sigma_1^{\mathcal{G}_1} \leq \mu_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_1} \leq \mu_2^{\mathcal{G}_2}$ and $\sigma_3^{\mathcal{G}_1} \geq \mu_3^{\mathcal{G}_2}$, we have $\sigma_1^{\mathcal{G}_2} \geq \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_2} \geq \mu_2^{\mathcal{G}_1}$ and $\sigma_3^{\mathcal{G}_2} \leq \mu_3^{\mathcal{G}_1}$.
For any $(u_1, u_2) \in \mathcal{N}_1 \times \mathcal{N}_2$,

$$\begin{aligned}
d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= \sum_{(u_1,u_2)(v_1,v_2) \in E} (\mu_1 \times_a \mu_2)((u_1, u_2)(v_1, v_2)) \\
&= \sum_{u_1 = v_1, u_2v_2 \in \mathcal{L}_2} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2) \\
&+ \sum_{u_2 = v_2, u_1v_1 \in \mathcal{L}_1} \sigma_1^{\mathcal{G}_2}(u_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1) \\
&+ \sum_{u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2} \sigma_1^{\mathcal{G}_2}(u_2) \wedge \sigma_1^{\mathcal{G}_2}(v_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1) \\
&+ \sum_{u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \sigma_1^{\mathcal{G}_1}(v_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2) \\
&= \sum_{u_1 = v_1, u_2v_2 \in \mathcal{L}_2} \sigma_1^{\mathcal{G}_1}(u_1) + \sum_{u_2 = v_2, u_1v_1 \in \mathcal{L}_1} \mu_1^{\mathcal{G}_1}(u_1v_1) \\
&+ \sum_{u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2} \mu_1^{\mathcal{G}_1}(u_1v_1) + \sum_{u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \sigma_1^{\mathcal{G}_1}(v_1) \\
&= \sigma_1^{\mathcal{G}_1}(u_1)\, d_1^{G_2^*}(u_2) + d_1^{\mathcal{G}_1}(u_1) + \left|\overline{E_2}\right| d_1^{\mathcal{G}_1}(u_1) + \left|\overline{E_1}\right| \sigma_1^{\mathcal{G}_1}(u_1)\, d_1^{G_2^*}(u_2) \\
&= \sigma_1^{\mathcal{G}_1}(u_1)\, d_1^{G_2^*}(u_2)\left[1 + \left|\overline{E_1}\right|\right] + d_1^{\mathcal{G}_1}(u_1)\left[1 + \left|\overline{E_2}\right|\right]
\end{aligned}$$

The α-product of this two spherical fuzzy graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is a regular spherical fuzzy graphs.

Then for any two points (u_1, u_2) and (v_1, v_2) in $\mathcal{N}_1 \times \mathcal{N}_2$,

$$\begin{aligned}
d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(v_1, v_2) \\
\sigma_1^{\mathcal{G}_1}(u_1) d_1^{G_2^*}(u_2)[1 + |\overline{E_1}|] + d_1^{\mathcal{G}_1}(u_1)[1 + |\overline{E_2}|] & \\
&= \sigma_1^{\mathcal{G}_1}(v_1) d_1^{G_2^*}(v_2)[1 + |\overline{E_1}|] + d_1^{\mathcal{G}_1}(v_1)[1 + [\overline{E_2}|] \\
\sigma_1^{\mathcal{G}_1}(u_1) n[1 + |\overline{E_1}|] &= \sigma_1^{\mathcal{G}_1}(v_1) n[1 + |\overline{E_1}|] \\
\sigma_1^{\mathcal{G}_1}(u_1) n &= \sigma_1^{\mathcal{G}_1}(v_1) n \\
\sigma_1^{\mathcal{G}_1}(u_1) &= \sigma_1^{\mathcal{G}_1}(v_1)
\end{aligned}$$

Similarly, $d_2^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) = d_2^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(v_1, v_2)$.
Which implies $\sigma_2^{\mathcal{G}_1}(u_1) = \sigma_2^{\mathcal{G}_1}(v_1)$ and $d_3^{\mathcal{G}_1 \times \mathcal{G}_2}(u_1, u_2) = d_3^{\mathcal{G}_1 \times \mathcal{G}_2}(v_1, v_2)$.
Which implies $\sigma_3^{\mathcal{G}_1}(u_1) = \sigma_3^{\mathcal{G}_1}(v_1)$.
This is true for all vertices $u_1, v_1 \in \mathcal{N}_1$.
Hence $\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right)$ is a constant function. □

Theorem 3.2. *Let* $\mathcal{G}_1 = (\mathcal{N}_1, \mathcal{L}_1) = \left(\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_1}, \mu_3^{\mathcal{G}_1}\right)\right)$ *and*
$\mathcal{G}_2 = (\mathcal{N}_2, \mathcal{L}_2) = \left(\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_2}, \mu_3^{\mathcal{G}_2}\right)\right)$ *be two spherical fuzzy graphs such that* $\sigma_1^{\mathcal{G}_2} \leq \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_2} \leq \mu_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_2} \geq \mu_3^{\mathcal{G}_1}$ *and* $\left(\sigma_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right)$ *is a constant function. Then* $\mathcal{G}_1 \times_a \mathcal{G}_2$ *is a regular spherical fuzzy graph if and only if* G_1^* *is a regular graph and* $\mathcal{G}_2$ *is a regular spherical fuzzy graph.*

Proof. Let $\sigma_1^{\mathcal{G}_2}(u) = c_1, \sigma_2^{\mathcal{G}_2}(u) = c_2, \sigma_3^{\mathcal{G}_2}(u) = c_3$ for all $u \in \mathcal{N}_2$.
Since $\sigma_1^{\mathcal{G}_2} \leq \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1} \leq \mu_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_2} \geq \mu_3^{\mathcal{G}_1}$, we have $\sigma_1^{\mathcal{G}_1} \geq \mu_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_1} \geq \mu_2^{\mathcal{G}_2}$ and $\sigma_3^{\mathcal{G}_1} \leq \mu_3^{\mathcal{G}_2}$.
Let $\mathcal{G}_2$ be a regular spherical fuzzy graph of degree (k_1, k_2, k_3) and G_1^* is a regular graph of degree n.

For any $(u_1, u_2) \in \mathcal{N}_1 \times \mathcal{N}_2$.

$$
\begin{aligned}
d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= \sum_{(u_2,u_2)(v_1,v_2)\in E} (\mu_1 \times_\alpha \mu_2)\left((u_1, u_2)(v_1, v_2)\right) \\
&= \sum_{u_1=v_1, u_2v_2 \in L_2} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2) \\
&+ \sum_{u_2=v_2, u_1v_1 \in L_1} \sigma_1^{\mathcal{G}_2}(u_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1) \\
&+ \sum_{u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2} \sigma_1^{\mathcal{G}_2}(u_2) \wedge \sigma_1^{\mathcal{G}_2}(v_2) \wedge \mu_1^{\mathcal{G}_1}(u_1v_1) \\
&+ \sum_{u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2} \sigma_1^{\mathcal{G}_1}(u_1) \wedge \sigma_1^{\mathcal{G}_1}(v_1) \wedge \mu_1^{\mathcal{G}_2}(u_2v_2) \\
&= \sum_{u_1=v_1, u_2v_2 \in \mathcal{L}_2} \mu_1^{\mathcal{G}_2}(u_2v_2) + \sum_{u_2=v_2, u_1v_1 \in \mathcal{L}_1} \sigma_1^{\mathcal{G}_2}(u_2) \\
&+ \sum_{u_1v_1 \in \mathcal{L}_1, u_2v_2 \notin \mathcal{L}_2} \sigma_1^{\mathcal{G}_2}(u_2) \wedge \sigma_1^{\mathcal{G}_1}(v_2) + \sum_{u_1v_1 \notin \mathcal{L}_1, u_2v_2 \in \mathcal{L}_2} \mu_1^{\mathcal{G}_2}(u_2v_2) \\
d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= d_1^{\mathcal{G}_2}(u_2) + \sum_{u_2=v_2, u_1v_1 \in \mathcal{L}_1} \sigma_2^{\mathcal{G}_2}(u_2) + c_1 d_1^{G_1^*}(u_1)|\overline{E_2}| + |\overline{E_1}| d_1^{\mathcal{G}_2}(u_2)
\end{aligned}
$$

Where $\left|\overline{E_1}\right|$ the degree of a vertex in complement of G_1^* and $\left|\overline{E_2}\right|$ is the degree of a vertex in complement of G_2^*.

$$
\begin{aligned}
&= \left[1 + |E_1|\right] d_1^{\mathcal{G}_2}(u_2) + c_1 n \left[1 + \left|\overline{E_2}\right|\right] \\
&= \left[1 + |E_1|\right] k_1 + c_1 n \left[1 + \left|\overline{E_2}\right|\right]
\end{aligned}
\tag{3.1}
$$

Since G_1^* is a regular graph of degree n and $d_1^{\mathcal{G}_2}(u_2) = k_1$, for all $u_2 \in \mathcal{N}_2$, Similarly,

$$
\begin{aligned}
d_2^{\mathcal{G}_1 \times_\alpha \mathrm{G}_2}(u_1, u_2) &= \left[1 + \left|\overline{E_1}\right|\right] k_2 + c_2 n \left[1 + \left|\overline{E_2}\right|\right] \\
d_3^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= \left[1 + \left|\overline{E_1}\right|\right] k_3 + c_3 n \left[1 + \left|\overline{E_2}\right|\right]
\end{aligned}
$$

Hence α-product of two spherical fuzzy graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is regular spherical fuzzy graphs. Conversely, assume that $\mathcal{G}_1 \times_\alpha \mathcal{G}_2$ is a regular spherical fuzzy graph. Then for any two points (u_1, u_2) and (v_1, v_2) in

$\mathcal{N}_1 \times \mathcal{N}_2$.

(3.2)
$$d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) = d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(v_1, v_2)$$
$$\left[1 + \left|\overline{E_1}\right|\right] d_1^{G_1^*}(u_2) + c_1 d_1^{G_1^*}(u_1)\left[1 + \left|\overline{E_2}\right|\right]$$
$$= \left[1 + |E_1|\right] d_1^{G_1^*}(v_2) + c_1 d_1^{G_1^*}(v_1)\left[1 + \left|\overline{E_2}\right|\right]$$

(using 3.1).
Fix $u \in V_1$ and consider (u, u_2) and (u, v_2) in $\mathcal{N}_1 \times \mathcal{N}_2$, where $u_2, v_2 \in \mathcal{N}_2$ are arbitrary. From (3.2),

$$\left[1 + |E_1|\right] d_1^{\mathcal{G}_2}(u_2) + c_1 d_1^{\mathcal{G}_1}(u)\left[1 + \left|\overline{E_2}\right|\right] = \left[1 + \left|\overline{E_1}\right|\right] d_1^{\mathcal{G}_2}(v_2) + c_1 d_1^{\mathcal{G}_1}(u)\left[1 + \left|\overline{E_2}\right|\right]$$
$$d_1^{\mathcal{G}_2}(u_2) = d_1^{\mathcal{G}_2}(v_2)$$

Similarly, $d_2^{\mathcal{G}_2}(u_2) = d_2^{\mathcal{G}_2}(v_2), d_3^{\mathcal{G}_2}(u_2) = d_3^{\mathcal{G}_2}(v_2)$.
This is true for all $u_2, v_2 \in \mathcal{N}_2$. Hence $\mathcal{G}_2$ is a regular spherical fuzzy graph of degree (k_1, k_2, k_3). Now fix $v \in \mathcal{N}_2$ and consider (u_1, v) and (v_1, v) in $\mathcal{N}_1 \times \mathcal{N}_2$, where $u_1, v_1 \in \mathcal{N}_1$ are arbitrary.
From (3.2),

$$= \left[1 + \left|\overline{E_1}\right|\right] d_1^{\mathcal{G}_2}(v) + c_1 d_1^{G_1^*}(v_1)\left[1 + \left|\overline{E_2}\right|\right].$$

Which implies $d_1^{G_1^*}(u_1) = d_1^{G_1^*}(v_1)$.
Similarly, $d_2^{G_1^*}(u_1) = d_2^{G_1^*}(v_1), d_3^{G_1^*}(u_1) = d_3^{G_1^*}(v_1)$.
This is true for all $u_1, v_1 \in \mathcal{N}_1$.
Hence G_1^* is a regular graph of degree n. □

Theorem 3.3. *Let* $\mathcal{G}_1 = (\mathcal{N}_1, \mathcal{L}_1) = \left(\left(\sigma_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_1}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_1}, \mu_3^{\mathcal{G}_1}\right)\right)$ *and* $\mathcal{G}_2 = (\mathcal{N}_2, \mathcal{L}_2) = \left(\left(\sigma_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right), \left(\mu_1^{\mathcal{G}_1}, \mu_2^{\mathcal{G}_2}, \mu_3^{\mathcal{G}_2}\right)\right)$ *be two spherical fuzzy graphs such that* $\sigma_1^{\mathcal{G}_2} \leq \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_2} \leq \mu_2^{\mathcal{G}_1}, \sigma_3^{\mathcal{G}_2} \geq \mu_3^{\mathcal{G}_1}$. *If* $\mathcal{G}_1 \times_\alpha \mathcal{G}_2$ *and* $\mathcal{G}_2$ *are regular spherical fuzzy graphs and if* G_1^* *is a regular graph, then* $\left(\sigma_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_2}, \sigma_3^{\mathcal{G}_2}\right)$ *is a constant function.*

Proof. Suppose that $\mathcal{G}_2$ is a regular spherical fuzzy graph of degree (k_1, k_2, k_3) and G_1^* is a regular graph of degree n. Since $\sigma_1^{\mathcal{G}_1} \leq \mu_1^{\mathcal{G}_1}, \sigma_2^{S_1} \leq \mu_2^{\mathcal{G}_1}$ and $\sigma_3^{\mathcal{G}_2} \geq \mu_3^{\mathcal{G}_1}$. we have $\sigma_1^{\mathcal{G}_1} \geq \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_1} \geq \mu_2^{\mathcal{G}_2}$ and $\sigma_3^{\mathcal{G}_1} \leq \mu_3^{\mathcal{G}_1}$.

For any $(u_1,u_2)\in\mathcal{N}_1\times\mathcal{N}_2$,

$$\begin{aligned}
d_1^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(u_1,u_2) &= \sum_{(u_1,u_2)(v_1,v_2)\in E}(\mu_1\times_\alpha\mu_2)\left((u_1,u_2)(v_1,v_2)\right)\\
&= \sum_{u_1=v_1,u_2v_2,\mathcal{L}_2}\sigma_1^{\mathcal{G}_1}(u_1)\wedge\mu_1^{\mathcal{G}_2}(u_2v_2)\\
&+ \sum_{u_2=v_2,u_1v_1\in\mathcal{L}_1}\sigma_1^{\mathcal{G}_2}(u_2)\wedge\mu_1^{\mathcal{G}_1}(u_1v_1)\\
&+ \sum_{u_1v_1\in\mathcal{L}_1,u_2v_2\notin\mathcal{L}_2}\sigma_1^{\mathcal{G}_2}(u_2)\wedge\sigma_1^{\mathcal{G}_2}(v_2)\wedge\mu_1^{\mathcal{G}_1}(u_1v_1)\\
&+ \sum_{u_1v_1\notin L_1,u_2v_2\in\mathcal{L}_2}\sigma_1^{\mathcal{G}_1}(u_1)\wedge\sigma_1^{\mathcal{G}_1}(v_1)\wedge\mu_1^{\mathcal{G}_2}(u_2v_2)\\
&= \left[1+\left|\overline{E_1}\right|\right]d_1^{\mathcal{G}_2}(u_2)+\sigma_2^{\mathcal{G}_2}(u_2)\,d_1^{G_1^*}(u_1)\left[1+\left|\overline{E_2}\right|\right]
\end{aligned}$$

Where $\left|\overline{E_1}\right|$ is the degree of a vertices in complement graph G_1^* and $\left|\overline{E_2}\right|$ is the degree of a vertex in complement graph G_2^*. Since $\mathcal{G}_1\times_\alpha\mathcal{G}_2$ is a regular spherical fuzzy graph.
For any two points (u_1,u_2) and (v_1,v_2) in $\mathcal{N}_1\times\mathcal{N}_2$, we have

$$\begin{aligned}
d_1^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(u_1,u_2) &= d_1^{\mathcal{G}_1\times_\alpha\mathcal{G}_2}(v_1,v_2)\\
[1+|\overline{E_1}|d_1^{\mathcal{G}_2}(u_2)+\sigma_1^{\mathcal{G}_2}(u_2)d_1^{G_1^*}(u_1)[1+|\overline{E_2}|]&\\
&= [1+|\overline{E_1}|d_1^{\mathcal{G}_2}(v_2)+\sigma_1^{\mathcal{G}_2}(v_2)d_1^{G_1^*}(v_1)[1+|\overline{E_2}|]\\
\sigma_1^{\mathcal{G}_2}(u_2)n[1+|\overline{E_2}|]+[1+|\overline{E_1}|]k_1 &= \sigma_1^{\mathcal{G}_2}(v_2)n[1+|\overline{E_2}|]+[1+|\overline{E_1}|]k_1\\
\sigma_1^{\mathcal{G}_2}(u_2)n &= \sigma_1^{\mathcal{G}_2}(v_2)n\\
\sigma_1^{\mathcal{G}_2}(u_2) &= \sigma_1^{\mathcal{G}_2}(v_2)
\end{aligned}$$

Similariy, $\sigma_2^{\mathcal{G}_2}(u_2)=\sigma_2^{\mathcal{G}_2}(v_2),\sigma_3^{\mathcal{G}_2}(u_2)=\sigma_3^{\mathcal{G}_2}(v_2)$.
This is true for all $u_2,v_2\in\mathcal{N}_2$.
Hence $\left(\sigma_1^{\mathcal{G}_2},\sigma_2^{\mathcal{G}_2},\sigma_3^{\mathcal{G}_2}\right)$ is a constant function.

$\square$

Theorem 3.4. *Let* $\mathcal{G}_1=(\mathcal{N}_1,\mathcal{L}_1)=\left(\left(\sigma_1^{\mathcal{G}_1},\sigma_2^{\mathcal{G}_1},\sigma_3^{\mathcal{G}_1}\right),\left(\mu_1^{\mathcal{G}_1},\mu_2^{\mathcal{G}_1},\mu_3^{\mathcal{G}_1}\right)\right)$ *and* $\mathcal{G}_2=(\mathcal{N}_2,\mathcal{L}_2)=\left(\left(\sigma_1^{\mathcal{G}_2},\sigma_2^{\mathcal{G}_2},\sigma_3^{\mathcal{G}_2}\right),\left(\mu_1^{\mathcal{G}_2},\mu_2^{\mathcal{G}_2},\mu_3^{\mathcal{G}_2}\right)\right)$ *be two spherical fuzzy graphs such that underlying crisp graph* G_1^* *is complete graph. If* $\mathcal{N}_1$ *and* $\mathcal{L}_1$ *are constants and* $\sigma_1^{\mathcal{G}_1}\le\mu_1^{\mathcal{G}_2},\sigma_2^{\mathcal{G}_1}\le\mu_2^{\mathcal{G}_2},\sigma_3^{\mathcal{G}_1}\ge\mu_3^{\mathcal{G}_2}$, *then*

$\mathcal{G}_1 \times_a \mathcal{G}_2$ is a regular spherical fuzzy graph if and only if $\mathcal{G}_1$ is a regular spherical fuzzy graph and G_2^* is a regular graph.

Proof. Let the constant value of $\mathcal{N}_1$ and $\mathcal{L}_1$ be (c_1, c_2, c_3). Let $\mathcal{G}_1 \times_a \mathcal{G}_2$ be a regular spherical fuzzy graph. We have $\sigma_1^{\mathcal{G}_1} \le \mu_1^{\mathcal{G}_2}, \sigma_2^{\mathcal{G}_1} \le \mu_2^{\mathcal{G}_2}$ and $\sigma_3^{\mathcal{G}_1} \ge \mu_3^{\mathcal{G}_2}$, hence $\sigma_1^{\mathcal{G}_2} \ge \mu_1^{\mathcal{G}_1}, \sigma_2^{\mathcal{G}_2} \ge \mu_2^{\mathcal{G}_1}$ and $\sigma_3^{\mathcal{G}_1} \le \mu_3^{\mathcal{G}_1}$, then for any points (u_1, u_2) and (v_1, v_2) in $\mathcal{N}_1 \times \mathcal{N}_2$,

$$d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) = d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(v_1, v_2)$$

$$\sigma_1^{\mathcal{G}_1}(u_1)\, d_1^{\mathcal{G}_2^*}(u_2)\left[1 + \left|\overline{E_1}\right|\right] + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) \tag{3.3}$$
$$= \sigma_1^{\mathcal{G}_1}(v_1)\, d_1^{\mathcal{G}_2}(v_2)\left[1 + \left|\overline{E_1}\right|\right] + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1)$$

Where $\left|\overline{E_1}\right|$ degree of a vertex in complement of G_1^* and $\left|\overline{E_2}\right|$ is the degree of a vertex in complement of $\mathcal{G}_2$.
Suppose that G_1^* is complete graph, therefore $\left|\overline{E_1}\right| = 0$. Then (3.3) becomes,

$$\sigma_1^{\mathcal{G}_1}(u_1)\, d_1^{G_1^*}(u_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1)$$
$$= \sigma_1^{\mathcal{G}_1}(v_1)\, d_1^{G_1^*}(v_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1)$$

$\mathcal{N}_1$ is a constant function, say (c_1, c_2, c_3)

$$c_1 d_1^{G_2^*}(u_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) = c_1 d_1^{G_2^*}(v_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1) \tag{3.4}$$

Fix $u \in \mathcal{N}_1$ and consider (u, u_2) and (u, v_2) in $\mathcal{N}_1 \times \mathcal{N}_2$, where $u_2, v_2 \in \mathcal{N}_2$ are arbitrary.
From (3.4), $\quad c_1 d_1^{G_1^*}(u_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) = c_1 d_1^{G_2^*}(v_2) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1)$
Which implies $d_1^{G_2^*}(u_2) = d_1^{G_2^*}(v_2)$.
Similarly, $d_2^{G_2^*}(u_2) = d_2^{G_2^*}(v_2), d_3^{G_3^*}(u_2) = d_3^{G_2^*}(v_2)$.
This is true for all $u_2, v_2 \in \mathcal{N}_2$. Hence $\mathcal{G}_2^*$ is a regular spherical fuzzy graph of degree n. Fis $u \in \mathcal{N}_1$ and consider (u_1, v) and (v_1, v) in $\mathcal{N}_1 \times \mathcal{N}_2$, where $u_1, v_1 \in \mathcal{N}_1$ are arbitrary.
From (3.4). $\quad c_1 d_1^{G_2^*}(v) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) = c_1 d_1^{G_2^*}(v) + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1)$.
Which implies $\left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) = \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(v_1)$.
Which implies $d_1^{\mathcal{G}_1}(u_1) = d_1^{\mathcal{G}_1}(v_1)$.
This is true for all $u_1, v_1 \in \mathcal{N}_1$.
Thus $\mathcal{G}_1$ is a regular spherical fuzzy graph of degree (k_1, k_2, k_3).
Conversely, assume that $\mathcal{G}_1$ is a regular spherical fuzzy graph and G_2^* is a regular graph.
Let $d_1^{\mathcal{G}_1}(u) = k_1, d_2^{\mathcal{G}_1}(u) = k_2, d_3^{\mathcal{G}_1}(u) = k_3$ for all $u \in \mathcal{N}_1$ and G_2^* is a

regular graph of degree n.
Then for any vertex (u_1, u_2) of $\mathcal{N}_1 \times {}_a\mathcal{N}_2$.

$$\begin{aligned} d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= \sigma_1(u_1)\, d_1^{G_2^*}(u_2) + \left[1 + \left|\overline{E_1}\right|\right] + \left[1 + \left|\overline{E_2}\right|\right] d_1^{\mathcal{G}_1}(u_1) \\ &= c_1 n \left[1 + \left|\overline{E_1}\right|\right] + \left[1 + \left|\overline{E_2}\right|\right] k_1. \end{aligned}$$

Given that G_1^* is complete graph, hence $\left|\overline{E_1}\right| = 0$.
Therefore, $d_1^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) = c_1 n + \left[1 + \left|\overline{E_2}\right|\right] k_1$.
Similarly,

$$\begin{aligned} d_2^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= c_2 n + \left[1 + \left|\overline{E_2}\right|\right] k_2; \\ d_3^{\mathcal{G}_1 \times_\alpha \mathcal{G}_2}(u_1, u_2) &= c_3 n + \left[1 + \left|\overline{E_2}\right|\right] k_3. \end{aligned}$$

This is true for all $(u_1, u_2) \in \mathcal{N}_1 \times \mathcal{N}_2$. Hence $\mathcal{G}_1 \times {}_\alpha\mathcal{G}_2$ is a regular spherical fuzzy graph. □

4. Conclusion

Spherical fuzzy graphs are more flexible than the picture fuzzy graphs. The spherical fuzzy model is more useful than the picture fuzzy model as it broadens the space of uncertain information, due to its outstanding characteristic of vast space of participation of acceptable triplets. In this paper, some regularity properties of α-product on spherical fuzzy graphs are studied. Theorems related to α-product of two spherical fuzzy graphs are proved. Furthermore, we will introduce some new operations on spherical fuzzy settings.

Acknowledgments

The authors thankful to Editor in Chief and the fruitful referees for their valuable comments and constructive suggestions to improve the paper.

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