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An example of conformally Osserman manifold

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Abstract. In this paper, we investigate pseudo-Riemannian manifolds those eigenvalues of the Weyl conformal Jacobi operators are constant on the unit sphere bundles. Using a result of [4], we give an explicit construction of conformally Osserman manifold which is not locally conformally flat.

Keywords: conformal Jacobi operator, conformally Osserman manifold, Weyl conformal curvature operator.

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1. **Introduction**

Let (M, g) be a pseudo-Riemannian manifold of dimension m with Levi-Civita connection ∇. Let $\mathcal{R}(X, Y) := \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$ be the curvature operator and $R(X, Y, Z, T) = g(\mathcal{R}(X, Y)Z, T)$ its Riemann curvature tensor. The Jacobi operator is defined by $J_{\mathcal{R}}(X) : Y \mapsto \mathcal{R}(Y, X)X$. It is a self-adjoint operator and it plays an important role in the curvature theory. The geodesic deviation is measured by this part of the curvature tensor and because of its fundamental role in the Jacobi equation, many geometric properties can be derived from the knowledge of the Jacobi operators [10, 11]. Since for each vector X, the Jacobi operator is a self-adjoint operator, the study of its eigenvalues is of great interest. In the Lorentzian case especially, they play a fundamental role in the construction of mathematical models in general relativity. On the other hand, the eigenvalues of the Jacobi operator depend both on a point $p \in M$ and a direction $X \in T_pM$.

Let $S^{\pm}(M, g)$ be the pseudo-sphere bundles of unit spacelike and timelike tangent vectors. Then (M, g) is said to be spacelike Osserman (respectively timelike Osserman) if the eigenvalues of $J_{\mathcal{R}}(\cdot)$ are constant on the unit sphere bundles $S^+(M, g)$ (respectively $S^-(M, g)$). The notions of spacelike Osserman and timelike Osserman are equivalent and if (M, g) is either of them, then (M, g) is said to be Osserman. Many mathematicians have studied Osserman manifolds (see e.g. [11] for the Riemannian case and [8, 9, 10] for pseudo-Riemannian case).

Let $\{e_i\}$ be a local frame for the tangent bundle. We set $g_{ij} := g(e_i, e_j)$ and let g^{ij} be the inverse matrix. The Ricci operator ρ, the associated Ricci tensor $\rho(\cdot, \cdot)$, the scalar curvature τ and the Weyl conformal curvature operator $\mathcal{W}$ are given by

$$
\begin{aligned}
\rho X &:= \sum_{i,j} g^{ij} \mathcal{R}(X, e_i)e_j, \\
\rho(X, Y) &:= g(\rho X, Y), \\
\tau &:= \sum_{i,j} g^{ij} \rho(e_i, e_j), \\
\mathcal{W}(X, Y)Z &:= \mathcal{R}(X, Y)Z + \frac{\tau}{(m-1)(m-2)} \mathcal{R}^0(X, Y)Z \\
&\quad + \frac{1}{(m-2)} \mathcal{L}(X, Y)Z.
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{R}^0(X, Y)Z &:= g(Y, Z)X - g(X, Z)Y, \\
\mathcal{L}(X, Y)Z &:= g(\rho Y, Z)X - g(\rho X, Z)Y + g(Y, Z)\rho X - g(X, Z)\rho Y,
\end{aligned}
$$

for $X, Y, Z \in \mathfrak{X}(M)$. The Weyl conformal curvature $\mathcal{W}$ as a conformal invariant is important in the understanding of conformal pseudo-Riemannian geometry.

It is well known that an m-dimensional pseudo-Riemannian manifold, $m \geq 4$, is conformally flat if and only if its Weyl conformal curvature vanishes. Note that, a Riemannian manifold (M, g) is said to be locally conformal flat if around every point $p \in M$, there exists a metric g' which is conformal to g and g' is flat. In [5], multidimensional cosmological solutions which are locally conformally flat are described. These solutions correspond to generalizations of the Friedmann-Robertson-Walker cosmology. Moreover, the authors [6], construct new examples of complete locally conformally flat manifolds of negative curvature by means of warped product and multiply warped product structures.

The Weyl conformal Jacobi operator $J_{\mathcal{W}}(\cdot)$ is defined by

$$J_{\mathcal{W}}(X)Y = \mathcal{W}(Y, X)X, \tag{1.1}$$

for $X, Y \in S^{\pm}(M, g)$. We say that (M, g) is *conformally Osserman* if the eigenvalues of $J_{\mathcal{W}}(\cdot)$ are constant on $S^{\pm}(M, g)$ and (M, g) is *nilpotent conformally Osserman* if the conformal Jacobi operator of (M, g) is nilpotent.

In the paper [2], the authors characterize manifolds which are locally conformally equivalent to either complex projective space or to its negative curvature dual. In particular, they classify the conformally complex space forms if the dimension is at least 8. The authors [3] proves that a 4-dimensional oriented Riemannian manifold is conformally Osserman if and only if it is self-dual or anti-self dual. It is shown in [4] that the conformally Osserman condition is a conformal invariant and that any Lorentzian [1] or odd-dimensional Riemannian [2] conformally Osserman manifold is locally conformally flat. Recently, in the paper [7], an example of a nilpotent conformally Osserman manifold of signature $(2, 2)$ and geodesically complete is exhibited.

Motivated by [7], we exhibit an example of pseudo-Riemannian metric of signature $(2, 2)$ which is conformally Osserman and is not locally conformally flat. In section 2, we will present some results concerning conformally spacelike and timelike Jordan Osserman manifolds. In section 3, we describe the metric that we considered.

2. **Conformally Osserman manifolds**

The study of conformally Osserman manifolds was started in [2] and then continued in [1, 3, 4, 12]. We say that (M, g) is conformally Osserman if the eigenvalues of the Weyl conformal Jacobi operator J_W are constant on the unit fiber spheres.

Recall that two metrics g_1 and g_2 are said to be conformally equivalent if there is a positive scaling function $\alpha \in \mathcal{C}^{\infty}(M)$ so that $g_1 = \alpha g_2$. We let

$[g]$ be the set of all pseudo-Riemannian metrics on M which are conformally equivalent to g.

Theorem 2.1. *Let g_1 and g_2 be conformally equivalent metrics on M. Then (M, g_1) is conformally Osserman if and only if (M, g_2) is conformally Osserman.*

We have the following result for later use.

Theorem 2.2. *If (M, g) is Einstein, then (M, g) is conformally spacelike (respectively timelike) Jordan Osserman if and only if (M, g) is pointwise spacelike (respectively timelike) Jordan Osserman.*

The classification is complete in certain settings:

Theorem 2.3. [4] *Assume either that (M, g) is an odd dimensional Riemannian manifold or that (M, g) is a Lorentzian manifold. Then (M, g) is conformally spacelike Jordan Osserman if and only if (M, g) is conformally flat.*

Any local rank 1 Riemannian symmetric space is necessarily conformally Osserman since the group of local isometries acts transitively on the unit sphere bundle. The authors [4] conjecture that the converse holds; this is the analogue of the Osserman conjecture in this setting:

Conjecture: A connected Riemannian manifold (M, g) is conformally Osserman if and only if (M, g) is locally conformally equivalent to a rank 1 symmetric space.

This conjecture holds if m is odd. The situation is considerably more complicated in the higher signature setting. There are conformally spacelike Jordan Osserman manifolds which are not conformally flat.

In the paper [2], the authors characterize manifolds which are locally conformally equivalent to either complex projective space or to its negative curvature dual.

Theorem 2.4. [2] *Let (M, g) be a conformally Osserman Riemannian manifold of dimension m.*

(1) *If m is odd, then (M, g) is conformally flat.*
(2) *If $m = 4k + 2 \geq 10$ and if p is a point of M where $W_p \neq 0$, then there is an open neighborhood of p in M which is conformally equivalent to an open subset of either complex projective space with the Fubini-Study metric or its negative curvature dual*

In [3], the following characterization in dimension four is obtained :

Theorem 2.5. [3] *Let (M, g) be a 4-dimensional oriented Riemannian manifold. The followin conditions are equivalent:*

(1) (M, g) *is conformally Osserman.*
(2) (M, g) *is self-dual or anti-self dual.*

In [1], the author proves that in Lorentzian manifolds of dimension greater than three, the conformally Osserman condition is equivalent to the conformally flat condition. Also it is proved that, in the Lorentzian setting, timelike conformally Osserman manifolds are spacelike conformally Osserman manifolds and vice versa.

Recently, The author of the paper [12] answers some part of the conjecture made by Blazic and Gilkey that a conformally Osserman manifold of dimension $n \neq 3, 4, 16$ is locally conformally equivalent either to a Euclidean space or to a rank-one symmetric space.

Theorem 2.6. [12] *A connected* C^∞ *Riemannian conformally Osserman manifold of dimension* $n \neq 3, 4, 16$ *is locally conformally equivalent to a Euclidean space or to a rank-one symmetric space*

3. Four dimensional of conformally Osserman manifolds

Let $M = \mathbb{R}^4$ be the 4-dimensional Euclidean space with usual coordinates (u_1, u_2, u_3, u_4). Then $\mathcal{D}_1 = \text{span}\{\partial_1, \partial_2\}$ and $\mathcal{D}_2 = \text{span}\{\partial_3, \partial_4\}$ define two distributions of TM. The splitting $TM = \mathcal{D}_1 \oplus \mathcal{D}_2$ is just the usual splitting $T\mathbb{R}^4 = T\mathbb{R}^2 \oplus T\mathbb{R}^2$. We define a pseudo-Riemannian metric of neutral sinature $(2, 2)$ by setting

$$\begin{aligned} g_{(f_1,f_2,h)} &= u_3 f_1(u_2) du_1 \otimes du_1 + u_4 f_2(u_1) du_2 \otimes du_2 \\ &\quad + [du_1 \otimes du_2 + du_2 \otimes du_1] \\ &\quad + [du_1 \otimes du_3 + du_3 \otimes du_1] \\ &\quad + [du_2 \otimes du_4 + du_4 \otimes du_2], \end{aligned} \tag{3.1}$$

where f_1 and f_2 are smooth real valued functions satisfying

$$\partial_2 f_1 + \partial_1 f_2 = 0. \tag{3.2}$$

Furthermore, the distribution $\mathcal{D}_2$ is totally isotropic with respect to the pseudo-Riemannian metric $g_{(f_1,f_2,h)}$.

The non zero Christoffel symbols of the pseudo-Riemannian $g_{(f_1,f_2,h)}$ are given by

$$\begin{aligned}
\Gamma^1_{11} &= -\frac{f_1}{2},\\
\Gamma^3_{11} &= \frac{u_3 f_1^2}{2},\\
\Gamma^4_{11} &= -\frac{u_3\partial_2 f_1}{2} + \frac{f_1}{2},\\
\Gamma^3_{12} &= \frac{u_3\partial_2 f_1}{2},\\
\Gamma^4_{12} &= \frac{u_4\partial_1 f_2}{2},\\
\Gamma^3_{13} &= \frac{f_1}{2},
\end{aligned}$$

$$\begin{aligned}
\Gamma^4_{24} &= \frac{f_2}{2}\\
\Gamma^2_{22} &= -\frac{f_2}{2},\\
\Gamma^3_{22} &= -\frac{u_4\partial_1 f_2}{2} + \frac{f_2}{2},\\
\Gamma^4_{22} &= \frac{u_4 f_2^2}{2}.
\end{aligned} \tag{3.3}$$

From (3.2) and (3.3), the only non zero components of the covariant derivatives are given by

$$\begin{aligned}
\nabla_{\partial_1}\partial_1 &= -\frac{f_1}{2}\partial_1 + \frac{u_3 f_1^2}{2}\partial_3 + \Big(-\frac{u_3\partial_2 f_1}{2} + \frac{f_1}{2}\Big)\partial_4;\\
\nabla_{\partial_1}\partial_3 &= \frac{f_1}{2}\partial_3;\\
\nabla_{\partial_2}\partial_2 &= -\frac{f_2}{2}\partial_2 + \Big(\frac{u_4\partial_2 f_1}{2} + \frac{f_2}{2}\Big)\partial_3 + \frac{u_4 f_2^2}{2}\partial_4;\\
\nabla_{\partial_2}\partial_4 &= \frac{f_2}{2}\partial_4\\
\nabla_{\partial_1}\partial_2 &= \frac{u_3\partial_2 f_1}{2}\partial_3 - \frac{u_4\partial_2 f_1}{2}\partial_4.
\end{aligned} \tag{3.4}$$

From (3.2) and (3.4), we obtain that the non zero components of the curvature operator are

$$\begin{aligned}
\mathcal{R}(\partial_1,\partial_2)\partial_1 &= \frac{\partial_2 f_1}{2}\partial_1 - \frac{u_3 f_1 \partial_2 f_1}{2}\partial_3 \\
&\quad +\Big(\frac{u_3\partial_{22}^2 f_1}{2} + \frac{u_3 f_2\partial_2 f_1}{4} - \frac{u_4 f_1 \partial_2 f_1}{4} - \frac{\partial_2 f_1}{2} - \frac{f_1 f_2}{4}\Big)\partial_4,\\
\mathcal{R}(\partial_1,\partial_2)\partial_2 &= \frac{\partial_2 f_1}{2}\partial_2 - \frac{u_4 f_2 \partial_2 f_1}{2}\partial_4 \\
&\quad -\Big(\frac{u_3\partial_{22}^2 f_1}{2} + \frac{u_3 f_2\partial_2 f_1}{4} - \frac{u_4 f_1 \partial_2 f_1}{4} - \frac{\partial_2 f_1}{2} - \frac{f_1 f_2}{4}\Big)\partial_3,\\
\mathcal{R}(\partial_1,\partial_2)\partial_3 &= -\frac{\partial_2 f_1}{2}\partial_3,\\
\mathcal{R}(\partial_1,\partial_2)\partial_4 &= -\frac{\partial_2 f_1}{2}\partial_4,\\
\mathcal{R}(\partial_1,\partial_3)\partial_1 &= \frac{\partial_2 f_1}{2}\partial_4,\\
\mathcal{R}(\partial_1,\partial_3)\partial_2 &= -\frac{\partial_2 f_1}{2}\partial_3,\\
\mathcal{R}(\partial_2,\partial_4)\partial_1 &= \frac{\partial_2 f_1}{2}\partial_4,\\
\mathcal{R}(\partial_2,\partial_4)\partial_2 &= -\frac{\partial_2 f_1}{2}\partial_3. \qquad (3.5)
\end{aligned}$$

And from (3.5), we obtain that the non zero components of the $(0,4)-$ curvature tensor are given by

$$\begin{aligned}
R(\partial_1,\partial_2,\partial_1,\partial_2) &= \frac{u_3\partial_{22}^2 f_1}{2} + \frac{u_3 f_2\partial_2 f_1}{4} - \frac{u_4 f_1 \partial_2 f_1}{4} - \frac{\partial_2 f_1}{2} - \frac{f_1 f_2}{4},\\
R(\partial_1,\partial_2,\partial_2,\partial_1) &= -\Big(\frac{u_3\partial_{22}^2 f_1}{2} + \frac{u_3 f_2\partial_2 f_1}{4} - \frac{u_4 f_1 \partial_2 f_1}{4} - \frac{\partial_2 f_1}{2} - \frac{f_1 f_2}{4}\Big)\\
R(\partial_1,\partial_2,\partial_1,\partial_3) &= \frac{\partial_2 f_1}{2},\\
R(\partial_1,\partial_2,\partial_3,\partial_1) &= -\frac{\partial_2 f_1}{2},\\
R(\partial_1,\partial_2,\partial_2,\partial_4) &= \frac{\partial_2 f_1}{2},\\
R(\partial_1,\partial_2,\partial_4,\partial_2) &= \frac{\partial_2 f_1}{2}. \qquad (3.6)
\end{aligned}$$

From (3.6), we can see that all components of the Ricci tensor vanishes. Hence the scalar curvature is zero. We have the following result.

Proposition 3.1. *The pseudo-Riemannian metric (3.1) is Einstein.*

If $X = \sum_{i=1}^4 \alpha_i\partial_i$ is a tangent vector on M, then the associated Jacobi operator $J_{\mathcal{R}}(X) = \mathcal{R}(\cdot, X)X$ defines a self-adjoint endomorphism of the tangent space at each point of M given by

$$\begin{aligned} J_{\mathcal{R}}(X)(\cdot) &= \alpha_1^2\mathcal{R}(\cdot,\partial_1)\partial_1+\alpha_1\alpha_2\mathcal{R}(\cdot,\partial_1)\partial_2+\alpha_1\alpha_3\mathcal{R}(\cdot,\partial_1)\partial_3+\alpha_1\alpha_4\mathcal{R}(\cdot,\partial_1)\partial_4\\ &+ \alpha_1\alpha_2\mathcal{R}(\cdot,\partial_2)\partial_1+\alpha_2^2\mathcal{R}(\cdot,\partial_2)\partial_2+\alpha_2\alpha_3\mathcal{R}(\cdot,\partial_2)\partial_3+\alpha_2\alpha_4\mathcal{R}(\cdot,\partial_2)\partial_4\\ &+ \alpha_1\alpha_3\mathcal{R}(\cdot,\partial_3)\partial_1+\alpha_2\alpha_3\mathcal{R}(\cdot,\partial_3)\partial_2+\alpha_3^2\mathcal{R}(\cdot,\partial_3)\partial_3+\alpha_3\alpha_4\mathcal{R}(\cdot,\partial_3)\partial_4\\ &+ \alpha_1\alpha_4\mathcal{R}(\cdot,\partial_4)\partial_1+\alpha_2\alpha_4\mathcal{R}(\cdot,\partial_4)\partial_2+\alpha_3\alpha_4\mathcal{R}(\cdot,\partial_4)\partial_3+\alpha_4^2\mathcal{R}(\cdot,\partial_4)\partial_4. \end{aligned}$$

The matrix associated to Jacobi operator $J_{\mathcal{R}}(X)$ with respect to the basis $\{\partial_i, i=1,2,3,4\}$ is given by

$$(J_{\mathcal{R}}(X)) = \begin{pmatrix} j_{11} & j_{12} & j_{13} & j_{14}\\ j_{21} & j_{22} & j_{23} & j_{24}\\ j_{31} & j_{32} & j_{33} & j_{34}\\ j_{41} & j_{42} & j_{43} & j_{44} \end{pmatrix}, \tag{3.7}$$

where

$$\begin{aligned} j_{11} &= \alpha_1\alpha_2\frac{\partial_2 f_1}{2};\\ j_{21} &= -\alpha_2^2\frac{\partial_1 f_2}{2};\\ j_{31} &= \alpha_2^2\frac{\partial_1 f_2}{2}-\alpha_2\alpha_3\partial_2 f_1-\alpha_2^2\frac{u_3\partial_2^2 f_1}{2}-\alpha_2^2\frac{u_3 f_2\partial_2 f_1}{4}\\ &\quad -\alpha_1\alpha_2\frac{u_3 f_1\partial_2 f_1}{2}-\alpha_2^2\frac{u_4\partial_1^2 f_2}{2}-\alpha_2^2\frac{u_4 f_1\partial_1 f_2}{4}-\alpha_2^2\frac{f_1 f_2}{4}; \end{aligned}$$

$$\begin{aligned}
j_{41} &= \alpha_2\alpha_4\frac{\partial_1 f_2}{2} - \alpha_1\alpha_2\frac{\partial_2 f_1}{2} + \alpha_1\alpha_3\frac{\partial_2 f_1}{2} + \alpha_1\alpha_2\frac{u_3\partial_2^2 f_1}{2} \\
&\quad +\alpha_1\alpha_2\frac{u_3 f_2\partial_2 f_1}{4} + \alpha_1\alpha_2\frac{u_4\partial_1^2 f_2}{2} + \alpha_1\alpha_2\frac{u_4 f_1\partial_1 f_2}{4} \\
&\quad +\alpha_2^2\frac{u_4 f_2\partial_1 f_2}{2} - \alpha_1\alpha_2\frac{f_1 f_2}{4}; \\
j_{12} &= -\alpha_1^2\frac{\partial_2 f_1}{2}; \\
j_{22} &= \alpha_1\alpha_2\frac{\partial_1 f_2}{2}; \\
j_{32} &= -\alpha_1\alpha_2\frac{\partial_1 f_2}{2} + \frac{\alpha_2\alpha_4\partial_1 f_2}{2} + \alpha_1\alpha_3\frac{\partial_2 f_1}{2} + \alpha_1\alpha_2\frac{u_3\partial_2^2 f_1}{2} \\
&\quad +\alpha_1^2\frac{u_3 f_1\partial_2 f_1}{2} + \alpha_1\alpha_2\frac{u_3 f_2\partial_2 f_1}{4} + \alpha_1\alpha_2\frac{u_4\partial_1^2 f_2}{2} \\
&\quad +\alpha_1\alpha_2\frac{u_4 f_1\partial_1 f_2}{4} - \alpha_1\alpha_2\frac{f_1 f_2}{4}; \\
j_{42} &= \alpha_1^2\frac{\partial_2 f_1}{2} - \alpha_1\alpha_4\partial_1 f_2 - \alpha_1^2\frac{u_3\partial_2^2 f_1}{2} - \alpha_1^2\frac{u_3 f_2\partial_2 f_1}{4} \\
&\quad -\alpha_1^2\frac{u_4\partial_1^2 f_2}{2} - \alpha_1^2\frac{u_4 f_1\partial_1 f_2}{4} - \alpha_1\alpha_2\frac{u_4 f_2\partial_1 f_2}{2} + \alpha_1^2\frac{f_1 f_2}{4}; \\
j_{13} &= 0; \\
j_{23} &= 0; \\
j_{33} &= \alpha_1\alpha_2\frac{\partial_2 f_1}{2}; \\
j_{43} &= -\alpha_1^2\frac{\partial_2 f_1}{2}; \\
j_{14} &= 0; \\
j_{24} &= 0; \\
j_{34} &= -\alpha_2^2\frac{\partial_1 f_2}{2}; \\
j_{44} &= \alpha_1\alpha_2\frac{\partial_1 f_2}{2}.
\end{aligned}$$

It follows from the matrix of the Jacobi operator $J_{\mathcal{R}}(X)$, where X is a non null vector on M that te characteristic polynomial satisfies $P_\lambda(J_{\mathcal{R}}(X)) = \lambda^4$.

Proposition 3.2. *The pseudo-Riemannian metric (3.1) is Osserman.*

From Theorem 2.2, by using Propositions 3.1 and 3.2 the pseudo-Riemannian metric (3.1) is conformally Osserman.

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