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Research Paper

ON REGULAR SPHERICAL FUZZY GRAPHS

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Abstract. A spherical fuzzy set is an advanced extension of classical fuzzy set in which it has an added advantage to deal with a wider sense of applicability in uncertain situations. In this paper, regular spherical fuzzy graphs and totally regular spherical fuzzy graphs are introduced. A necessary and sufficient condition for regular spherical fuzzy graphs is given. Some properties of regular and totally regular spherical fuzzy graphs are studied.

Key Words: regular, total degree, totally regular, spherical fuzzy graphs.
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1. Introduction

Mahmood et al. [6] introduced the concept of spherical fuzzy set which gives an additional strength to the concept of picture fuzzy set by enlarging the space for the grades of all the four parameters. Kifayat et al. [5] studied the geometrical comparison of fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets with spherical fuzzy sets. Zuo et al. [12] introduced some new concepts of picture fuzzy graph. Akram et.al [1] introduced the notion of spherical fuzzy graphs and Guleria [2] also introduced generalized version spherical fuzzy graphs using T-spherical fuzzy sets. Gani and Radha [4] discussed regular fuzzy graphs. S. Saxena et. al [11] discussed total fuzzy coloring in fuzzy graphs. Some recent works in intuitionistic fuzzy graphs and spherical

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fuzzy graphs can be found in [7, 8, 9, 10]. In this paper, regular spherical fuzzy graphs and totally regular spherical fuzzy graphs are introduced. A necessary and sufficient condition for regular spherical fuzzy graphs is given. Some properties of regular and totally regular spherical fuzzy graphs are derived.

2. Preliminaries

In this section, some basic definitions are presented which are very useful to understand the concepts of the paper.

Definition 2.1. [6] A spherical fuzzy set S in U (universe of discourse) is given by $S = \{< \alpha, \mu_S(\alpha), \eta_S(\alpha), \nu_S(\alpha) >: \alpha \in U\}$ where $\mu_S : U \to [0,1]$, $\eta_S : U \to [0,1]$ and $\nu_S : U \to [0,1]$ denote degree of membership, degree of neutral membership and degree of non-membership respectively, and for each $\alpha \in U$ satisfying the condition $0 \leq \mu_S^2(\alpha) + \eta_S^2(\alpha) + \nu_S^2(\alpha) \leq 1, \forall \alpha \in U$. The degree of refusal for any spherical fuzzy set S and $\alpha \in U$ is given by $r_S(\alpha) = \sqrt{1 - (\mu_S^2(\alpha) + \eta_S^2(\alpha) + \nu_S^2(\alpha))}$.

Definition 2.2. [1] A spherical fuzzy graph(SFG) $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ where

- $\mathcal{N} = \{v_1, v_2, ..., v_n\}$ such that $\sigma_1 : \mathcal{N} \to [0,1], \sigma_2 : \mathcal{N} \to [0,1]$ and $\sigma_3 : \mathcal{N} \to [0,1]$ denote the degree of membership, degree of neutral membership and degree of non-membership of each element $v_i \in \mathcal{N}$ respectively, and
$$0 \leq \sigma_1^2(v_i) + \sigma_2^2(v_i) + \sigma_3^2(v_i) \leq 1, \tag{2.1}$$
for every $v_i \in \mathcal{N}, (i = 1, 2, 3..., n)$
- $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ where $\mu_1 : \mathcal{L} \to [0,1], \mu_2 : \mathcal{L} \to [0,1]$ and $\mu_3 : \mathcal{L} \to [0,1]$ are such that
$$\begin{aligned}
&\mu_1(u_i, u_j) \leq \min\{\sigma_1(u_i), \sigma_1(u_j)\},\\
&\mu_2(u_i, u_j) \leq \min\{\sigma_2(u_i), \sigma_2(u_j)\},\\
&\mu_3(u_i, u_j) \leq \max\{\sigma_3(u_i), \sigma_3(u_j)\}, \text{ and}\\
&0 \leq \mu_1^2(u_i, u_j) + \mu_2^2(u_i, u_j) + \mu_3^2(u_i, u_j) \leq 1
\end{aligned} \tag{2.2}$$
for every $(u_i, u_j) \in \mathcal{L}, (i, j = 1, 2, 3, ..., n)$.

Definition 2.3. [1] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. The degree of a vertex u of a SFG is $d_{\mathcal{G}}(u) = (\sum_{u \neq v} \mu_1(u, v), \sum_{u \neq v} \mu_2(u, v), \sum_{u \neq v} \mu_3(u, v))$.

Definition 2.4. [1] The minimum degree of a SFG $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is defined by $\delta(\mathcal{G}) = (\delta_1(\mathcal{G}), \delta_2(\mathcal{G}), \delta_3(\mathcal{G}))$, where

$$\delta_1(\mathcal{G}) = \min\{d_1(u)/u \in \mathcal{N}\},$$
$$\delta_2(\mathcal{G}) = \min\{d_2(u)/u \in \mathcal{N}\},$$
$$\delta_3(\mathcal{G}) = \min\{d_3(u)/u \in \mathcal{N}\}.$$

Definition 2.5. [1] The maximum degree of a SFG $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is defined by $\Delta(\mathcal{G}) = (\Delta_1(\mathcal{G}), \Delta_2(\mathcal{G}), \Delta_3(\mathcal{G}))$, where

$$\Delta_1(\mathcal{G}) = \max\{d_1(u)/u \in \mathcal{N}\},$$
$$\Delta_2(\mathcal{G}) = \max\{d_2(u)/u \in \mathcal{N}\},$$
$$\Delta_3(\mathcal{G}) = \max\{d_3(u)/u \in \mathcal{N}\}.$$

Definition 2.6. [1] Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. The total degree of a vertex u of SFG $\mathcal{G}$ is denoted by $td_{\mathcal{G}}(u) = (td_1(u), td_2(u), td_3(u))$ and defined as

$$td_{\mathcal{G}}(u) = (\sum_{u \neq v} \mu_1(u,v) + \sigma_1(u), \sum_{u \neq v} \mu_2(u,v) + \sigma_2(u), \sum_{u \neq v} \mu_3(u,v) + \sigma_3(u))$$

3. Main Results

Definition 3.1. Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. If $d_{\mathcal{G}}(u) = (c_1, c_2, c_3), \forall u \in \mathcal{N}$. (i.e) each vertex has same degree (c_1, c_2, c_3), then $\mathcal{G}$ is said to be regular spherical fuzzy graph(RSFG) of degree (c_1, c_2, c_3) or a $(c_1, c_2, c_3)-$regular spherical fuzzy graph((c_1, c_2, c_3)-RSFG).

Example 3.2. Consider the spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ on $G^* = (V, E)$ given in Figure 1 such that $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$. Here, the degree of a vertex $d(v_i) = (0.2, 0.4, 0.6)$, for all $i = 1, 2, 3, 4, 5$.

Definition 3.3. Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. If $td_{\mathcal{G}}(u) = (r_1, r_2, r_3), \forall u \in \mathcal{N}$. (i.e) each vertex has same total degree (r_1, r_2, r_3), then $\mathcal{G}$ is said to be totally regular spherical fuzzy graph(TRSFG) of total degree (r_1, r_2, r_3) or a $(r_1, r_2, r_3)-$ totally regular spherical fuzzy graph((r_1, r_2, r_3)-TRSFG).

Example 3.4. Consider the spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ on $G^* = (V, E)$ given in Figure 2 such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Here, the total degree of a vertex $td(v_i) = (0.6, 1.2, 1.8)$, for all $i = 1, 2, 3, 4$.

Remark 3.5. Every connected SFG $\mathcal{G}$ with two vertices is always RSFG.

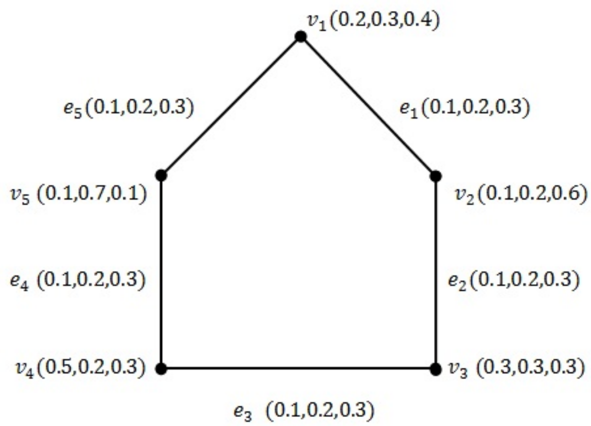

Figure 1. $(0.2, 0.4, 0.6)$-Regular Spherical Fuzzy Graph

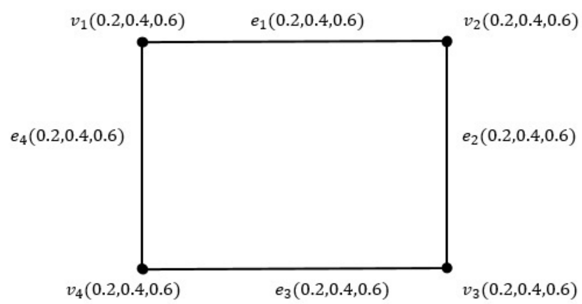

Figure 2. (0.6,1.2,1.8) -Totally Regular Spherical Fuzzy Graph

Example 3.6. Consider the spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ on $G^* = (V, E)$ given in Figure 3 such that $V = \{v_1, v_2\}$ and $E = \{e_1\}$.

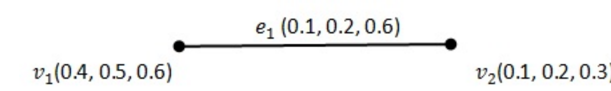

Figure 3. Illustration of Remark 3.5

Remark 3.7. A SFG $\mathcal{G}$ is (c_1, c_2, c_3)-RSFG if and only if $\delta(\mathcal{G}) = \Delta(\mathcal{G}) = (c_1, c_2, c_3)$.

Definition 3.8. Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG, then the order of $\mathcal{G}$ is denoted by $O(\mathcal{G})$ and defined by

$$O(\mathcal{G}) = (\sum_{v\in\mathcal{N}} \sigma_1(v), \sum_{v\in\mathcal{N}} \sigma_2(v), \sum_{v\in\mathcal{N}} \sigma_3(v)).$$

Definition 3.9. Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG, then the size of $\mathcal{G}$ is denoted by $S(\mathcal{G})$ and defined by

$$S(\mathcal{G}) = (\textstyle\sum_{uv\in\mathcal{L}} \mu_1(uv), \sum_{uv\in\mathcal{L}} \mu_2(uv), \sum_{uv\in\mathcal{L}} \mu_3(uv)).$$

Definition 3.10. A path in a spherical fuzzy graph $\mathcal{G}$ is a sequence of distinct vertices $u_0, u_1, u_2, ..., u_n$ such that

$$\mu_1(u_{i-1}, u_i) = \sigma_1(u_{i-1}) \wedge \sigma_1(u_i), 1 \leq i \leq n, n > 0$$
$$\mu_2(u_{i-1}, u_i) = \sigma_2(u_{i-1}) \wedge \sigma_2(u_i), 1 \leq i \leq n, n > 0$$
$$\mu_3(u_{i-1}, u_i) = \sigma_3(u_{i-1}) \vee \sigma_3(u_i), 1 \leq i \leq n, n > 0$$

is called the length of the path. The path in a spherical fuzzy graph is called a spherical fuzzy cycle if $u_0 = u_n, n \geq 3$.

Theorem 3.11. *Let $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a SFG. Then $\mathcal{N}$ is a constant function if and only if the following are equivalent*

(1) *$\mathcal{G}$ is a RSFG,*
(2) *$\mathcal{G}$ is a TRSFG.*

Proof. Suppose that $\mathcal{N}$ is a constant function.
Let $\sigma_1(u) = c_1, \sigma_2(u) = c_2, \sigma_3(u) = c_3, \forall u \in \mathcal{N}$.
Assume that $\mathcal{G}$ is a $(k_1, k_2, k_3)-$ regular spherical fuzzy graph.
Then $d_1(u) = k_1, d_2(u) = k_2, d_3(u) = k_3 \, \forall u \in \mathcal{N}$.
So that

$$\begin{aligned} td_1(u) &= d_1(u) + \sigma_1(u) \\ td_2(u) &= d_2(u) + \sigma_2(u) \\ td_3(u) &= d_3(u) + \sigma_3(u), \forall u \in \mathcal{N} \\ \implies td_1(u) &= k_1 + c_1 \\ td_2(u) &= k_2 + c_2 \\ td_3(u) &= k_3 + c_3, \forall u \in \mathcal{N}. \end{aligned}$$

Hence $\mathcal{G}$ is a totally regular spherical fuzzy graph.
Thus (1) $\implies$ (2)

Now, suppose that $\mathcal{G}$ is a $(r_1, r_2, r_3)-$is a totally regular spherical fuzzy graph.
Then

$$\begin{aligned}
& td_1(u) = r_1 \\
& td_2(u) = r_2 \\
& td_3(u) = r_3, \forall u \in \mathcal{N} \\
\implies & d_1(u) + \sigma_1(u) = r_1 \\
& d_2(u) + \sigma_2(u) = r_2 \\
& d_3(u) + \sigma_3(u) = r_3, \forall u \in \mathcal{N} \\
\implies & d_1(u) + c_1 = r_1 \\
& d_2(u) + c_2 = r_2 \\
& d_3(u) + c_3 = r_3, \forall u \in \mathcal{N} \\
\implies & d_1(u) = r_1 - c_1 \\
& d_2(u) = r_2 - c_2 \\
& d_3(u) = r_3 - c_3, \forall u \in \mathcal{N}.
\end{aligned}$$

So $\mathcal{G}$ is a regular spherical fuzzy graph.
Thus (2) $\implies$ (1) is proved.
Hence (1) and (2) are equivalent.
Conversely, Assume that (1) and (2) are equivalent.
$\mathcal{G}$ is *RSFG* if and only if $\mathcal{G}$ is *TRSFG*.
Suppose $\mathcal{N}$ is not a constant function.
Then $\sigma_i(u) \neq \sigma_i(v)$ for $i = 1, 2, 3$ at least one pair of vertices $u, v \in \mathcal{N}$.
Let $\mathcal{G}$ be a (k_1, k_2, k_3)-*RSFG*.
Then $d_i(u) = d_i(v) = k_i$ where $i = 1, 2, 3$.
So

$$\begin{aligned}
td_1(u) &= d_1(u) + \sigma_1(u) = k_1 + \sigma_1(u), \\
td_2(u) &= d_2(u) + \sigma_2(u) = k_2 + \sigma_2(u), \\
td_3(u) &= d_3(u) + \sigma_3(u) = k_3 + \sigma_3(u),
\end{aligned} \tag{3.1}$$

and

$$\begin{aligned}
td_1(w) &= d_1(w) + \sigma_1(w) = k_1 + \sigma_1(w), \\
td_2(w) &= d_2(w) + \sigma_2(w) = k_2 + \sigma_2(w), \\
td_3(w) &= d_3(w) + \sigma_3(w) = k_3 + \sigma_3(w).
\end{aligned} \tag{3.2}$$

Since

$$\begin{aligned}\sigma_1(u) &\neq \sigma_1(w),\\ \sigma_2(u) &\neq \sigma_2(w),\\ \sigma_3(u) &\neq \sigma_3(w),\end{aligned} \tag{3.3}$$

Therefore,

$$\begin{aligned}td_1(u) &\neq td_1(w),\\ td_2(u) &\neq td_2(w),\\ td_3(u) &\neq td_3(w).\end{aligned} \tag{3.4}$$

So $\mathcal{G}$ is not *TRSFG* which is a contradiction to our assumption.
Hence $\mathcal{N}$ is a constant function. □

Theorem 3.12. *If a spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is both regular and totally regular, then $\mathcal{N}$ is a constant function.*

Proof. Let $\mathcal{G}$ *be a* $(k_1, k_2, k_3)-$ *RSFG* and $(c_1, c_2, c_3)-$ *TRSFG*. Then

$$\begin{aligned}
&d_1(u) = k_1,\\
&d_2(u) = k_2,\\
&d_3(u) = k_3, \forall u \in \mathcal{N}.\\
&td_1(u) = c_1,\\
&td_2(u) = c_2,\\
&td_3(u) = c_3, \forall u \in \mathcal{N}.\\
\Longrightarrow\ &d_1(u) + \sigma_1(u) = c_1,\\
&d_2(u) + \sigma_2(u) = c_2,\\
&d_3(u) + \sigma_3(u) = c_3, \forall u \in \mathcal{N}.\\
\Longrightarrow\ &k_1 + \sigma_1(u) = c_1,\\
&k_2 + \sigma_2(u) = c_2,\\
&k_3 + \sigma_3(u) = c_3, \forall u \in \mathcal{N}.\\
\Longrightarrow\ &\sigma_1(u) = c_1 - k_1,\\
&\sigma_2(u) = c_2 - k_2,\\
&\sigma_3(u) = c_3 - k_3, \forall u \in \mathcal{N}.
\end{aligned}$$

Hence $\mathcal{N}$ is a constant function. □

Remark 3.13. Converse of the above theorem 3.12 need not be true.

Example 3.14. Consider the spherical fuzzy graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ on $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$ given in Figure 4. Here $\sigma(v_i) = (0.1, 0.2, 0.3)$ for all $i = 1, 2, 3, 4$.

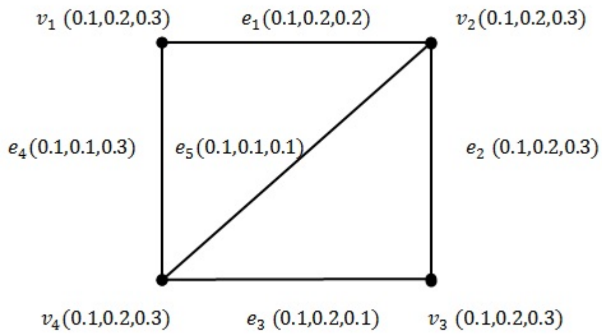

FIGURE 4. Illustration of Remark 3.13

Therefore $\mathcal{N}$ is a constant function.
But $d_G(v_1) = (0.2, 0.3, 0.5) \neq (0.2, 0.4, 0.4) = d_G(v_3)$,
Also $td_G(v_1) = (0.3, 0.5, 0.8) \neq (0.3, 0.6, 0.7) = td_G(v_3)$,
So $\mathcal{G}$ is neither RSFG nor TRSFG.

Theorem 3.15. *If $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a spherical fuzzy graph on G^* where $G^* = (V, E)$ is an odd cycle, then G is regular spherical fuzzy graph iff $\mathcal{L}$ is a constant function.*

Proof. Let $\mathcal{L} = (\mu_1, \mu_2, \mu_3)$ be a constant function.
Then $\mu_1(uv) = c_1, \mu_2(uv) = c_2, \mu_3(uv) = c_3$ where c_1, c_2, c_3 are constants for all $u \in \mathcal{L}$. Therefore, $d_1(v) = 2c_1, d_2(v) = 2c_2, d_3(v) = 2c_3$ for all $v \in \mathcal{N}$.
Hence all the vertices of $\mathcal{G}$ has same degree $(2c_1, 2c_2, 2c_3)$.
Thus $\mathcal{G}$ is regular spherical fuzzy graph.
Conversely, suppose that $\mathcal{G}$ is a (c_1, c_2, c_3) regular spherical fuzzy graph.
Let $e_1, e_2, ..., e_{2n+1}$ be the edges of G^* in that order.
Let $\mu_1(e_1) = k_1, \mu_2(e_1) = k_2, \mu_3(e_1) = k_3$, since $\mathcal{G}$ is $(c_1, c_2, c_3)-$ RSFG.
$\mu_1(e_2) = c_1 - k_1, \mu_2(e_2) = c_2 - k_2, \mu_3(e_3) = c_3 - k_3$
$\mu_1(e_3) = c_1 - (c_1 - k_1), \mu_2(e_3) = c_2 - (c_2 - k_2), \mu_3(e_3) = c_3 - (c_3 - k_3)$
and so on...
Thus,

$$\mu_j(e_i) = \left\{ \begin{array}{rcc} k_j, & \text{if} \quad i & \text{is odd} \\ c_j - k_j, & \text{if} \quad i & \text{is even} \end{array} \right\},$$

for $j = 1, 2, 3$.
Therefore $\mu_j(e_1) = \mu_j(e_{2n+1}) = k_j$.
So if e_1 and e_{2n+1} are incident at a vertex u,then $d_j(u) = c_j$.
Therefore $\mu_j(e_1) + \mu_j(e_{2n+1}) = c_j$
i.e. $k_j + k_j = c_j$ or $k_j = \frac{c_j}{2}$.
Hence $c_j - k_j = c_j - \frac{c_j}{2} = \frac{c_j}{2}$.
Thus $\mu_j(e_i) = \frac{c_j}{2}$ for all $i, j = 1, 2, 3$.
Hence $\mathcal{L} = (\mu_1, \mu_2, \mu_3)$ is a constant function. □

Remark 3.16. The above theorem does not hold for totally regular spherical fuzzy graph($TRSFG$) .

Example 3.17. Consider the spherical fuzzy graph $\mathcal{G}$ given in Figure 5. Here, the total degree of each vertex $td_{\mathcal{G}}(v) = (0.4, 1.0, 1.3), \forall v \in \mathcal{N}$.

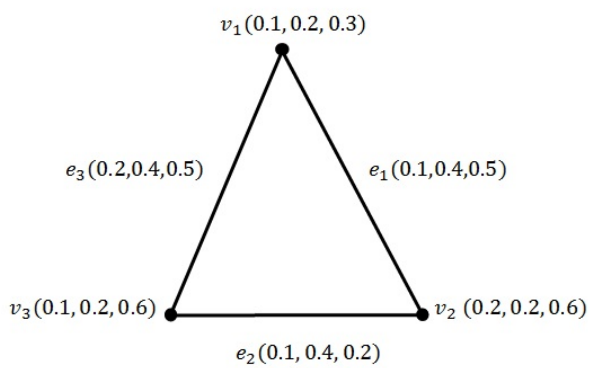

FIGURE 5. Illustration of Remark 3.16

Hence $\mathcal{G}$ is $TRSFG$, but $\mathcal{L}$ is not a constant function.

Theorem 3.18. *If $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ be a spherical fuzzy graph where G^* is an even cycle.Then $\mathcal{G}$ is regular iff either $\mathcal{L}$ is a constant function or alternate edges have same membership values.*

Proof. If either $\mathcal{L}$ is a constant function or alternate edges have same membership values, then $\mathcal{G}$ is regular spherical fuzzy graph.
Conversely,
Suppose $\mathcal{G}$ is a $(c_1, c_2, c_3)-$ regular spherical fuzzy graph.
Let $e_1, e_2, ..., e_{2n}$ be the edges of the even cycle G^* in that order.

Proceeding from the Theorem (3.15),

$$\mu_j(e_i) = \left\{ \begin{array}{rll} k_j, & \text{if} \quad i & \text{is odd} \\ c_j - k_j, & \text{if} \quad i & \text{is even} \end{array} \right\},$$

for $j = 1, 2, 3$.
If $k_j = c_j - k_j$, then $\mathcal{L}$ is a constant function.
If $k_j \neq c_j - k_j$, then alternate edges have same membership values. $\square$

Remark 3.19. The above theorem does not hold for totally regular spherical fuzzy graph.

Example 3.20. Consider the spherical fuzzy graph given in Figure 6. The

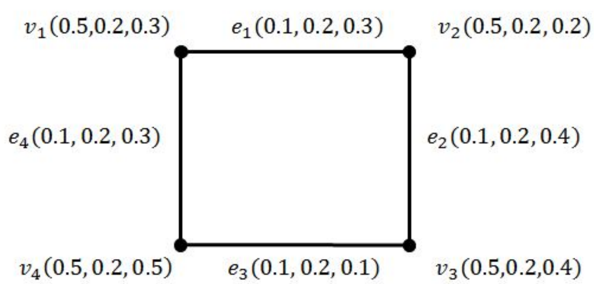

FIGURE 6. Illustration of Remark 3.19

total degree of each vertex $td_{\mathcal{G}}(v) = (0.7, 0.6, 0.9)$.
Therefore, $\mathcal{G}$ is *TRSFG*, but neither L is a constant nor alternate edges have the same membership degrees.

Theorem 3.21. *Let G be a (k_1, k_2, k_3)- regular spherical fuzzy graph on $G^* = (V, E)$. Then $S(\mathcal{G}) = (\frac{pk_1}{2}, \frac{pk_2}{2}, \frac{pk_3}{2})$ where $p = |V|$.*

Proof. The size of $\mathcal{G}$ is

$$S(\mathcal{G}) = (\sum_{uv \in \mathcal{L}} \mu_1(uv), \sum_{uv \in \mathcal{L}} \mu_2(uv), \sum_{uv \in \mathcal{L}} \mu_3(uv))$$

Since (k_1, k_2, k_3)- regular spherical fuzzy graph,

$$\begin{aligned} d_{\mathcal{G}}(u) &= (d_1(u), d_2(u), d_3(u)) \\ &= (k_1, k_2, k_3), \forall u \in \mathcal{N}, \end{aligned}$$

we have,

$$\begin{aligned}\sum_{u\in\mathcal{N}} d_{\mathcal{G}}(u) &= \sum_{u\in\mathcal{N}}(d_1(u), d_2(u), d_3(u))\\ &= (\sum_{u\in\mathcal{N}} d_1(u), \sum_{u\in\mathcal{N}} d_2(u), \sum_{u\in\mathcal{N}} d_3(u))\\ &= (2\sum_{uv\in\mathcal{L}}\mu_1(uv), 2\sum_{uv\in\mathcal{L}}\mu_2(uv), 2\sum_{uv\in\mathcal{L}}\mu_3(uv))\\ &= 2S(\mathcal{G}).\end{aligned}$$

Therefore,

$$\begin{aligned}2S(\mathcal{G}) &= \sum_{u\in\mathcal{N}} d_{\mathcal{G}}(u)\\ &= \sum_{u\in\mathcal{N}}(k_1, k_2, k_3)\\ &= p(k_1, k_2, k_3).\end{aligned}$$

Hence $S(\mathcal{G}) = (\frac{pk_1}{2}, \frac{pk_2}{2}, \frac{pk_3}{2})$. □

Theorem 3.22. *If $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ is a $(r_1, r_2, r_3)-$ totally regular spherical fuzzy graph on $G^* = (V, E)$, then $2S(\mathcal{G}) + O(\mathcal{G}) = p(r_1, r_2, r_3)$ where $p = |V|$.*

Proof. Since $\mathcal{G}$ is a (r_1, r_2, r_3) totally regular spherical fuzzy graph,

$$\begin{aligned}r_1 &= td_1(v)\\ &= d_1(v) + \sigma_1(v),\\ r_2 &= td_2(v)\\ &= d_2(v) + \sigma_2(v),\\ r_1 &= td_3(v)\\ &= d_3(v) + \sigma_3(v), \forall v \in \mathcal{N}.\end{aligned}$$

Therefore,

$$\begin{aligned}\sum_{v\in\mathcal{N}} r_1 &= \sum_{v\in\mathcal{N}} d_1(v) + \sum_{v\in\mathcal{N}}\sigma_1(u)\\ &= 2S_1(\mathcal{G}) + O_1(\mathcal{G}).\end{aligned}$$

Similarly,

$$\sum_{v\in\mathcal{N}} r_2 = 2S_2(\mathcal{G}) + O_2(\mathcal{G}),$$
$$\sum_{v\in\mathcal{N}} r_3 = 2S_3(\mathcal{G}) + O_3(\mathcal{G}).$$

Therefore, $2S_i(\mathcal{G}) + O_i(\mathcal{G}) = p(r_i), \forall i = 1, 2, 3.$
Hence $2S(\mathcal{G}) + O(\mathcal{G}) = p(r_1, r_2, r_3).$ □

Corollary 3.23. *If $\mathcal{G}$ is both (k_1, k_2, k_3)-regular spherical fuzzy graph and a $(r_1, r_2, r_3)-$totally regular spherical fuzzy graph on $G^* = (V, E)$, then $O_i(\mathcal{G}) = p(r_i - k_i), \forall i = 1, 2, 3$ where $p = |V|$.*

Proof. Let $\mathcal{G}$ be both (k_1, k_2, k_3)-regular spherical fuzzy graph and a $(r_1, r_2, r_3)-$totally regular spherical fuzzy graph on $G^* = (V, E)$.
By Theorem 3.21,

$$S_i(\mathcal{G}) = \frac{pk_i}{2},$$
$$2S_i(\mathcal{G}) = pk_i, \forall i = 1, 2, 3.$$

By Theorem 3.22, $2S_i(\mathcal{G}) + O_i(\mathcal{G}) = pr_i, \forall i = 1, 2, 3.$
Therefore, $O_i(\mathcal{G}) = pr_i - 2S_i(\mathcal{G}) = pr_i - pk_i = p(r_i - k_i), \forall i = 1, 2, 3.$ □

4. Conclusion

The spherical fuzzy graphs are more useful than graph structures to tackle with uncertainty. In this article, we have discussed the regular and totally regular spherical fuzzy graphs. Also, we have established the necessary and sufficient conditions for regular and totally regular spherical fuzzy graphs. Further, we will extend this work to some operations on spherical fuzzy graphs.

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