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A Resilience-Oriented Graph-Based Method for Restoration of Critical Loads in Distribution Networks Using Microgrids

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Abstract— *This paper presents a resilience-based approach for critical load restoration in distribution networks using microgrids during extreme events when the main supply is disrupted. Reconfiguration of the distribution network using graph theory is investigated, for which Dijkstra's algorithm is first used to determine the shortest paths between microgrids and critical loads, and then the feasible restoration trees are established by combining the restorable paths. A mixed-integer linear programming (MILP) model is then used to find the optimal selection of feasible restoration trees to make a restoration scheme. The service restoration is implemented with the objectives of maximizing the energy delivered to the critical loads and minimizing the number of switching operations. The limited fuel storage of the generation sources in microgrids, the operational constraints of the network and microgrids, as well as the radiality constraint of the restored sub-networks, are considered the constraints of the optimization problem. The presented method can be used for optimal restoration of critical loads including the number of switching operations which is essential for the ease of implementation of a restoration plan. The results of simulations on a 118-bus distribution network demonstrate the efficiency of the procedure.*

Keywords—Electric vehicles, optimization, particle swarm optimization, cuckoo search algorithm, load demand.

1. INTRODUCTION

Extreme weather events such as hurricanes, and earthquakes, cause extended outages to customers and great economic losses. According to studies conducted in [1], approximately 78% of the outages from 1992 to 2010 were caused by extreme disasters affecting around 178 million metered customers in the US [2]. In these conditions, restoring critical loads (CLs) such as hospitals, water stations, and airports in the shortest possible time is a crucial issue for distribution network operators which has received great attention from researchers in the literature [3]. In this situation, microgrids or distributed energy sources can be used as emergency sources to supply critical loads.

Mathematical optimization approaches have been widely used to solve service restoration problems in the literature [4–9]. In [4], a method based on MILP is used to formulate the problem of critical load restoration by forming microgrids, considering the constraints of network operation and the on/off status of switches. In [5], a method based on stochastic optimization is proposed by creating self-supplied microgrids to continuously supply restored loads by distributed generation sources. An optimization method for locating switching equipment and service restoration simultaneously in the distribution network is proposed in [6]. The objective function is to minimize the total cost of unsupplied required energy and the cost of switching equipment. In [7], the resilience of a distribution network with microgrids in extreme conditions is evaluated by considering factors such as the number of disconnected lines, the probability of load not being fully supplied, and the amount of expected demand not supplied. In [8], the optimal size, number, and location of distributed energy sources for optimal restoration of the distribution network, considering service reliability and investment and maintenance costs, have been investigated. A decentralized multi-agent network approach is proposed in [9], to solve the service restoration problem with the objectives of the maximum amount of restored load and the minimum number of switching operations. The mathematical optimization methods can find the optimal solution but their implementation is much more complicated compared to other optimization approaches.

Meta-heuristic algorithms have been also used for service restoration in distribution networks [10–13]. Meta-heuristics are not problem-specific, but their solution optimality cannot be guaranteed.

Some references have used graph theory-based approaches for load restoration in distribution networks [14–20]. In [14], a spanning tree search algorithm based on graph theory is presented to maximize the restored loads and minimize the number of switching operations. A spanning tree search algorithm is also provided in [15] to minimize out-of-service customers and reduce the operating costs for service restoration in the distribution network. Operating costs include the open or close status of switches using the switching cost coefficients assigned to all branches in the de-energized area. In [16], the spanning tree search algorithm is used to restore critical loads and non-critical loads as much as possible with the objective of minimizing the number of switching operations and total network losses. A graph-based heuristic method for restoring critical loads by microgrids is proposed in [17]. The objectives include maximizing restored energy and minimizing voltage violation of critical loads from the permissible value during restoration. In [18], a critical load restoration strategy based on a graph shortest path algorithm is proposed to maximize the cumulative weighted restoration time by considering the dynamic constraints of microgrids. In [19], a service restoration method for distribution networks is presented by combining intentional islanding of distributed energy sources with network reconfiguration to maximize the restoration of out-of-service loads. In [20], a critical load restoration method is proposed for the formation of microgrids considering master-

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slave distributed generators and topology reconfiguration. The availability of distributed generation sources and the reliability of the restoration plan is considered the objectives of the restoration problem in some references [21], [22]. In [23], A service restoration method for the distribution network using distributed generation sources, electric vehicles, and plug-in hybrid electric vehicles is proposed. The first part of the objective is to maximize the expected supply loads. And in the second part, the risk management tool is included, which can determine the energy plan of the customers. In [24], A service restoration strategy is proposed with the objectives of maximizing the restored energy with load priority, minimizing the path preparation time, and the number of switching operations in different scenarios. In [25], a service restoration strategy using microgrids is proposed considering the demand curve of critical load. The objectives are to maximize the total energy of critical loads and to minimize the cost of switching operations. In [26], A proposed critical load restoration method with the objectives of minimizing the expected cost of importing power from the upstream network and generating power from distributed generation sources, as well as minimizing the expected cost of unused energy after extreme events, has been proposed. In [27], A dual-objective critical load restoration method including maximizing weighted restored loads while minimizing responsive loads is proposed.

The critical and non-critical load restoration methods based on graph theory can be divided into three categories. In references [14–16] and [19], the spanning tree search method is used for the load restoration. The references [17], [22] use a heuristic search method to find all feasible restoration paths and the references [18], [21], [24], [25] use the shortest path search method for critical load restoration. The methods of spanning trees and feasible restoration paths involve a large network topology and a huge number of operation paths compared to the shortest path search method, particularly in large-scale distribution networks.

There are different approaches to choosing the objective function for the restoration problem in the literature. In some studies [19] and [20] the fault duration or restoration time is not considered in the restoration plan. Although the time of fault clearance and reconnecting of the main power supply is not known in advance, however, regarding the fault intensity and the experiences from past events, an approximated duration can be considered for the restoration of critical loads for the optimal use of the fuel reserve of generation sources [17]. It should be noted that the fuel storage of generation sources is restricted and each microgrid can supply critical loads for a limited time besides its local loads. In [18], the objective function is defined in terms of maximizing the total restoration time of the weighted critical loads while increasing the power or energy of the restored loads is not of the objectives of the optimization problem. In this way, the restoration plan will tend to select small critical loads with longer restoration time, and thus the probability of restoring larger ones is reduced. Controlling the number of switching operations required to perform the restoration plan has been considered in some references [6], [9], [11], [13], [13–16], [24] and [25]. Reducing the total number of switching operations is one of the essential factors in facilitating the implementation of the restoration plan and reducing the related operational costs.

The objectives of the service restoration imply that restoration is a complex, multi-objective, and, multi-constraint optimization problem that is not easy to solve and requires special solution approaches. In this paper, a resilience-oriented restoration approach based on the graph theory is presented to maximize the energy delivered to critical loads and minimize the number of switching operations using the extra capacity of microgrids. The presented strategy includes the use of Dijkstra's algorithm to determine optimal restoration paths and find the best restoration plan with the minimum number of switching operations by a MILP optimization model.

The major contributions of this paper are:

1) With the objective of effective restoration of critical loads, weighting factor, rated power, and restoration time of critical loads are considered.
2) The limited amount of power and fuel storage of accessible generation sources in microgrids, as well as the network operational constraints, have been considered in determining the restoration paths.
3) The number of switching operations that is important in reducing operational costs and speeding up the implementation of the restoration plan has been considered.
4) It can be used for emergency conditions with multiple faults, where many areas of the network are de-energized due to severe events.

In this study, it is assumed that the available microgrids in the network, in addition to supplying power to their local loads, have excess power to supply the critical loads of the network. This is done by a central analyzer for fault conditions in the network. The assumptions used in this case are in line with the ones used by previous studies for centralized critical load restoration [17], [18], [22]. Decentralized methods are also used in the literature for service restoration in the distribution network [9], that are beyond the scope of this study.

The remainder of this paper is organized as follows:

The problem formulation for critical load restoration is established in section 2. In section 3, the steps of the proposed method to find the restoration trees are described. Section 4 provides a problem solution using MILP for the optimal selection of restoration trees. section 5 presents simulation results for a 118-bus distribution network. The conclusion is given in section 6.

2. PROBLEM FORMULATION

The objective of this study is to use the capacity of microgrids to supply power to critical loads during extreme events when the main power utility supplying the distribution network is unavailable and the fault continues for a specific time. Microgrids have limited power and energy. Each microgrid must first supply its local load, and if there is additional capacity, it can be used for restoring the grid's critical loads. Note that during the restoration of critical loads, non-critical loads are also restored along the path between the source and each critical load. The main objective is to choose the correct status of the switches (temporary reconfiguration of the network) to provide optimal paths for the restoration of prioritized critical loads by microgrids. For this purpose, the restoration problem is formulated as an optimization problem that is subjected to the constraints related to the power and energy of microgrids, as well as the operational and topological constraints of the network.

2.1. Objective function

A) Energy delivered to critical loads:

The first objective is to maximize the amount of energy supplied by microgrids to restore critical loads in distribution feeders, i.e.,

$$Min. \quad -\sum_{c\in C} W_c P_c T_c. \tag{1}$$

where c is the set of the critical loads and W_c, P_c and T_c are the weighting factor, the rated active power, and the restoration time of the critical load c, respectively.

According to Eq. (1), the objective of the optimization is to maximize the energy delivered to all critical loads considering the weighting factor of restoring critical loads.

B) Number of switching operations:

The number of switching operations determines the efficiency of the restoration plan, as it is closely related to the time required to perform the restoration plan. In addition, increasing the number of switching operations gives rise to the maintenance cost. Therefore, it is desirable to minimize the number of switching operations so that the restoration plan is performed efficiently and timely; Therefore, the second objective function is to minimize the total number of switching operations, i.e.,

$$Min.\ W_{sw} \sum_{i \in S^R} S_i. \tag{2}$$

where S^R and W_{sw} are the set of the switchable lines and the cost factor of switching operation, respectively. S_i, is the switch status of line i with a value equal to 0 or 1.

According to Eq. (2), the objective of the optimization is to minimize the total number of switching operations required to establish all restoration paths.

2.2. Constraints

A) Total energy or fuel storage constraint of generation sources:

The amount of energy that can be delivered by a microgrid to external loads during a major outage is limited.

$$\sum_{i \in G_k^{gen}} P_i T_i \le E_k^{ex} \qquad \forall k \in M. \tag{3}$$

where M is the set of the microgrids and E_k^{ex} is the amount of available energy of the microgrid k. P_i and T_i are the power and service time related to tree i from the set of the restoration trees G_k^{gen}.

B) Load flow constraints:

The load flow equations should be satisfied in restoration paths.

$$P_i^{(a)} = \sum_{j=1}^{n} \left|V_i^{(a)}\right| \left|V_j^{(a)}\right| |Y_{i,j}| \cos(\theta_{i,j} - \delta_i^{(a)} + \delta_j^{(a)}), \tag{4}$$

$$Q_i^{(a)} = -\sum_{j=1}^{n} \left|V_i^{(a)}\right| \left|V_j^{(a)}\right| |Y_{i,j}| \sin(\theta_{i,j} - \delta_i^{(a)} + \delta_j^{(a)}), \tag{5}$$

$$\Delta P_i^{(a)} = P_i^{sch} - P_i^{(a)} < \varepsilon, \tag{6}$$

$$\Delta Q_i^{(a)} = Q_i^{sch} - Q_i^{(a)} < \varepsilon. \tag{7}$$

In Eqs. (4) and (5), P_i^a and Q_i^a are the calculated active and reactive power, V_i^a, V_j^a, δ_i^a, and δ_j are the magnitude and angle of voltages of buses i and j, and $Y_{i,j}$ and $\theta_{i,j}$ are the magnitude and angle of the admittance of the line connected between buses i and j, all related to the iteration a of the Newton_Raphson Algorithm. In Eqs. (6) and (7) P_i^{sch} and Q_i^{sch} represent the planned active and reactive power, and $dPi^{(a)}$ and $dQi^{(a)}$ represent the residual active and reactive power which need to be less than a preset small value of ε to have the convergence being achieved. It should be noted that in load flow calculations, bus 1, is the slack bus, and the other buses are load buses.

C) Operating constraints:

Operating constraints, including bus voltage limits, maximum line current, and maximum active and reactive power generations, are based on the results obtained from load flow as follows.

$$V_i^{\min} \le V_i \le V_i^{\max} \qquad \forall i \in B, \tag{8}$$

$$I_l \le I_l^{\max} \qquad \forall l \in L, \tag{9}$$

$$P_k \le P_k^{\max},\ Q_k \le Q_k^{\max} \qquad \forall k \in M. \tag{10}$$

Eq. (8) imposes the permissible range of bus voltages, where B is the set of restored buses and V_i is the voltage of bus i. $V_i^{\min}$ and $V_i^{\max}$ denote the minimum and maximum bus voltage values, respectively. Due to the limitation of the thermal capacity, the line currents should not exceed a certain value according to Eq. (9) where L is the set of lines in service, I_l is the current flowing through line l and $I_l^{\max}$ is the upper limit of the current of line l. Eq. (10) specifies the limitation of the active and reactive power generations of microgrids. In Eq. (10), M is the set of microgrids, $P_k^{\max}$ and $Q_k^{\max}$ are the upper limits of the active and reactive power of microgrid k, and P_k and Q_k are the active and reactive output power of microgrid k.

D) Topological constraint:

In the restoration problem, the radial structure of the network should be maintained, in other words, each critical load should not be fed by more than one microgrid.

3. PROPOSED METHOD

The steps of the proposed method include determining the critical loads that can be restored by each microgrid, forming all feasible restoration trees to serve critical loads by each microgrid, and finally, determining the best combination of restoration trees for supplying critical loads. The objectives of the optimization include maximizing the total energy delivered to critical loads and minimizing the total number of switching operations.

3.1. Form feasible restoration paths:

In the first step, the feasible restoration paths between microgrids and critical loads are determined. In order to distinguish the paths between each microgrid and critical loads, the distribution network is modeled as an undirected graph G=(V,E) where V is the set of nodes and E is the set of edges [28]. Nodes in V represent loads and microgrids, and edges in E represent switchable lines.

For ease of explanation, the single-line diagram of a simple distribution network is shown in Fig. 1. In normal conditions (without fault), the network is operated with a radial structure. The network is equipped with two tie-line switches with normally open status. The network has two microgrids and four critical loads. A fault occurred in the main feeder and led to an outage of all loads. The objective is to determine the best restoration configuration for supplying critical loads with the extra power of microgrids.

The equivalent graph of the distribution network of Fig. 1 is shown in Fig. 2. According to Fig. 2, the buses and lines in the network's single-line diagram become nodes and edges in the network's equivalent graph, respectively. The weight of an edge is equal to the power of the load being restored by the corresponding line along the restoration path from a microgrid to a specific critical load.

Dijkstra's algorithm [29] is used to find the shortest path from a microgrid to each of the critical loads. The total load power restored along each path is considered as the length of the path. Among all the feasible paths between the microgrid and the desired critical load, the shortest path is the path that has the minimum total weight of the edges, or in other words, the minimum amount of supplied load power. In Fig. 2, out of the 4 feasible paths between microgrid 1 and critical load 1 (dotted lines), the shortest path (highlighted in green) is determined by Dijkstra's algorithm and is the path where the total power of non-critical loads is the minimum.

The paths found are considered feasible for restoration if the operating constraints including the bus voltage limit, the maximum line current, and the maximum power capacity of the microgrid are met, otherwise, the determined path is unfeasible. In order to reduce the number of load flow calculations, it is first checked

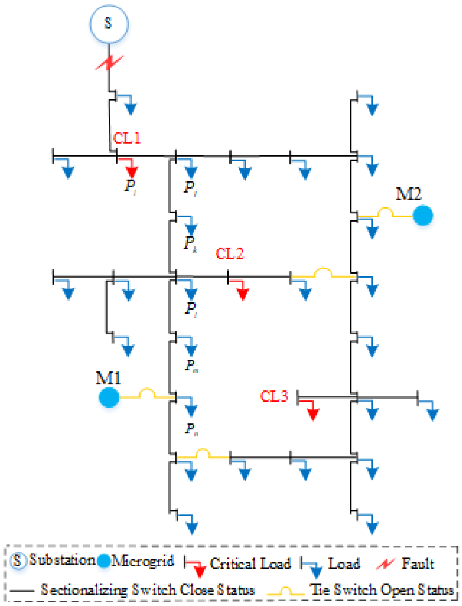

Fig. 1. One-line diagram of the example distribution network (including two microgrids and three critical loads).

whether the total load value of the determined path is greater than the maximum power available in the corresponding microgrid or not. If yes, the determined path is unfeasible restoration without the need to perform load flow calculation. Otherwise, a load flow calculation will be performed.

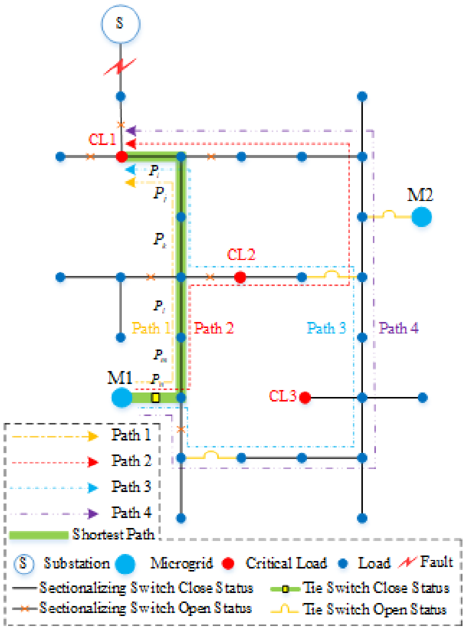

Fig. 2. Converting the single-line diagram of the simple distribution network to a graph and determining the shortest restoration path from each microgrid to each critical load (the dashed lines represent all feasible paths and the solid line represents the shortest restoration path from microgrid 1 to critical load 1).

3.2. Dijkstra's algorithm

Given a graph and a source node in the graph, Dijkstra's algorithm finds the shortest paths from the source to all nodes in the given graph. The algorithm is formulated as below.

Table 1. Dijkstra's algorithm.

```
function Dijkstra(Graph, source):

    for each vertex v in Graph.Vertices:
        dist[v] ← INFINITY
        prev[v] ← UNDEFINED
        add v to Q
    dist[source] ← 0

    while Q is not empty:
        u ← vertex in Q with min dist[u]
        remove u from Q

        for each neighbor v of u still in Q:
            alt←dist[u]+Graph.Edges(u,v)
            if alt < dist[v] and dist[u] is not INFINITY:
                dist[v] ← alt
                prev[v] ← u

return dist[], prev[]
```

1) Mark all nodes unvisited. Create a set of all the unvisited nodes called the unvisited set, Q.
2) Assign to every node a tentative distance value: set it to zero for the initial node and to infinity for all other nodes. The tentative distance of node v is the length of the shortest path between node v and the starting node. Set the initial node as current.
3) For the current node, consider all of its unvisited neighbors and calculate their tentative distances through the current node. Compare the newly calculated tentative distance to the one currently assigned to the neighbor and assign it to the smaller one.
4) When considering all of the unvisited neighbors of the current node, mark the current node as visited and remove it from the unvisited set. A visited node will never be checked again.
5) If the destination node has been marked visited (when planning a route between two specific nodes) or if the smallest tentative distance among the nodes in the unvisited set is infinity (when planning a complete traversal; occurs when there is no connection between the initial node and remaining unvisited nodes), then stop. The algorithm has finished.
6) Otherwise, select the unvisited node that is marked with the smallest tentative distance, set it as the new current node, and go back to step 3.

In the pseudocode shown in Table 1, dist[u] is the current distance from the source to the vertex u. The prev[v] contains the predecessors of vertex v. Thus, after the implementation of the algorithm, following the previous vertices from the destination to the source, the shortest path between two points is found. The code min dist[u], searches for the vertex u in the vertex set Q that has the least dist[u] value. Graph.Edges(u,v) returns the distance between the two neighbor nodes u and v. The variable alt is the length of the path from the root node to the neighbor node v if it were to go through u. If this path is shorter than the current shortest path recorded for v, that current path is replaced with this alt path.

3.3. Form feasible restoration trees:

The second step is to restore two or more critical loads using a microgrid through a restoration tree. A restoration tree for each microgrid is determined from the combination of two or more feasible restoration paths related to that microgrid. Using feasible restoration paths obtained in the first step, all feasible restoration trees for each microgrid are determined. Then, load flow calculation is applied in order to check the feasibility of critical load restoration in each feasible restoration tree.

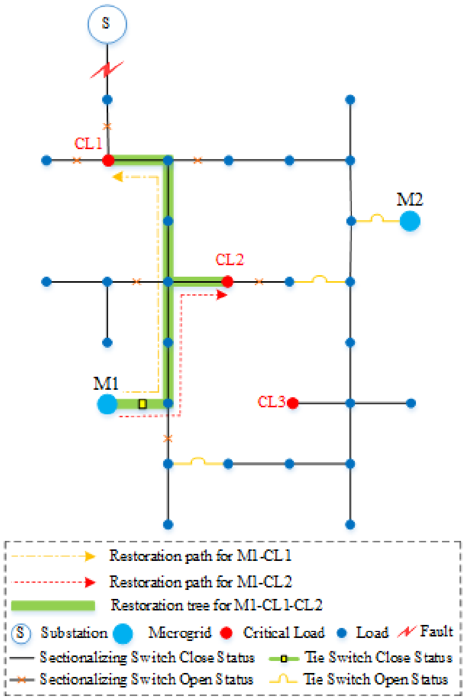

Fig. 3. Formation of the restoration tree for each microgrid in order to feed critical loads (for example, the restoration tree of microgrid 1 to feed critical loads 1 and 2).

Similar to the formation of feasible restoration paths, all operational constraints must be satisfied for each feasible restoration tree, otherwise, it is removed from the list of feasible restoration trees. This way, a set of feasible restoration trees is established with all the operational constraints met. For example, according to Fig. 3, from the combination of feasible restoration paths of critical loads 1 and 2 (dotted lines), a restoration tree (green highlight) is determined for microgrid 1. If all the operational constraints are fulfilled according to load flow calculations, this tree is considered a feasible restoration tree for supplying critical loads 1 and 2 by microgrid 1.

4. CRITICAL LOAD RESTORATION: PROBLEM SOLUTION

In this section, the symbol j is used for indexing feasible restoration trees. The restoration time of tree j, T_j^R, is calculated as $T_j^R = E_{ext}^k / P_j$. E_{ext}^k and P_j are the maximum available energy of the corresponding microgrid, and the total power of the selected feasible restoration tree, respectively. In this way, the energy constraint of generation sources is considered in the service restoration scheme.

The optimal selection of the feasible restoration trees can be treated as a MILP optimization problem with linear objective function subject to linear unequal constraints as follows.

4.1. Objective function:

In the first scenario, the objective function of the problem is to maximize the amount of energy delivered to the critical loads, considering the weighting factor of the loads, i.e.,

$$Min.\ -\sum_{j:1}^{G_{uni}} Z_j T_j^R \sum_{c:1}^{C_j} P_j^c W_j^c . \tag{11}$$

where G_{uni} is the universal set of the feasible restoration trees and C_j is the set of the critical loads energized within the feasible restoration tree j. Z_j is the binary variable used for the selection status of the feasible restoration tree j and T_j^R is the service time of the feasible restoration tree j. P_j^c, and W_j^c are the nominal active power, and the weighting factor of the critical load i related to the feasible restoration tree j, respectively.

In the second scenario, the objective function is developed to include the number of switching operations besides the energy delivered to the critical loads, i.e.,

$$Min.\ -\sum_{j:1}^{G_{uni}} Z_j T_j^R \sum_{c:1}^{C_j} P_j^c W_j^c + W_{sw} \sum_{j:1}^{G_{uni}} Z_j S_j . \tag{12}$$

where W_{sw} is the switching cost factor, and S_j is the number of switching operations required to isolate the feasible restoration tree j. It is worth mentioning that W_{sw} is selected deliberately with a process of trial and error, so that the first term of the objective function, i.e. maximizing the energy delivered to critical loads, has a higher priority than the second term of the objective function, i.e. minimizing the number of switching operations.

4.2. Unequal constraints:

1) Each microgrid can supply critical loads through only one restoration tree, i.e.,

$$\sum_{j:1}^{G_k^{gen}} Z_j \leq 1 \qquad \forall k \in M. \tag{13}$$

Where M is the set of microgrids and G_k^{gen} is the set of the feasible restoration trees supplied by microgrid k.

Applying condition Eq. (13) causes only one restoration tree to be selected among all feasible restoration trees related to a microgrid.

2) Each critical load is supplied by only one feasible restoration tree. In other words, one critical load is not supplied by two or more microgrids, i.e.,

$$\sum_{j=1}^{G_i^{cl}} Z_j \leq 1 \qquad \forall i \in C. \tag{14}$$

Where G_i^{cl} is the set of the feasible restoration trees restoring the critical load i and C is the set of the critical loads.

3) Each non-critical load energized by a microgrid inside a restoration tree cannot be restored by other restoration trees, i.e.,

$$\sum_{j=1}^{G_n^{l}} Z_j \leq 1 \qquad \forall n \in B. \tag{15}$$

Where G_n^l is the set of the feasible restoration trees restoring the non-critical load n and B is the set of all non-critical loads which can be served by the feasible restoration trees.

This condition prevents restoration trees from intersecting or having a common node. The radiality constraint of the problem is held by Dijkstra's algorithm (loop-free restoration paths) and using isolated restoration trees.

The flowchart of the proposed method for critical load restoration is illustrated in Fig. 4.

5. RESULTS AND DISCUSSION

The proposed method has been implemented for the restoration of the critical loads in a 118-bus distribution network [30] using the capacity of the microgrids. The load flow calculation for restoration paths and restoration trees is performed using the Newton-Raphson algorithm by the MatPower 7.1 toolbox

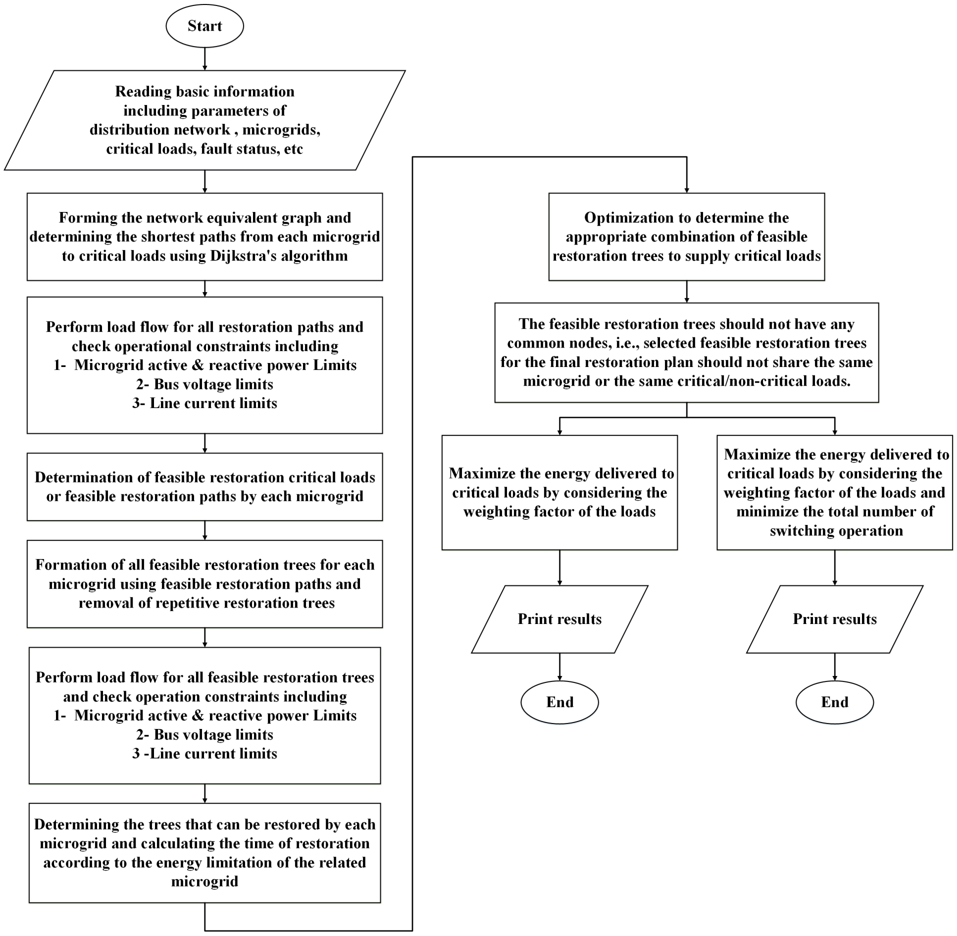

Fig. 4. Flowchart of the proposed model for the restoration of critical loads.

[31]. To solve the optimization problem with MILP formulation, the “Intlinprog” command is used in MATLAB R2018b. The simulation is carried out on a PC with an Intel Core i7-8550 @ 3.4 GHz processor and 12 GB RAM.

Fig. 5 shows the equivalent graph of the studied 118-bus distribution network. The network contains 12 critical loads on buses 4, 7, 14, 46, 47, 55, 60, 63, 73, 79, 92, and 103. The parameters of critical loads, including weighting factor and active and reactive powers, are given in Table 2. The critical loads 7, 47, and 103 have high priority with a weighting factor of 3, the critical loads 14, 63, and 92 have average priority with a weighting factor of 2, and the critical loads 4, 46, 55, 60, 73, and 79 have low priority with a weighting factor of 1. The weighting factor for non-critical loads is set to zero. Six microgrids are connected to buses 27, 28, 62, 65, 77, and 110. Table 3 shows the capacity of generation sources and their local loads with a power factor of 0.9. The equivalent circuit used for each microgrid includes a source and a local load, as shown in Fig. 6. After an extreme event, the main supply is unavailable and six faults occurred in the distribution network. The corresponding faulted lines are 5-6, 20-21, 25-26, 32-33, 67-68, and 101-102, as shown in Fig. 5. The outage duration is estimated to be 16 hours.

5.1. Form feasible restoration paths

According to Table 4, 35 feasible restoration paths for supplying 12 critical loads can be formed by the available microgrids. As can be seen, due to the network's operational constraints and the limited power of the generation sources, the number of restorable critical loads for each microgrid is different.

Table 2. Critical load parameters.

Critical Loads	P (kW)	Q (kVar)	Load Weight Factor
CL4	34.315	21.845	1
CL7	104.47	61.725	3
CL14	141.9	117.5	2
CL46	39.653	20.758	1
CL47	66.195	42.361	3
CL55	62.1	26.86	1
CL60	80.551	49.156	1
CL63	478.8	463.74	2
CL73	52.699	22.482	1
CL79	294.55	162.47	1
CL92	114.57	81.748	2
CL103	408.43	168.46	3

Table 3. Capacity of generation sources and local loads within microgrids.

Microgrid ID	Maximum Real Power (MW)	Maximum Reactive Power (MVAr)	Fuel reserve (MWh)	Demand (MW) PF=0.9
M1	4.52	2.17	35	2.26
M2	5.57	2.7	45	2.78
M3	7.02	3.4	80	3.51
M4	5.3	2.62	40	2.65
M5	3.6	1.72	30	1.8
M6	7.37	3.55	65	3.68

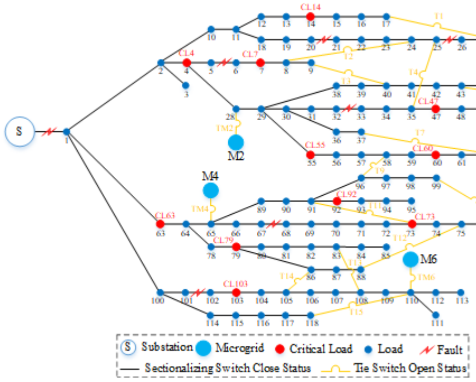

Fig. 5. Equivalent graph of the studied 118-bus distribution network, including microgrids, critical loads, and incident faults.

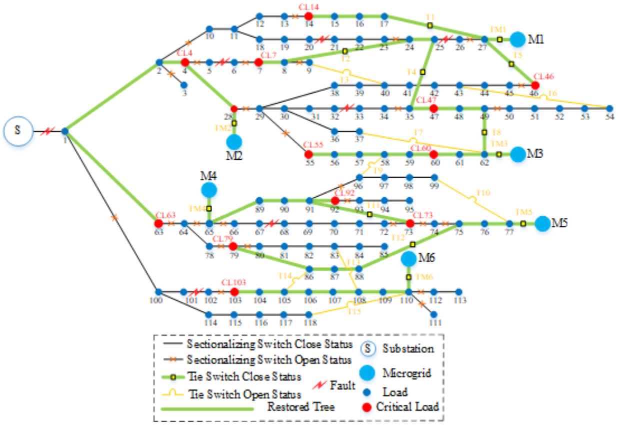

Fig. 7. The restored 118-bus distribution network (first scenario).

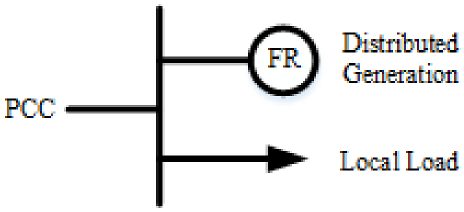

Fig. 6. Microgrid equivalent circuit.

5.2. Form feasible restoration trees

For microgrids 1, 2, 3, 4, 5, and 6, respectively, 9, 15, 17, 42, 3, and 6 feasible restoration trees can be formed from the combination of the feasible restoration paths related to each microgrid. A total of 92 feasible restoration trees are established for all microgrids. The weighted energy supplied to the critical loads, and the related restoration time have been calculated for each of the feasible restoration trees.

5.3. Simulation results and discussion

In this section, the performance of the proposed method for critical load restoration in two different scenarios is investigated.

Scenario I - Optimization for Energy of Critical Loads:

In this scenario, the objective of the service restoration is to maximize the amount of energy delivered to critical loads, regardless of the number of switching operations. Fig. 7. shows the arrangement of the restored network with green highlights for the restored trees. Restored critical loads, restoration times, and the amount of weighted and unweighted energy delivered to critical loads by each microgrid are given in Table 5. In Table 6, the operated sectionalizing and tie switches, as well as the total number of switching operations required for isolating the restoration trees, are shown.

In this scenario, critical loads 14 and 46 are restored by microgrid 1, and critical loads 4 and 63 are restored by microgrid 2. Similarly, critical loads 7, 47, 55, and 60 are restored by microgrid 3, critical loads 63 and 92 are restored by microgrid 4, critical load 79 is restored by microgrid 5, and finally, critical load 103 is restored by microgrid 6. All critical loads by the respective supplied by any of the microgrids in this scenario.

According to Tables 4 and 5, microgrid 3 supplies three critical loads 7, 47, and 60, with the maximum number of switching operations to make the corresponding restoration tree, i.e., 10 switching operations, including opening 6 sectionalizing switches and closing 4 tie switches. According to Table 6, the total number of switching operations required for restoring critical loads by all microgrids is 40. Therefore, the restoration tree associated with microgrid 3 includes nearly a quarter of the total number of switching operations. However, according to Table 5, the energy restored by this restoration tree is equal to 4.83 kWh, which is only 21.21% of the total energy restored.

Fig. 8 shows the weighted energy restored per unit number of switching operations in each restoration tree. As can be seen, the corresponding value for the feasible restoration tree associated with microgrid 3 is low compared with other feasible restoration trees, indicating the inefficiency of this tree in terms of the required number of switching operations. The total number of switching operations is significant for performing the restoration plan in the shortest possible time. The selection of restoration trees with relatively high restored energy but extensive switching operations can be avoided by including the number of switching operations in the objective function. As an alternative, restoration trees with lower restored energy but a smaller number of switching operations can be used.

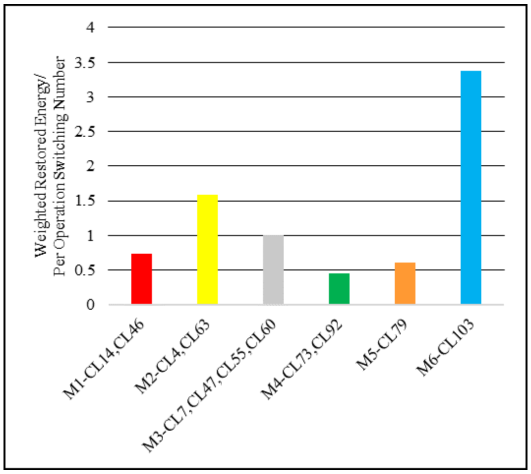

Fig. 8. Comparison of weighted energy restored per unit number of switching operations in each restoration tree (first scenario).

Scenario II - Optimization for Energy of Critical Loads and Switching Operations:

In this scenario, the main objectives are to maximize the weighted energy delivered to the critical loads and minimize the number of switching operations simultaneously.

The arrangement of the restored network is illustrated in Fig. 9. Restored critical loads, restoration time, and restored energy for each microgrid are given in Table 7. Details of the switching operations are also provided in Table 8. Similarly, critical loads 7, 47, 55, and 60 are restored by microgrid 3, critical load 63, 79, and 92 is restored by microgrid 4, and finally, critical load

Table 4. Feasible restoration paths for each microgrid.

Microgrids	Number Path	Restoration Paths
M1	1	M1-27-17-16-15-14-13-12-11-10-2-4
	2	M1-27-17-16-15-14
	3	M1-27-46
	4	M1-27-17-16-15-14-13-12-11-10-2-1-63
M2	1	M2-28-4
	2	M2-28-29-38-39-40-9-8-7
	3	M2-28-4-2-10-11-12-13-14
	4	M2-28-4-2-10-11-12-13-14-15-16-17-27-46
	5	M2-28-29-55
	6	M2-28-29-55-56-57-58-59-60
	7	M2-28-4-2-1-63
M3	1	M3-62-49-48-47-35-25-24-8-7
	2	M3-62-49-48-47
	3	M3-62-61-60-59-58-57-56-55
	4	M3-62-61-60
	5	M3-62-61-60-59-58-96-91-92
	6	M3-62-61-60-59-58-96-91-73
M4	1	M4-65-64-63-1-2-4
	2	M4-65-64-63-1-2–10-11-12-13-14
	3	M4-65-64-63-1-2–10-11-12-13-14-15-16-17-27-46
	4	M4-65-89-90-91-96-58-59-60-61-62-49-48-47
	5	M4-65-89-90-91-96-58-57-56-55
	6	M4-65-89-90-91-96-58-59-60
	7	M4-65-89-90-91-92
	8	M4-65-64-63
	9	M4-65-89-90-91-73
	10	M4-65-64-78-79
	11	M4-65-64-78-79-86-105-104-103
M5	1	M5-77-99-98-97-96-91-92
	2	M5-77-76-75-74-73
	3	M5-77-76-75-85-88-87-86-79
M6	1	M6-110-118-117-116-115-114-100-1-2-4
	2	M4-110-118-117-116-115-114-100-1-2-10-11-12-13-14
	3	M4-110-118-117-116-115-114-100-1-63
	4	M4-110-109-108-107-106-105-104-103

Table 5. Optimal restoration plan parameters (first scenario).

Micro Grids	Restoration Critical Loads	Restoration Time (h)	Restored Energy (MWh)	Weighted Restored Energy (MWh)
M1	CL14,CL46	13.68	2.48	4.42
M2	CL4,CL63	11.16	5.73	11.07
M3	CL7,CL47,CL55,CL60	15.42	4.83	10.09
M4	CL73,CL92	12.85	2.14	3.62
M5	CL79	10.52	3.09	3.09
M6	CL103	11.03	4.5	13.5

Table 6. Switching operations required for restoration (first scenario).

Micro Grids	Sectionalizing Switch Operation	Tie Switch Operation	Switching Operation
M1	13-14,26-27,45-46	TM1,T1,T5	6
M2	28-29,4-5,2-10,2-3,1-100,63-64	TM2	7
M3	29-55,49-50,34-35,23-24,8-9,6-7	TM3,T8,T4,T2	10
M4	64-65,65-66,72-73,73-74,92-93,91-96	TM4,T11	8
M5	74-75,78-79,79-80	TM5,T12	5
M6	111-112,102-103,110-111	TM6	4

103 is restored by microgrid 6. As is clear, by including the cost of switching operations in the objective function, critical loads 4, 46, and 73 with low priority (weighting factor of 1) are not restored by any of the microgrids. Also, microgrids 2 and 5 do not participate in the load restoration plan.

Fig. 10 shows the weighted energy restored per unit number of switching operations in each restoration tree in the second scenario. As can be seen from this figure, considering the number of switching operations can prevent the selection of restoration trees with a large number of switching operations.

The total weighted restored energy as well as the total number of switching operations for the two studied scenarios are compared in Table 9. According to this table, the total weighted energy supplied to critical loads has decreased in the second scenario, but instead, the number of switching operations has reduced from 40 to 25, indicating an almost 40% reduction in switching operations. Reducing switching operations facilitates the implementation of the restoration plan in the faulty network.

As explained in Section 2, in the multi-objective optimization approach (second scenario), the switching cost coefficient W_{sw} is selected in such a way that maximizing the weighted energy delivered to critical loads has a higher priority than minimizing the number of switching operations. By increasing W_{sw}, the use of restoration trees with a lower number of switching operations becomes more important, whereas by decreasing W_{sw}, the importance of the number of switching operations is reduced so that forW_{sw} equal to zero, the only priority is to maximize the weighted restored energy in the critical loads.

Table 7. Optimal restoration plan parameters (second scenario).

Micro Grids	Restored Critical Loads	Restoration Time (h)	Restored Energy (MWh)	Weighted Restored Energy (MWh)
M1	CL14	13.9	1.97	3.94
M3	CL7,CL47,CL55,CL60	15.42	4.83	10.09
M4	CL63,CL79,CL92	9.52	8.45	14.1
M6	CL103	11.03	4.5	13.5

Table 8. Switching operations required for restoration (second scenario).

Micro Grids	Sectionalizing Switch Operation	Tie Switch Operation	Switching Operation
M1	13-14,26-27	TM1,T1	4
M3	29-55,49-50,34-35,23-24,8-9,6-7	TM3,T2,T4,T8	10
M4	1-63,65-66,79-80,79-86,91-96,92-93	TM4	7
M6	111-112,102-103,110-111	TM6	4

Table 9. Total restored energy and the total number of switching operations in the first and second scenario.

States Parameters	Optimization Results without Switching Operations	Optimization Results with Switching Operations
Total Restored Energy (MWh)	22.77	19.75
Total Weighted Restored Energy (MWh)	45.79	41.63
Total Number of Switching Operations	40	25

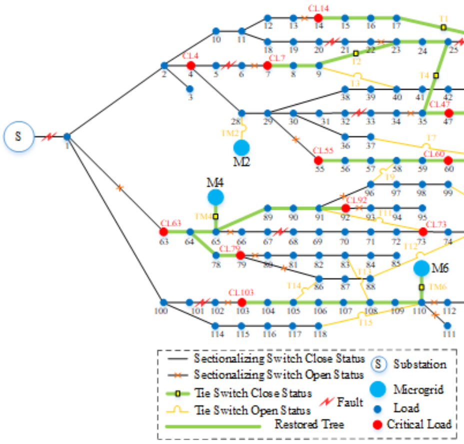

Fig. 9. The restored 118-bus distribution network (second scenario).

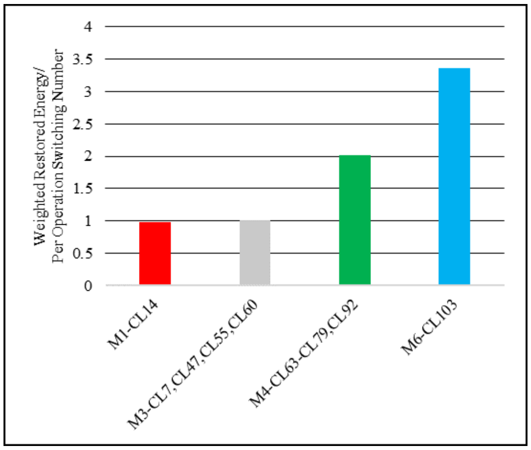

Fig. 10. Comparison of weighted energy restored per unit number of switching operations in each restoration tree (second scenario).

5.4. Performance comparison

To better understand the efficiency of the method presented in the current study, the results of the critical load restoration in the 118-bus distribution network have been compared with the results of the method presented in [18]. The network parameters and the fault condition are the same ones used in previous sections. The comparison results for 5 distinct cases with different coefficients applied to the active power of the microgrids are shown in Table 10. According to Table 10, in the first case, when the power reduction coefficient of all microgrids is equal to 1 (nominal power values), and also in the second case, for a moderate reduction in the output power of the microgrids, the power, energy and weighted energy restored in critical loads are the same for both methods. With the further reduction of the output powers, that is, when the additional capacity of the microgrids to feed the critical loads in the network is reduced, the power, energy and weighted energy restored in critical loads using the proposed method is more than of the method of [18] (Cases 3, 4, and 5). It should be noted that the power reduction coefficients are randomly created in the range of 0.6-0.8 for Case 2 and in the range of 0.4-0.6 for Cases 3 to 5. As can be seen, the proposed method is superior in critical load restoration with the limited capacity of generation resources. Note that since the objective function in [18] is formulated based on maximizing the weighted sum of the restoration times of the critical loads, smaller critical loads with longer restoration times are preferred to be supplied by the restoration procedure in [18]. This feature can result in non-optimal solutions especially in case of limited generation capacities as can also be seen from Table 10.

6. CONCLUSIONS

This paper presents a resilience-oriented method for critical load restoration using the capacity of microgrids after an extreme event based on the graph shortest path theory. The restoration objectives are to maximize the energy delivered to critical loads considering the load priority and minimize the number of switching operations. The method includes finding the shortest feasible restoration paths by Dijkstra's algorithm, forming the feasible restoration trees using the feasible restoration paths, and optimal selection of the restoration trees using a MILP optimization approach. The simulation results on a 118-bus distribution network showed that as a result of using the proposed method, it is possible to restore the total critical load of 22.77 MWh by using the excess capacity of microgrids observing the network's operational constraints. Also, considering the number of switching operations in the objective function, although the total amount of restored load is reduced to 19.75 MWh, the total number of switching operations decreased from 40 to 25, demonstrating an approximately 40% reduction in the total number of switching operations. The simulation results prove the high efficiency of the proposed method in restoring the

Table 10. Comparison of the proposed method with [18].

Case no.	Microgrids Active Power Coefficients (f)	Total Restored Power (MW)		Total Restored Energy (MWh)		Total Weighted Restored Energy (MWh)	
		Proposed Method	[18]	Proposed Method	[18]	Proposed Method	[18]
1	fM1=1, fM2=1 fM3=1, fM4=1 fM5=1, fM6=1	1.8782	1.8782	22.8017	22.8017	45.8382	45.8382
2	fM1=0.8, fM2=0.6 fM3=0.6, fM4=0.7 fM5=0.8, fM6=0.7	1.4698	1.4698	17.7018	17.7018	31.7194	31.7194
3	fM1=0.5, fM2=0.4 fM3=0.4, fM4=0.6 fM5=0.5, fM6=0.6	0.8653	0.5538	9.9619	6.9876	21.4074	14.8427
4	fM1=0.6, fM2=0.4 fM3=0.4, fM4=0.5 fM5=0.6, fM6=0.5	0.8653	0.5538	9.7224	6.9585	20.9355	14.818
5	fM1=0.4, fM2=0.5 fM3=0.5, fM4=0.6 fM5=0.4, fM6=0.6	1.008	0.6965	11.6776	8.7033	23.1121	16.5474

critical loads considering the limited fuel storage of microgrids and the operational and topological constraints of the restored sub-networks. Using the proposed method it is also possible to optimize the total number of switching operations which can help implement the restoration plan more effectively in a shorter time. The proposed method is applicable to large distribution networks for optimal use of the extra capacity of microgrids to restore the critical loads of the network with the minimum number of switching operations.

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