

On Riemannian and Ricci curvatures of Ingarden-Támassy metrics

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Abstract. In this paper, we study reversibility of Riemann Curvature and Ricci curvature for the Ingarden-Támassy metric and prove two global results. First, we prove that a Ingarden-Támassy metric is R-reversible if and only if $s_i = 0, s_{ij|k} = 0$. Then we show that a Ingarden-Támassy metric is Ricci-reversible if and only if $s_i = 0$.

Keywords: Ingarden-Támassy metric, Riemannian curvature, Ricci curvature.

1. Introduction

In this paper, we study the following Finsler metric

$$F = \alpha + \frac{\beta^2}{\alpha}, \quad (1.1)$$

where $\alpha = \sqrt{a_{ij}(x)y^i y^j}$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a 1-form on M . It is remarkable that, this metric was introduced by R. Ingarden and S. Tamássy in [5], when they were studying physical applications of Finsler metrics in electron optic and thermodynamic. Then the Finsler metric (1.1) is called the Ingarden-Tamássy metric.

Recently, Crampin proved that a Randers metric $F = \alpha + \beta$ has reversible geodesics if and only if β is parallel with respect to α [2]. Then Masa-Sabau-Shimada investigate (α, β) -metrics with reversible geodesics and projectively

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reversible geodesics [7][8]. In general, the Finsler metrics might not be reversible. In spite of the non-reversibility of Finsler metrics, the geodesics and curvatures might be reversible.

In [10], Shen-Yang introduced R-reversibility and Ricci-reversibility. They proved that Randers metrics are R-reversible or Ricci-reversible if and only if they are R-quadratic or Ricci-quadratic, respectively. Recently, in [14], Tayebi-Tabatabaeifar studied the reversibility of Riemann curvature and Ricci curvature for a Matsumoto metric $F = \alpha^2/(\alpha - \beta)$ which is called by Matsumoto's slope-of-a-mountain metric, also. This metric was introduced by Matsumoto as a realization of Finsler's idea "a slope measure of a mountain with respect to a time measure" [11][12]. He gave an exact formulation of a Finsler surface to measure the time on the slope of a hill and introduced the Matsumoto metrics in [9][13].

The Riemann curvature $\mathbf{R}_y : T_x M \rightarrow T_x M$ is a family of linear maps on tangent spaces. A Finsler metric F on a manifold M is said to be R-quadratic if it's Riemann curvature \mathbf{R}_y is quadratic in $y \in T_x M$ [6][10]. Also, F is called R-reversible if $\mathbf{R}(x, y) = \mathbf{R}(x, -y)$ (see [10]).

A Finsler metric F is called Ricci-quadratic if it's Ricci curvature \mathbf{Ric}_y , is quadratic in $y \in T_x M$. F is called Ricci-reversible if $\mathbf{Ric}_y = \mathbf{Ric}_{-y}$ (see [10]). In this paper, we are going to prove the following:

Theorem 1.1. *The Ingarden-Támassy metric is R-reversible if and only if β satisfies $s_i = 0$ and $s_{ij|k} = 0$.*

Theorem 1.2. *The Ingarden-Támassy metric is Ricci-reversible if and only if β satisfies $s_i = 0$.*

2. Preliminaries

Let M be an n -dimensional C^∞ -manifold. Denote by $T_x M$ the tangent space at $x \in M$ and by $TM := \cup_{x \in M} T_x M$ the tangent bundle of M . Each element of TM has the form (x, y) , where $x \in M$ and $y \in T_x M$. Let $TM_0 = TM \setminus \{0\}$ [15]. A Finsler metric on M is a function $F : TM \rightarrow [0, \infty)$, with the following properties:

- (i) F is C^∞ on TM_0 ,
- (ii) F is positively 1-homogeneous on the fibers of tangent bundle TM ;
- (iii) The Hessian of $F^2/2$ with element $g_{ij} = \frac{1}{2} \frac{\partial^2 F^2}{\partial y^i \partial y^j}$ is positive definite on TM_0 .

The pair $F^n = (M, g)$ is called a Finsler space of dimension n . F is called fundamental function and g_{ij} is called the fundamental tensor of the Finsler space F^n . By the homogeneity of F , we have $F(x, y) = \sqrt{g_{ij}(x, y)y^i y^j}$. An important class of Finsler metrics are Riemann metrics, which are in the form of $F(x, y) = \sqrt{g_{ij}(x)y^i y^j}$. Another important class of Finsler metrics are Minkowski metrics, which are in the form of $F(x, y) = \sqrt{g_{ij}(y)y^i y^j}$ [4].

Given a Finsler manifold (M, F) , then a global vector field \mathbf{G} is induced by F on TM_0 , which in a standard coordinate (x^i, y^i) for TM_0 is given by

$$\mathbf{G} = y^i \frac{\partial}{\partial x^i} - 2G^i(x, y) \frac{\partial}{\partial y^i},$$

where

$$G^i = \frac{1}{4}g^{il} \left\{ [F^2]_{x^m y^l} y^m - [F^2]_{x^l} \right\}. \quad (2.1)$$

$G = G^i(x, y)$ are called the spray coefficients. \mathbf{G} is called the spray associated with F .

For a non-zero vector $y \in T_x M_0$, the Riemann curvature is a family of linear transformation $\mathbf{R}_y : T_x M \rightarrow T_x M$ with homogeneity $\mathbf{R}_{\lambda y} = \lambda^2 \mathbf{R}_y, \forall \lambda > 0$ which is defined by

$\mathbf{R}_y(u) := R^i_k(y) u^k \frac{\partial}{\partial x^i}$, where

$$R^i_k(y) := 2 \frac{\partial G^i}{\partial x^k} - \frac{\partial^2 G^i}{\partial x^j \partial y^k} y^j + 2G^j \frac{\partial^2 G^i}{\partial y^j \partial y^k} - \frac{\partial G^i}{\partial y^j} \frac{\partial G^j}{\partial y^k}. \quad (2.2)$$

The family $\mathbf{R} := \{\mathbf{R}_y\}_{y \in TM_0}$ is called the Riemann curvature.

The Ricci curvature is the trace of Riemann curvature means $\mathbf{Ric}(x, y) := R^m_m(x, y)$ and the Ricci tensor is defined by

$$\mathbf{Ric}_{ij} := \frac{1}{2} \mathbf{Ric}_{y^i y^j}.$$

Let $F = \alpha\phi(s)$, $s = \beta/\alpha$, be an (α, β) -metric on a manifold M , where $\phi = \phi(s)$ is a scalar function on the interval $(-b_0, b_0)$, $\alpha = \sqrt{a_{ij}(x)y^i y^j}$ is a Riemannian metric and $\beta = b_i(x)y^i$ is a 1-form on M .

$$r_{ij} := \frac{1}{2}(b_{i|j} + b_{j|i}), \quad s_{ij} := \frac{1}{2}(b_{i|j} - b_{j|i}), \quad (2.3)$$

where ' $|$ ' denotes the covariant derivative with respect to the Levi-Civita connection of α . Let

$$\begin{aligned} r^i_j &:= a^{im} r_{mj}, & r_{i0} &:= r_{ij} y^j, & r_{00} &:= r_{ij} y^i y^j, & r_j &:= b^m r_{mj}, & r_0 &:= r_j y^j, \\ r &:= b^i r_i, & s_j &:= b^m s_{mj}, & s^i_j &:= a^{im} s_{mj}, & s_{i0} &:= s_{ij} y^j, & s_0 &:= s_j y^j. \end{aligned}$$

Lemma 2.1. [1, 3] *Let $G^i = G^i(x, y)$ and $\widehat{G}^i = \widehat{G}^i(x, y)$ denote the geodesic coefficients of F and α respectively. Then we have*

$$G^i = \widehat{G}^i + \alpha Q s^i_0 + (r_{00} - 2Q\alpha s_0)(\theta l^i + \psi b^i), \quad (2.4)$$

where

$$Q := \frac{\phi'}{\phi - s\phi'}, \quad \theta := \frac{\phi\phi' - s(\phi\phi'' + \phi'^2)}{2\phi[\phi - s\phi' + (B - s^2)\phi'']}, \quad \psi := \frac{\phi''}{2[\phi - s\phi' + (B - s^2)\phi'']},$$

Here $l^i := y^i/\alpha$ and $B := b^2$. By (2.4), we can rewrite the geodesic coefficients of an (α, β) -metric as

$$G^i = \widehat{G}^i + T^i, \quad (2.5)$$

where

$$T^i = \alpha Q s^i_{0|0} + \theta \{r_{00} - 2Q\alpha s_0\} l^i + \psi \{r_{00} - 2Q\alpha s_0\} b^i.$$

From (2.2) and by use of a technique for computing Riemannian curvature (see Proposition 3.1 in [4]) and (2.5), we have

$$R^i_j = \widehat{R}^i_j + RT^i_j, \quad (2.6)$$

where \widehat{R}^i_j denotes the Riemann curvature of α .

Also, the Ricci curvature of F is related to the Ricci curvature $\widehat{\text{Ric}}$ of α (see Proposition 3.3 in [4]) by

$$\text{Ric} = \widehat{\text{Ric}} + T^m_m. \quad (2.7)$$

3. Proof of Theorem 1.1

The Riemannian curvature of Ingarden-Támassy metric $F = \alpha + \beta^2/\alpha$ is given by

$$R^i_j = \left(\frac{1}{\alpha(\alpha-\beta)^3(\alpha+\beta)^3(\alpha^2+\beta^2)^3(2B\alpha^2+\alpha^2-3\beta^2)^4} \right) \sum_{k=0}^{21} t_k \alpha^k, \quad (3.1)$$

where

$$t_0 := 81s^i_{0|0}\beta^{20}y_j.$$

All the coefficients of t_k are tedious, listed in Appendix 1.

Let F be R-reversible, $R(y) = R(-y)$. Then by contracting both sides (3.1) with $\alpha(\alpha-\beta)^3(\alpha+\beta)^3(\alpha^2+\beta^2)^3(2B\alpha^2+\alpha^2-3\beta^2)^4$ and by a quite long computational procedure using Maple program, we obtain

$$\sum_{i=0}^{10} t'_{2i} \alpha^{2i} = 0, \quad (3.2)$$

where

$$\begin{cases} t'_0 := -81s^i_{0|0}\beta^{20}y_j, \\ \cdot \\ \cdot \\ \cdot \\ t'_{20} := y_j (2B+1)^3 \left(-8b^i s^2_0 + (2B+1)s^i_{0|0} \right), \end{cases}$$

and other coefficients of t'_{2i} are listed in Appendix 2. By (3.2), it follows that α^2 divides t'_0 , which is impossible. Therefore $t'_0 = 0$, we get

$$s^i_{0|0} = 0. \quad (3.3)$$

Putting it in others implies that $t'_2 = 0$. Put $s^i_{0|0} = 0$ in others. Then, one can show that

$$s^2_0 = 0. \quad (3.4)$$

Conversely, by placing $s_i = 0$ and $s_{ij|k} = 0$, in (3.1), we have

$$\sum_{i=0}^{10} t''_{2i} \alpha^{2i} = 0, \quad (3.5)$$

where

$$\begin{aligned} t''_0 := & 18\beta^{16} \left(-24\beta^2 y^i y_j r_{k0} s^k_0 + 4B\beta y^i y_j r_{00|0} - 10By^i y_j r_{00}^2 + 3\beta^3 \delta^i_j r_{00|0} - 5\beta^2 \delta^i_j r_{00}^2 \right. \\ & + 27s^i_0 r_{00} \beta^2 y_j - 3r^i_0 r_{00} \beta^2 y_j - 3\beta^2 y^i b_j r_{00|0} + 4\beta y^i b_j r_{00}^2 + y^i r_{00} \beta^2 r_{j0} + 6\beta^3 y^i r_{j0|0} \\ & - 6\beta^3 y^i r_{00|j} + 9y^i r_{00} \beta^2 s_{0j} + 4\beta y^i y_j r_{00} r_0 - 3b^i y_j r_{00|0} \beta^2 + 8b^i y_j \beta r_{00}^2 - 4\beta y^i y_j r_{00|0} \\ & \left. + 12y^i y_j r_{00}^2 \right) - 81\hat{R}^i_j \beta^{20}, \end{aligned}$$

and other coefficients of t''_2 are listed in Appendix 3. By (3.5), F is R-reversible.

4. Proof of Theorem 1.2

The Ricci curvature of Ingarden-Támassy metric $F = \alpha + \beta^2/\alpha$ is given by

$$\mathbf{Ric} = \left(\frac{1}{(\alpha^4 - \beta^4)^2 (\alpha^2 - \beta^2) (2B\alpha^2 + \alpha^2 - 3\beta^2)^4} \right) \sum_{k=0}^{18} d_k \alpha^k, \quad (4.1)$$

where

$$d_0 := 18\beta^{16} \left(3(n-2)\beta r_{00|0} - (5n-22)r_{00}^2 \right) - 81\widehat{\mathbf{Ric}}\beta^{18},$$

and other coefficients of d_k are listed in Appendix 4.

Let the Ricci curvature of F be reversible, i.e., $\mathbf{Ric}(y) = \mathbf{Ric}(-y)$. Then by contracting both of (4.1) with $(\alpha^4 - \beta^4)^2 (\alpha^2 - \beta^2) (2B\alpha^2 + \alpha^2 - 3\beta^2)^4$ and by computational procedure using Maple program, we obtain

$$\sum_{i=0}^7 d'_{2i} \alpha^{2i} = 0, \quad (4.2)$$

where

$$\left\{ \begin{array}{l} d'_0 := 1296(n-1)s^2_0 \beta^{15} \\ \vdots \\ d'_{14} := 16(n-1)(2B+1)^4 s^2_0 \beta, \end{array} \right.$$

and other coefficients of d'_{2i} are listed in Appendix 5. By (4.2), it follows that α^2 divides d'_0 , which is impossible. Thus $d'_0 = 0$, we get

$$s^2_0 = 0. \quad (4.3)$$

Conversely, by placing $s_i = 0$ in (4.1), we have

$$\sum_{i=0}^9 d''_{2i} \alpha^{2i} = 0, \quad (4.4)$$

where

$$d''_0 := 18\beta^{16} \left(3(n-2)\beta r_{00|0} - (5n-22)r_{00}^2 \right) - 81\widehat{\mathbf{Ric}}\beta^{18}, \quad (4.5)$$

and other coefficients of d''_{2i} are listed in Appendix 6. By (4.4) F is Ricci-reversible.

5. Appendix 1

$$\begin{aligned}
t_1 &:= 18\beta^{16} \left(-24\beta^2 y^i y_j r_{k0} s^k_0 + 4\beta y^i y_j B r_{00|0} - 10y^i y_j B r^2_{00} - 9s^i_0 \beta^3 y_j + 3\beta^3 \delta^i_j r_{00|0} - 5\beta^2 \delta^i_j r^2_{00} + 27s^i_0 r_{00} \beta^2 y_j \right. \\
&\quad \left. - 3r^i_0 r_{00} \beta^2 y_j - 3\beta^2 y^i b_j r_{00|0} + 4\beta y^i b_j r^2_{00} + y^i r_{00} \beta^2 r_{j0} + 6\beta^3 y^i r_{j0|0} - 6\beta^3 y^i r_{00|j} + 9y^i r_{00} \beta^2 s_{0j} + 24\beta^2 y^i y_j s_{0j} \right. \\
&\quad \left. + 4\beta y^i y_j r_{00|r0} - 28\beta y^i y_j s_{0r0} - 3b^i y_j r_{00|0} \beta^2 + 8b^i y_j \beta r^2_{00} - 4\beta y^i y_j r_{00|0} + 12y^i y_j r^2_{00} \right) - 81\hat{R}^i_j \beta^{20} \\
t_2 &:= -108(2B+1)s^i_0 \beta^{18} y_j \\
t_3 &:= -6\beta^{14} \left(45s^i_0 \beta^3 y_j - 27s^i_0 \beta^4 b_j - 243s^i_0 \beta^4 s_{0j} - 36\beta^4 y^i s_{0j|0} + 72\beta^4 y^i s_{0j|j} + 8By^i r_{00} \beta^2 r_{j0} - 240\beta^2 y^i y_j B s^k_0 r_{k0} \right. \\
&\quad \left. - 24\beta y^i y_j B r_{0r00} + 104\beta y^i y_j B r_{00s0} - 30\beta^2 y^i b_j r_{00|0} + 64\beta y^i b_j r^2_{00} - 3b^i r_{00} \beta^3 r_{j0} - 144\beta^3 y^i y_j s_k s^k_0 - 54s^i_0 \beta^5 \right. \\
&\quad \left. + 27s^i_0 \beta^5 + 7r^i_0 \beta^3 s_{0yj} - 6r^i_0 r_{00} \beta^2 y_j - 162s^i_0 r_{00} \beta^2 y_j - 27b^i r_{00} \beta^3 s_{0j} + 6y^i r_{00} \beta^3 s_j - 18s^i_0 r_{00} \beta^3 y_j + 216y^i \beta^3 s_{0s0} \right. \\
&\quad \left. + 18y^i r_{00} \beta^2 s_{0j} - 72\beta^4 y^i r_{jk}s^k_0 + 36\beta^4 y^i r_{k0}s^k_0 + 108\beta^4 s^i_k s^k_0 y_j + 81s^i_0 r_{00} \beta^3 b_j - 72Bs^i_0 \beta^3 y_j - 324s^i_0 \beta^3 s_{0yj} \right. \\
&\quad \left. - 9r^i_0 r_{00} \beta^3 b_j - 12Br^i_0 r_{00} \beta^2 y_j + 18\beta^3 \delta^i_j B r_{00|0} - 24\beta^3 \delta^i_j r_{00|r0} - 16\beta^2 \delta^i_j B r^2_{00} + 72\beta^3 y^i b_j s_{0|0} - 80\beta y^i y_j s_{0r00} \right. \\
&\quad \left. + 96\beta^2 y^i y_j B s_{0|0} + 96\beta^2 y^i y_j r_{k0}s^k_0 - 96\beta^2 y^i y_j s_{0r0} + 16\beta y^i y_j B^2 r_{00|0} - 8\beta y^i y_j B r_{00|0} + 48\beta y^i y_j r_{00|r0} - 9r^i_0 r_{00} \beta^4 \right. \\
&\quad \left. - 36\beta^4 \delta^i_j s_{0|0} + 18\beta^3 \delta^i_j r_{00|0} - 38\beta^2 \delta^i_j r^2_{00} - 36\beta^3 y^i r_{00|j} + 36\beta^3 y^i r_{j0|0} - 72\beta^4 \delta^i_j r_{k0}s^k_0 + 12y^i r_{00} \beta^3 r_j \right. \\
&\quad \left. + 10y^i r_{00} \beta^2 r_{j0} - 36By^i \beta^3 r_{00|j} + 36By^i \beta^3 r_{j0|0} + 36\beta^3 y^i b_j r_{k0}s^k_0 - 6B\beta^2 y^i b_j r_{00|0} + 36\beta^2 y^i b_j r_{00|r0} - 162\beta^2 y^i b_j s_{0r00} \right. \\
&\quad \left. - 22\beta y^i b_j B r^2_{00} + 90By^i r_{00} \beta^2 s_{0j} + 162Bs^i_0 r_{00} \beta^2 y_j + 140y^i y_j r^2_{00} + 10b^i y_j B r^2_{00\beta} - 9b^i b_j r_{00|0} \beta^3 + 27b^i b_j r^2_{00\beta^2} \right. \\
&\quad \left. + 36b^i y_j s^k_0 r_{k0} \beta^3 - 12b^i y_j B r_{00|0} \beta^2 + 30b^i y_j r_{00|r0} \beta^2 - 228b^i y_j \beta^2 s_{0r00} + 72b^i y_j s_{0|0} \beta^3 - 6b^i y_j r_{00|0} \beta^2 + 8b^i y_j \beta r^2_{00} \right. \\
&\quad \left. - 12y^i y_j B^2 r^2_{00} + 16y^i y_j B r^2_{00} + 288\beta^2 y^i y_j s^2_0 + 48\beta^2 y^i y_j s_{0|0} - 44\beta y^i y_j r_{00|0} - 9b^i \beta^4 r_{00|j} + 9b^i \beta^4 r_{j0|0} + 9r^i_0 \beta^4 r_{j0} \right. \\
&\quad \left. - 36y^i r_{00} \beta^3 r_{j0} + 96\beta^3 \delta^i_j s_{0r00} \right) + 108\hat{R}^i_j \beta^{18} (2B+1) \\
t_4 &:= 27\beta^{16} y_j \left(8B^2 s^i_0 |0 + 24b^i s^2_0 + 8Bs^i_0 |0 - 7s^i_0 |0 \right), \\
t_5 &:= 2\beta^{12} \left(18b^i r_{00} \beta^4 r_j - 648y^i y_j s^k_0 \beta^4 s_k + 360b^i y_j B \beta^3 s_{0j} + 36b^i y_j s^k_0 r_{k0} \beta^3 - 432b^i y_j r_{00s0} \beta^3 - 12b^i y_j B^2 r_{00|0} \beta^2 \right. \\
&\quad \left. - 36b^i b_j B r_{00|0} \beta^3 + 288\beta^3 y^i b_j B s_{0|0} + 612\beta^3 y^i b_j r_{k0}s^k_0 - 504\beta^3 y^i b_j s_{0r0} + 12\beta^2 y^i b_j B^2 r_{00|0} - 36\beta y^i b_j B^2 r^2_{00} \right. \\
&\quad \left. - 114\beta^2 y^i b_j B r_{00|0} + 264\beta^2 y^i b_j r_{00|r0} + 96\beta^2 y^i y_j B^2 s_{0|0} + 16\beta y^i y_j B^3 r_{00|0} + 96\beta^2 y^i y_j B s_{0|0} + 1128\beta^2 y^i y_j r_{k0}s^k_0 \right. \\
&\quad \left. - 384\beta^2 y^i y_j s_{0r0} - 300\beta y^i y_j B r_{00|0} + 260\beta y^i y_j r_{00|r0} - 1900\beta y^i y_j s_{0r00} + 72b^i y_j B s^k_0 r_{k0} \beta^3 + 60b^i y_j B r_{00|r0} \beta^2 \right. \\
&\quad \left. - 420b^i y_j B r_{00} \beta^2 s_0 + 36y^i y_j B^2 r^2_{00} + 654y^i y_j B r^2_{00} - 1728\beta^2 y^i y_j s^2_0 + 24\beta^2 y^i y_j s_{0|0} - 148\beta y^i y_j r_{00|0} + 324B^2 s^i_0 r_{00} \beta^2 y_j \right. \\
&\quad \left. + 96\beta^2 y^i y_j B r_{0s0} - 64\beta y^i y_j B^2 r_{0r00} + 128\beta y^i y_j B^2 r_{00s0} + 56\beta y^i y_j B r_{00r0} - 280\beta y^i y_j B r_{00s0} - 288\beta^3 y^i y_j B s^k_0 s_k \right. \\
&\quad \left. + 162r^i_0 \beta^4 s_0 b_j - 18r^i_0 r_{00} \beta^3 b_j - 204y^i r_{00} \beta^3 r_{j0} + 18y^i r_{00} \beta^2 r_{j0} - 216B^2 s^i_0 |0 \beta^3 y_j - 672\beta^2 y^i y_j B^2 s^k_0 r_{k0} + 36b^i b_j r^2_{00} \beta^2 \right. \\
&\quad \left. - 6Bb^i r_{00} \beta^3 r_{j0} - 72\beta^3 y^i b_j B s^k_0 r_{k0} + 24\beta^2 y^i b_j B r_{0r00} + 168\beta^2 y^i b_j B r_{00s0} + 588y^i y_j r^2_{00} - 1944Bs^i_0 \beta^3 s_{0yj} \right. \\
&\quad \left. - 1134Bs^i_0 r_{00} \beta^2 y_j - 108Bb^i r_{00} \beta^3 s_{0j} + 864By^i \beta^3 s_{0s0j} + 216By^i s^k_0 \beta^4 r_{k0} - 144y^i \beta^4 s_{0rj} + 60y^i r_{00} \beta^3 r_j \right. \\
&\quad \left. - 18By^i r_{00} \beta^2 s_{0j} - 432By^i s^k_0 \beta^4 r_{jk} + 864Bs^i_k s^k_0 \beta^4 y_j + 486Bs^i_0 r_{00} \beta^3 b_j + 27s^i_0 |0 \beta^5 - 54s^i_0 |j \beta^5 - 12b^i y_j B r_{00|0} \beta^2 \right. \\
&\quad \left. + 30b^i y_j r_{00|r0} \beta^2 + 816b^i y_j \beta^2 s_0 r_{00} - 12b^i y_j B r^2_{00\beta} - 144\beta^3 y^i y_j s_k s^k_0 + 432\beta^5 \delta^i_j s_k s^k_0 - 216\beta^4 \delta^i_j B s_{0|0} + 1296\beta^3 y^i b_j s^2_0 \right. \\
&\quad \left. + 108\beta^4 y^i r_{k0}s^k_j + 144\beta^3 y^i b_j s_{0|0} - 114\beta^2 y^i b_j r_{00|0} + 462\beta y^i b_j r^2_{00} + 810b^i \beta^4 s_0 s_{0j} + 162s^i_k s^k_0 \beta^5 b_j - 972s^i_0 \beta^4 s_0 b_j \right. \\
&\quad \left. - 486s^i_0 r_{00} \beta^3 b_j + 252B^2 y^i r_{00} \beta^2 s_{0j} + 90\beta^3 \delta^i_j B r_{00|0} + 108b^i r_{00} \beta^3 s_{0j} - 216Bs^i_0 |0 \beta^4 b_j - 216\beta^4 \delta^i_j r_{k0}s^k_0 \right. \\
&\quad \left. + 36\beta^3 \delta^i_j B^2 r_{00|0} - 12\beta^2 \delta^i_j B^2 r^2_{00} - 216B^5 y^i s_k s^k_j - 120\beta^3 \delta^i_j r_{00|r0} + 432\beta^3 \delta^i_j s_{0r00} - 108\beta^2 \delta^i_j B r^2_{00} + 1584b^i y_j \beta^3 s^2_0 \right. \\
&\quad \left. - 144b^i y_j s_{0|0} \beta^3 + 78b^i y_j r_{00|0} \beta^2 - 222b^i y_j \beta r^2_{00} - 54b^i \beta^5 s_{0|0} - 54r^i_0 \beta^5 s_j - 54s^i_0 \beta^5 r_{j0} + 135s^i_0 |0 \beta^4 b_j + 162s^i_k s^k_j \beta^6 \right. \\
&\quad \left. + 288\beta^4 \delta^i_j s_0 r_{00} + 1215s^i_0 \beta^4 s_{0j} - 432Bs^i_0 |j \beta^5 + 108b^i \beta^5 s_{0|j} + 216Bs^i_0 |0 \beta^5 - 108\beta^4 y^i s_{0|0} + 216\beta^4 y^i s_{0|j} + 108s^i_0 |0 \beta^3 y_j \right. \\
&\quad \left. + 90b^i y_j r_{00|r0} \beta^3 - 738b^i y_j r_{00} \beta^3 s_0 + 36b^i y_j B r^2_{00\beta^2} + 360Br^i_0 \beta^3 s_0 y_j - 40\beta^4 s^i_k s^k_0 y_j - 12Br^i_0 r_{00} \beta^2 y_j + 2916s^i_0 \beta^3 s_{0yj} \right. \\
&\quad \left. + 624\beta^2 y^i y_j B s^k_0 r_{k0} + 36b^3 y^i r_{00|j} - 36b^3 y^i r_{j0|0} + 27b^i \beta^4 r_{j0|0} + 27r^i_0 \beta^4 r_{j0} - 432\beta^4 \delta^i_j s^2_0 - 108\beta^4 \delta^i_j s_{0|0} \right. \\
&\quad \left. - 18\beta^3 \delta^i_j r_{00|0} - 60\beta^2 \delta^i_j r^2_{00} + 108r^i_0 \beta^5 s_0 - 27r^i_0 r_{00} \beta^4 - 27b^i \beta^4 r_{00|j} - 432\beta^4 \delta^i_j B s^k_0 r_{k0} - 648\beta^4 y^i b_j s_k s^k_0 \right. \\
&\quad \left. + 18b^i \beta^4 s_0 r_{j0} + 54b^i s^k_0 \beta^5 r_{k0} - 12B^2 r^i_0 r_{00} \beta^2 y_j - 96\beta^3 \delta^i_j B r_{00|r0} + 28\beta^3 \delta^i_j B r_{00s0} + 162b^i b_j \beta^4 s_{0|0} - 18b^i b_j r_{00|r0} \beta^3 \right. \\
&\quad \left. - 18b^i r_{00} \beta^4 r_{j0} - 12b^i r_{00} \beta^3 r_{j0} - 72B^2 y^i \beta^3 r_{00|j} - 180By^i \beta^3 r_{00|j} + 72B^2 y^i \beta^3 r_{j0|0} + 180By^i \beta^3 r_{j0|0} - 36Br^i_0 r_{00} \beta^3 b_j \right. \\
&\quad \left. - 192By^i r_0 \beta^3 r_{j0} + 12B^2 y^i r_{00} \beta^2 r_{j0} + 60By^i r_{00} \beta^2 r_{j0} - 194Bs^i_0 \beta^4 s_{0j} + 72y^i r_{00} \beta^4 s_j + 18y^i r_{00} \beta^3 s_j + 90s^i_0 r_{00} \beta^3 y_j \right. \\
&\quad \left. + 432Bs^i_0 |0 \beta^3 y_j - 216By^i \beta^4 s_{0j} + 432By^i \beta^4 s_{0|j} - 108b^i s^k_0 \beta^5 r_{jk} - 864y^i \beta^3 s_{0s0j} - 234y^i r_{00} \beta^2 s_{0j} + 36b^i r_{00} \beta^4 s_j \right. \\
&\quad \left. - 216b^4 y^i r_{jk}s^k_0 - 162s^i_0 r_{00} \beta^2 y_j \right) - 27\hat{R}^i_j \beta^{16} \left(8B^2 + 8B - 7 \right) \\
t_6 &:= -24\beta^{14} y_j \left(4s^i_0 |0 B^3 + 42B^i s^2_0 + 6B^2 s^i_0 |0 - 24b^i s^2_0 - 24Bs^i_0 |0 - 13s^i_0 |0 \right),
\end{aligned}$$

$$\begin{aligned}
t_7 &:= 4\beta^{10} \left[-96B^2 r_0^i \beta^3 s_0 y_j + 192 Br_0^i \beta^3 s_0 y_j - 54 Br_0^i r_{00} \beta^2 y_j + 72 r_0^i \beta^4 s_0 b_j - 39 r_0^i r_{00} \beta^3 b_j - 50 y^i r_0 \beta^3 r_{j0} \right. \\
&\quad - 50 b^i y_j s_0^k \beta^4 s_k - 9 b^i y_j B^2 \beta^3 s_0|_0 - 600 b^i y_j B \beta^3 s_0^2 + 54 B b^i \beta^5 s_j|_0 + 192 b^i y_j B \beta^3 s_0|_0 + 186 b^i y_j s_0^k r_{k0} \beta^3 - 168 b^i y_j r_0 s_0 \beta^3 \\
&\quad - 18 b^i r_0 \beta^5 s_j + 864 b^i \beta^4 s_0 s_0 j - 90 b^i r_{00} \beta^3 s_0 j + 48 y^i \beta^4 s_0 r_j + 22 y^i r_{00} \beta^3 r_j + 72 \beta^2 y^i b_j s_k s_0^k - 594 b^i y_j \beta^2 s_0 r_{00} \\
&\quad + 2 y^i r_{00} \beta^2 r_{j0} + 45 b^i y_j Br_0^2 \beta - 4 \beta^2 y^i b_j B^3 r_{00}|_0 + 972 B s_0^i \beta^4 s_0 b_j + 117 \beta^2 y^i b_j Br_{00}|_0 - 86 \beta^2 y^i b_j r_{00} r_0 + 967 \beta^2 y^i b_j s_0 r_{00} \\
&\quad - 48 \beta^3 y^i b_j B s_0|_0 - 294 \beta^3 y^i b_j r_{k0} s_0^k + 264 \beta^3 y^i b_j s_0 r_0 + 24 \beta^2 y^i b_j B^2 r_{00}|_0 - 18 \beta y^i b_j B^2 r_0^2 + 12 B b^i r_0 \beta^4 r_{j0} \\
&\quad + 84 B s_0^i k s_0^k \beta^4 y_j - 18 B b^i r_{00} \beta^4 s_j - 66 b^i b_j B \beta^3 r_{00}|_0 + 6 b^i b_j B^2 r_{00}|_0 \beta^3 + 156 b^i y_j B r_{00}|_0 - 96 b^i y_j r_{00} r_0 + 1344 \beta y^i y_j s_0 r_{00} \\
&\quad - 162 B^2 s_0^i r_{00} \beta^3 b_j + 96 \beta^2 y^i y_j B^3 s_0^k r_{k0} + 192 \beta^3 y^i y_j B s_0^k s_k - 288 \beta^2 y^i y_j B^2 s_0^k r_{k0} - 96 \beta^2 y^i y_j B^2 r_0 s_0 - 126 \beta^2 y^i y_j B s_0^k r_{k0} \\
&\quad + 192 \beta^2 y^i y_j B r_0 s_0 + 144 \beta y^i y_j B^2 r_{00} s_0 - 264 \beta y^i y_j B r_{00} r_0 + 1752 \beta y^i y_j B r_{00} s_0 - 30 b^i b_j B r_0 r_{00} \beta^3 + 6 B^2 r_0^i r_{00} \beta^3 b_j \\
&\quad - 72 B^2 y^i s_j^k \beta^4 r_{k0} - 72 B y^i s_j^k \beta^4 r_{k0} - 54 B b^i s_j^k \beta^5 r_{k0} - 72 B s_0^i \beta^4 s_0 y_j + 60 B^2 s_0^i r_{00} \beta^3 y_j + 18 B s_0^i r_{00} \beta^4 b_j - 432 B^2 s_0^i k s_0^k \beta^4 y_j \\
&\quad + 108 s_0^i j_0 \beta^5 - 216 s_0^i j_0 \beta^5 - 216 B s_0^i k s_0^k \beta^6 + 54 B r_0^i \beta^5 s_j + 54 s_0^i \beta^5 s_0 b_j + 1134 s_0^i \beta^3 s_0 y_j - 369 s_0^i r_{00} \beta^2 y_j - 1458 s_0^i \beta^4 s_0 b_j \\
&\quad + 81 s_0^i r_{00} \beta^3 b_j - 21 s_0^i r_{00} \beta^3 y_j - 45 s_0^i r_{00} \beta^4 b_j + 972 B^2 s_0^i \beta^4 s_0 j - 1944 B s_0^i \beta^4 s_0 j + 54 \beta^4 s_0^i k s_0^k y_j + 216 s_0^i k s_0^k \beta^5 b_j \\
&\quad + 114 s_0^i j_0 \beta^5 y_j - 48 \beta^3 y^i b_j B^2 s_0|_0 - 144 \beta^3 y^i b_j B s_0^2 - 108 B^2 s_0^i j_0 \beta^5 + 90 \beta^4 y^i r_{k0} s_0^k - 36 B^3 s_0^i r_{00} \beta^2 y_j + 216 B^2 s_0^i j_0 \beta^5 \\
&\quad - 180 \beta^4 y^i r_{jk} s_0^k + 72 s_0^i \beta^4 s_0 y_j + 54 B s_0^i \beta^5 r_{j0} - 144 B^2 y^i \beta^4 s_0|_0 - 144 B y^i \beta^4 s_0|_0 - 252 y^i \beta^3 s_0 s_0 j + 81 y^i r_{00} \beta^2 s_0 \\
&\quad - 24 y^i r_0 \beta^4 s_j - 24 y^i \beta^4 s_0 s_j + 19 y^i r_{00} \beta^3 s_j + 72 B^2 y^i \beta^4 s_0|_0 + 72 B y^i \beta^4 s_0|_0 + 54 b^i \beta^5 s_0 s_j - 9 b^i r_{00} \beta^4 s_j - 27 B s_0^i r_{00} \beta^2 y_j \\
&\quad + 54 b^i s_0^i k \beta^6 s_k + 108 B^2 s_0^i j_0 \beta^4 b_j - 216 B s_0^i j_0 \beta^4 b_j - 252 B^2 s_0^i j_0 \beta^3 y_j - 72 B s_0^i j_0 \beta^3 y_j - 108 B b^i \beta^5 s_0|_j + 144 B^2 y^i s_0^i \beta^4 r_{jk} \\
&\quad + 144 B y^i s_0^i \beta^4 r_{jk} + 369 B y^i r_{00} \beta^2 s_0 j + 216 B y^i s_0^i j_0 \beta^5 s_k + 72 B^2 y^i r_{00} \beta^2 s_0 j - 48 B y^i r_{00} \beta^4 s_j - 48 B y^i \beta^4 s_0 s_j + 4 B^2 y^i r_{00} \beta^3 s_j \\
&\quad + 4 B y^i r_{00} \beta^3 s_j - 702 B b^i \beta^4 s_0 s_0 j + 18 B^2 b^i r_{00} \beta^3 s_0 j - 90 B b^i r_{00} \beta^3 s_0 j + 567 B s_0^i r_{00} \beta^3 b_j + 648 B^2 s_0^i \beta^3 s_0 y_j \\
&\quad - 3240 B s_0^i \beta^3 s_0 y_j + 432 B^2 s_0^i r_{00} \beta^2 y_j + 108 B b^i s_0^k \beta^5 r_{jk} + 36 b^i \beta^5 s_0 r_j - 6 b^i r_{00} \beta^4 r_j + 18 B^2 b^i \beta^4 r_{00}|_j + 18 B b^i \beta^4 r_{00}|_j \\
&\quad - 18 B^2 r_0^i \beta^4 r_{j0} - 18 B r_0^i \beta^4 r_{j0} + 228 r_0^i \beta^3 s_0 y_j - 27 r_0^i r_{00} \beta^2 y_j - 162 y^i y_j r_{00}^2 + 12 b^3 y^i b_j B^2 s_0^k r_{k0} - 528 \beta^3 y^i b_j B s_0^k r_{k0} \\
&\quad + 96 \beta^3 y^i b_j B r_0 s_0 + 16 b^2 y^i b_j B^2 r_{00} - 68 \beta^2 y^i b_j B^2 r_{00} s_0 - 92 \beta^2 y^i b_j B r_{00} r_0 + 28 \beta^2 y^i b_j B r_{00} s_0 - 36 r_j^i r_{00} \beta^4 - 36 b^i \beta^4 r_{00}|_j \\
&\quad + 36 b^i \beta^4 r_{j0}|_0 + 96 \beta^4 \delta^i_j s_0^2 - 90 \beta^4 \delta^i_j s_0|_0 + 49 \beta^3 \delta^i_j r_{00}|_0 - 27 \beta^2 \delta^i_j r_{00}^2 - 98 \beta^3 y^i r_{00}|_j + 98 \beta^3 y^i r_{j0}|_0 - 144 \beta^2 y^i y_j B s_0|_0 \\
&\quad - 32 \beta^3 \delta^i_j r_{00} s_0 + 360 \beta^4 y^i b_j s_0^k s_k + 36 b^i y_j B^2 r_{00} \beta^2 s_0 - b^i y_j B r_{00} \beta^2 s_0 - 120 B s_0^i r_{00} \beta^3 y_j + 336 \beta^3 y^i y_j s_k s_0^k + 576 \beta^2 y^i y_j B s_0^2 \\
&\quad - 240 \beta^2 y^i y_j r_{k0} s_0^k + 192 \beta^2 y^i y_j s_0 r_0 + 120 \beta y^i y_j B^2 r_{00}|_0 - 6 \beta y^i y_j r_{00}|_0 - 108 y^i y_j B^2 r_{00}^2 - 360 y^i y_j B r_{00}^2 + 48 B^3 s_0^i j_0 \beta^3 y_j \\
&\quad - 216 B s_0^i k s_0^k \beta^5 b_j + 36 r_0^i \beta^4 r_{j0} - 144 B^2 y^i \beta^3 s_0 s_0 j - 36 B^3 y^i r_{00} \beta^2 s_0 j + 720 B y^i \beta^3 s_0 s_0 j - 432 \beta^5 \delta^i_j B s_0^k s_k + 54 s_0^i k s_0^k \beta^6 \\
&\quad + 144 \beta^4 \delta^i_j B^2 s_0^k r_{k0} + 144 \beta^4 \delta^i_j B s_0^k r_{k0} - \beta^4 \delta^i_j r_{00} s_0 - b^i b_j B \beta^4 s_0|_0 - 18 b^i b_j s_0^k r_{k0} \beta^4 + 198 b^i b_j r_0 \beta^4 s_0 + 16 \beta^3 \delta^i_j B^2 r_0 r_{00} \\
&\quad + 64 \beta^3 \delta^i_j B r_0 r_{00} - 176 \beta^3 \delta^i_j B r_0 r_0 + 270 b^i s_0^k \beta^5 s_k - 54 s_0^i j_0 \beta^4 b_j - 54 s^i \beta^6 s_j - 90 \beta^4 y^i s_0|_0 + 66 B y^i \beta^3 r_{j0}|_0 + 180 \beta^4 y^i s_0|_j \\
&\quad - 399 b^i y_j r_{00} \beta^3 s_0 + 3 b^i b_j B r_{00}^2 \beta^2 - 54 b^i y_j B r_{00}|_0 \beta^2 + 135 b^i y_j r_{00} r_0 \beta^2 - 2304 \beta^2 y^i y_j s_0^2 - 72 \beta^2 y^i y_j s_0|_0 - 96 \beta^3 y^i y_j B s_0^k s_k \\
&\quad + 504 b^i y_j B s_0^k \beta^4 s_k + 24 b^i y_j B^2 s_0^k r_{k0} \beta^3 - 48 b^i y_j B s_0^k r_{k0} \beta^3 + 168 b^i y_j B r_{00} \beta^3 s_0 - 90 b^i b_j B s_0^k r_{k0} \beta^4 + 228 B b^i b_j r_{00} \beta^3 s_0 \\
&\quad - 15 b^i b_j r_{00} r_0 \beta^3 - 12 b^i r_{00} \beta^3 r_{j0} + 18 B^2 r_0^i r_{00} \beta^4 + 18 r_0^i r_{00} \beta^4 - 12 B^2 r_{00} r_j - 108 B r_0^i \beta^5 s_0 - 18 B^2 b^i \beta^4 r_{00}|_0 - 18 B^2 b^i \beta^4 r_{00}|_0 \\
&\quad + 3 B b^i r_{00} \beta^3 r_{j0} + 40 B^2 y^i r_{00} \beta^3 r_{j0} + 136 B y^i r_{00} \beta^3 r_{j0} - 12 B^2 y^i r_{00} \beta^2 r_{j0} + 96 B y^i \beta^4 s_0 r_j - 8 B^2 y^i r_{00} \beta^3 r_j - 32 B y^i r_{00} \beta^3 r_j \\
&\quad - 126 B r_0^i \beta^4 s_0 b_j + 6 B r_0^i r_{00} \beta^3 b_j + 72 \beta^4 \delta^i_j s_0|_0 + 192 \beta^4 \delta^i_j B s_0^2 - 4 \beta^3 \delta^i_j B^3 r_{00}|_0 - 810 b^i b_j \beta^4 s_0^2 + b^i b_j \beta^4 s_0|_0 - 8 B^3 y^i \beta^3 r_{j0}|_0 \\
&\quad - 39 b^i y_j r_{00}|_0 \beta^3 + 2004 b^i y_j \beta^3 s_0^2 + 228 b^i y_j s_0|_0 \beta^3 + \beta^4 \delta^i_j B s_0|_0 - 180 \beta^4 \delta^i_j r_{k0} s_0^k - \beta^4 \delta^i_j s_0 r_0 - \beta^3 \delta^i_j B^2 r_{00}|_0 + 792 \beta^3 y^i b_j s_0^2 \\
&\quad + 12 \beta^2 \delta^i_j B^2 r_{00}^2 + 33 \beta^3 \delta^i_j B r_{00}|_0 - \beta^3 \delta^i_j r_{00} r_0 - 8 \beta^3 \delta^i_j s_0 r_0 + \beta^2 \delta^i_j B r_{00}^2 - 12 \beta^3 y^i b_j s_0|_0 + 25 \beta^2 y^i b_j r_{00}|_0 - 48 B^2 y^i \beta^3 r_{j0}|_0 \\
&\quad - \beta y^i b_j r_{00}^2 + b^i b_j r_{00}^2 \beta^2 + 8 B^3 y^i \beta^3 r_{00}|_j + B^2 y^i \beta^3 r_{00}|_j - 27 b^i y_j r_{00}|_0 \beta^2 + 36 b^i y_j \beta r_{00}^2 + 6 b^i b_j B r_{00}|_0 \beta^3 - 351 \beta y^i b_j B r_{00}^2 \Big] \\
&\quad + 24 \hat{R}_j^i \beta^{14} (2 B + 1) (2 B^2 + 2 B - 13) \\
t_8 &:= 2 \beta^{12} y_j (8 B^4 s_0^i|_0 + 240 B^2 b^i s_0^2 + 16 B^3 s_0^i|_0 - 696 B b^i s_0^2 - 312 B^2 s_0^i|_0 - 624 b^i s_0^2 - 320 B s_0^i|_0 + 41 s_0^i|_0) \\
t_9 &:= -4 \beta^8 \left[-36 B^2 b^i r_{00} \beta^3 s_0 j - 90 B b^i r_{00} \beta^3 s_0 j - 27 B s_0^i r_{00} \beta^3 b_j + 144 B^3 s_0^i \beta^3 s_0 y_j - 2376 B^2 s_0^i \beta^3 s_0 y_j + 108 B^3 s_0^i r_{00} \beta^2 y_j \right. \\
&\quad - 162 B^2 s_0^i r_{00} \beta^2 y_j - 729 B s_0^i r_{00} \beta^2 y_j + 72 B^2 b^i s_0^k \beta^5 r_{jk} - 36 B b^i s_0^k \beta^5 r_{jk} - 48 B s_0^i r_{00} \beta^3 y_j - 96 B^2 s_0^i \beta^4 s_0 y_j + 16 \beta^3 y^i y_j s_k s_0^k \\
&\quad - 192 \beta^2 y^i y_j B^2 s_0|_0 - \beta^2 y^i y_j B s_0^2 + 32 \beta y^i y_j B^3 r_{00}|_0 - \beta^2 y^i y_j B s_0|_0 + 324 \beta^2 y^i y_j r_{k0} s_0^k + 96 \beta^2 y^i y_j s_0 r_0 + 96 \beta y^i y_j B^2 r_{00}|_0 \\
&\quad + 36 B s_0^i \beta^5 s_0 b_j - 36 B^3 s_0^i r_{00} \beta^3 b_j - 216 B^2 s_0^i k s_0^k \beta^5 b_j + 648 B s_0^i k s_0^k \beta^5 b_j - 24 B b^i r_0 \beta^5 s_j + 48 B b^i \beta^5 s_0 s_j - 4 B^2 b^i r_{00} \beta^4 s_j \\
&\quad + 288 B^2 y^i \beta^3 s_0 s_0 j + B^3 y^i r_{00} \beta^2 s_0 j - B y^i \beta^3 s_0 s_0 j - 288 \beta^5 \delta^i_j B^2 s_0^k s_k + 32 \beta^4 \delta^i_j B^3 s_0^k r_{k0} + 144 \beta^5 \delta^i_j B s_0^k s_k + 48 \beta^4 \delta^i_j B^2 s_0^k r_{k0} \\
&\quad - 64 \beta^4 \delta^i_j B^2 r_0 s_0 - 408 \beta^4 \delta^i_j B s_0^k r_{k0} - 64 \beta^4 \delta^i_j B r_0 s_0 - 600 b^i b_j B \beta^4 s_0^2 + 174 b^i b_j B \beta^4 s_0|_0 + 258 b^i b_j s_0^k r_{k0} \beta^4 - 162 b^i b_j r_0 \beta^4 s_0
\end{aligned}$$

$$\begin{aligned}
& +16\beta^3\delta_j^iB^2r_0r_{00}-32\beta^3\delta_j^iB^2r_{00}s_0-80\beta^3\delta_j^iBr_0r_{00}-32\beta^3\delta_j^iBr_{00}s_0-450b^ib_js_0^k\beta^5s_k-60b^ib_jB^2\beta^4s_{0|0}-54b^ib_jBr_{00|0}\beta^3 \\
& +135b^ib_jr_0r_{00}\beta^3-675b^ib_jr_{00}\beta^3s_0+54b^ib_jBr_{00}^2\beta^2-18b^iy_jBr_{00|0}\beta^2+45b^iy_jr_{00}r_0\beta^2+48Bb^i\beta^5s_0r_j-4B^2b^ir_{00}\beta^4r_j \\
& -4Bb^ir_{00}\beta^4r_j-9Bb^ir_{00}\beta^3r_{j0}+40B^2y^ir_0\beta^3r_{j0}-152By^ir_0\beta^3r_{j0}+6B^2y^ir_{00}\beta^2r_{j0}+54By^ir_{00}\beta^2r_{j0}+32B^2y^i\beta^4s_0r_j \\
& +32By^i\beta^4s_0r_j-8B^2y^ir_{00}\beta^3r_j+40By^ir_{00}\beta^3r_j-60B^2r_0^i\beta^4s_0b_j+174Br_0^i\beta^4s_0b_j-54Br_0^ir_{00}\beta^3b_j-16B^3r_0^i\beta^3s_0y_j \\
& +144B^2r_0^i\beta^3s_0y_j+300Br_0^i\beta^3s_0y_j-18B^2r_0^ir_{00}\beta^2y_j-18Br_0^ir_{00}\beta^2y_j-16b^iy_jB^2\beta^3s_{0|0}-144b^iy_jB^2\beta^3s_0^2-228b^iy_js_0^k\beta^4s_k \\
& +144b^iy_jB^2\beta^3s_{0|0}+2088b^iy_jB\beta^3s_0^2+300b^iy_jB\beta^3s_{0|0}+14b^iy_js_0^kr_{k0}\beta^3-464b^iy_jr_0s_0\beta^3-18b^iy_jB^2r_{00|0}\beta^2+156b^4b_js_ks^k_0 \\
& +720b^iy_js_0^2r_{00}-18b^iy_jBr_{00}^2\beta^2+69\beta^2y^ib_jBr_{00|0}+20\beta^2y^ib_jr_{00}r_0+1397\beta^2y^ib_js_0r_{00}-396\beta^2y^ib_jBr_{00}^2+576\beta^3y^ib_js_0^2 \\
& +3348Bs^i_0\beta^3s_0y_j+4\beta^2y^ib_jB^3r_{00|0}-144\beta^3y^ib_jBs_{0|0}+78\beta^3y^ib_jr_{k0}s^k_0-12\beta^3y^ib_js_0r_0+90\beta^2y^ib_jB^2r_{00|0}-120\beta y^ib_jB^2r_{00}^2 \\
& +4B^2b^ir_0\beta^4r_{j0}+4B^2b^i\beta^4s_0r_{j0}+4Bb^ir_0\beta^4r_{j0}+4Bb^i\beta^4s_0r_{j0}+648B^2s_0^i\beta^4s_0b_j-3240Bs_0^i\beta^4s_0b_j-16B^3y^is^k_0\beta^4r_{k0} \\
& -24B^2y^is^k_0\beta^4r_{k0}+204By^is^k_0\beta^4r_{k0}-16B^2y^ir_0\beta^4s_j-36B^2b^is^k_0\beta^5r_{k0}+18Bb^is^k_0\beta^5r_{k0}+108Bb^is^k_0\beta^6s_k-32B^2y^i\beta^4s_0s_j \\
& +16B^3s^i_0r_{00}\beta^3y_j+264Bs^i_0\beta^4s_0y_j-108B^2s^i_0r_{00}\beta^3y_j+24B^2s^i_0r_{00}\beta^4b_j-102B^2s^i_0r_{00}\beta^4b_j-192B^3s^i_0s^k_0\beta^4y_j+1008B^2s^i_0s^k_0\beta^4y_j \\
& -144Bs^i_0s^k_0\beta^4y_j-4Bb^ir_0\beta^4s_j-54\beta y^iy_jBr_{00|0}+58\beta y^iy_jr_{00}r_0+298\beta y^iy_js_0r_{00}+432B^2s^i_0r_{00}\beta^3b_j+32B^3y^is^k_0\beta^4r_{jk} \\
& +48B^2y^is^k_0\beta^4r_{jk}-408By^is^k_0\beta^4r_{jk}+9By^ir_{00}\beta^2s_{0j}+144B^2y^is^k_0\beta^5s_k-72By^is^k_0\beta^5s_k+306B^2y^ir_{00}\beta^2s_{0j}-16By^ir_0\beta^4s_j \\
& -32By^i\beta^4s_0s_j+4B^2y^ir_{00}\beta^3s_j+4By^ir_{00}\beta^3s_j-396B^2b^i\beta^4s_0s_j+1710Bb^i\beta^4s_0s_j+15r^i_0\beta^4s_0b_j-27r^i_0r_{00}\beta^3b_j \\
& +18s^i_0\beta^5s_0b_j+342s^i_0\beta^3s_0y_j+54s^i_0r_{00}\beta^2y_j+1134s^i_0\beta^4s_0b_j-369s^i_0r_{00}\beta^3b_j+59s^i_0r_{00}\beta^3y_j+51s^i_0r_{00}\beta^4b_j+288Bs^i_0|\beta^5 \\
& -108Bs^i_0\beta^6s_j+432B^3s^i_0\beta^4s_{0j}-B^2s^i_0\beta^4s_{0j}+648Bs^i_0\beta^4s_{0j}-\beta^4s^i_0s^k_0y_j-108s^i_0s^k_0\beta^5b_j-180\beta^5y^is_k^s_0+106\beta^4y^ir_{k0}s^k_0 \\
& -212B^4y^ir_{jk}s^k_0-168s^i_0\beta^4s_0y_j+36B^2s^i_0\beta^5r_{j0}-18Bs^i_0\beta^5r_{j0}-32B^2y^i\beta^4s_{0|j}-48B^2y^i\beta^4s_{0|j}+408By^i\beta^4s_{0|j}-4B^3b^i\beta^4r_{j0}|0 \\
& -136y^i\beta^4s_0r_j+58y^ir_{00}\beta^3r_j-174By^i\beta^3r_{00|j}-216B^2s^i_0s^k_0\beta^6+216Bs^i_0s^k_0\beta^6+36B^2r_0^i\beta^5s_j-18Br_0^i\beta^5s_j+96B^3s^i_0|\beta^5 \\
& +72y^i\beta^3s_0s_{0j}-108y^ir_{00}\beta^2s_{0j}+68y^ir_0\beta^4s_j-8y^i\beta^4s_0s_j+19y^ir_0\beta^3s_j+16B^3y^i\beta^4s_{j|0}+24B^2y^i\beta^4s_{j|0}-576Bs^i_0|\beta^5 \\
& -204By^i\beta^4s_{j|0}-144b^i s^k_0\beta^5r_{jk}-30b^i\beta^5s_0s_j+53b^ir_0\beta^3s_j-54b^is^k_0\beta^6s_k+48B^3s^i_0|\beta^4b_j-252B^2s^i_0|\beta^4b_j+162s^i_0s^k_0\beta^6 \\
& -72Bs^i_0|\beta^4b_j+8B^4s^i_0|\beta^3y_j+12B^2s^i_0|\beta^3y_j+328Bs^i_0|\beta^3y_j+72b^is^k_0|\beta^5r_{k0}-72B^2b^i\beta^5s_{0|j}+36Bb^i\beta^5s_{0|j}+48s^i_0|\beta^5 \\
& -12b^i\beta^5s_0r_j+26b^ir_0\beta^4r_{j0}+4B^3b^i\beta^4r_{00|j}+6B^2b^i\beta^4r_{00|j}-78Bb^i\beta^4r_{00|j}-4B^3r_0^i\beta^4r_{j0}-6B^2r_0^i\beta^4r_{j0}-96s^i_0|\beta^5 \\
& +78Br_0^i\beta^4r_{j0}-104r^i_0\beta^3s_0y_j+36r^i_0r_{00}\beta^2y_j-74\beta y^iy_jr_{00|0}-60y^iy_jB^2r_{00}^2-57y^iy_jBr_{00}^2-128B^3s^i_0|\beta^3y_j-72B^2r^i_0\beta^5s_0 \\
& +4B^3r^i_0r_{00}\beta^4-960B^2y^iy_js_0^2+60\beta^2y^iy_js_0|0-26b^ir_0\beta^4r_{j0}+28b^i\beta^4s_0r_{j0}-18b^ir_0\beta^3r_{j0}+6B^2r^i_0r_{00}\beta^4-78Br^i_0r_{00}\beta^4 \\
& +16b^4\delta_j^iB^3s_{0|0}+\beta^4\delta_j^iB^2s_0^2+\beta^5\delta_j^is^k_0+24\beta^4\delta_j^iB^2s_{0|0}+32\beta^4\delta_j^iB^2s_0^2-4\beta^3\delta_j^iB^3r_{00|0}+b^ib_j\beta^4s_0^2-194y^ir_0\beta^3r_{j0} \\
& +156b^ib_j\beta^4s_{0|0}-b^ib_jr_{00|0}\beta^3-b^iy_j\beta^3s_0^2-104b^iy_js_{0|0}\beta^3-204\beta^4\delta_j^iBs_{0|0}-212\beta^4\delta_j^ir_{k0}s^k_0+\beta^4\delta_j^is_0r_0+30y^ir_0\beta^2r_{j0} \\
& +30\beta^3\delta_j^iB^2r_{00|0}+6\beta^2\delta_j^iB^2r_{00}^2+87\beta^3\delta_j^ir_{00}r_0-\beta^3\delta_j^is_0r_{00}-18\beta^2\delta_j^ir_{00}^2-\beta^3y^ib_js_{0|0}+36B^2b^i\beta^5s_{j|0} \\
& -\beta^2y^ib_jr_{00|0}-\beta y^ib_jr_{00}^2+54b^ib_js_0^2+8B^3y^i\beta^3r_{00|j}-B^2y^i\beta^3r_{00|j}+36b^iy_jr_{00|0}\beta^2-b^iy_j\beta r_{00}^2-6B^2b^i\beta^4r_{j0|0} \\
& -2304B^3y^ib_js_0^2-8B^3y^i\beta^3r_{j0|0}+60B^2y^i\beta^3r_{j0|0}+174By^i\beta^3r_{j0|0}-18y^iy_jr_{00}^2-40b^iy^ir_{00|j}-40r^i_0r_{00}\beta^4+78Bb^i\beta^4r_{j0|0} \\
& +144r^i_0\beta^5s_0+40b^i\beta^4r_{j0|0}+40r^i_0\beta^4r_{j0|0}+8\beta^4\delta_j^is_0^2-106\beta^4\delta_j^is_0|0+22\beta^3\delta_j^ir_{00|0}-33\beta^2\delta_j^ir_{00}^2-128\beta y^iy_jB^2r_{0r00} \\
& -106\beta^4y^is_{j|0}+212\beta^4y^is_{0|j}-72r^i_0\beta^5s_j+216s^i_0|\beta^4s_{0j}+23s^i_0|\beta^3y_j-48B^3s^i_0|\beta^5+36B^2s^i_0|\beta^5-684\beta^3y^ib_js_{0|0} \\
& -96\beta^2y^iy_js_0^2r_{k0}+1056\beta^3y^iy_js_0^k-960\beta^2y^iy_js_0^2r_{k0}+96\beta^2y^iy_js_0^2r_{k0}-240\beta^2y^iy_js_0^2r_{k0}+672\beta^2y^iy_jBr_{0r0} \\
& +496B^2y^iy_jB^2r_{00}s_0-200\beta y^iy_jBr_{00}r_{00}+1096B^2y^iy_jBr_{00}s_0-64\beta^3y^iy_js_0^2r_{k0}-360\beta^3y^iy_js_0^2r_{k0}-48\beta^3y^ib_jB^2r_{0r0} \\
& +240\beta^3y^ib_jBr_{00}s_0-16\beta^2y^ib_js_0^2r_{00}+140\beta^2y^ib_js_0^2r_{00}s_0-184\beta^2y^ib_jBr_{00}r_{00}+1892\beta^2y^ib_jBr_{00}s_0+48\beta^4y^ib_jB^2s_0^k \\
& +120\beta^4y^ib_js_0^k-108b^iy_jB^2r_{00}\beta^2s_0+90b^iy_jBr_{00}\beta^2s_0-18b^iy_jBr_{00}\beta^2s_0+288\beta^3y^iy_js_0^2r_{k0}-240b^iy_jB^2s_0^k\beta^4s_k \\
& -984b^iy_js_0^2\beta^4s_k-72b^iy_js_0^2r_{k0}\beta^3+16b^iy_js_0^2r_{00}\beta^3s_0+204b^iy_js_0^2r_{k0}\beta^3-200b^iy_jBr_{00}\beta^3s_0-30b^iy_js_0^2r_{k0}\beta^4 \\
& -18Bb^i\beta^5s_{j|0}+126b^ir_0\beta^3s_{0j}-72B^2s^i_0|\beta^5+6b^ir_0\beta^5s_j-180b^i\beta^4s_0s_{0j}+144b^ib_jBr_0\beta^4s_0-468b^ib_jBr_{00}\beta^3s_0 \\
& +36B^2b^ir_0\beta^3s_0b_j+396b^ib_js_0^k\beta^5s_k-12b^ib_js_0^2r_{k0}\beta^4+48\beta^3y^ib_jB^3s_0^k\beta^3s_{0r}+16b^iy_jB^3s_0^k\beta^3s_{0r} \\
& -2\hat{R}_j^i\beta^{12}\left(8B^4+16B^3-312B^2-320B+41\right) \\
t_{10} & := 16y_j\beta^{10}\left(-4B^3b^i s_0^2+60B^2b^i s_0^2+18s^i_0|\beta^3+81Bb^i s_0^2+27B^2s^i_0|\beta^0-56b^i s_0^2-27B^2s^i_0|\beta^0-18s^i_0|\beta^0\right) \\
t_{11} & := -4\beta^6\left[-1080B^2b^i\beta^4s_0s_0+918Bb^i\beta^4s_0s_0+18B^2b^i\beta^3s_0s_0-198Bb^i\beta^3s_0s_0+729Bs^i_0r_{00}\beta^3b_j+576B^3s^i_0\beta^3s_0y_j\right. \\
& \quad \left.-B^2s^i_0\beta^3s_0y_j+72B^3s^i_0r_{00}\beta^2y_j+432Bs^i_0\beta^3s_0y_j+B^2s^i_0r_{00}\beta^2y_j-351Bs^i_0r_{00}\beta^2y_j-144B^3s^i_0\beta^4s_0b_j+48B^2b^i s_0^k\beta^5r_{jk}\right. \\
& \quad \left.+276Bb^i s_0^k\beta^5r_{jk}+8B^3b^i s_0^k\beta^5r_{jk}+64\beta^2y^iy_jB^3s_0|\beta^0-144B^2s^i_0r_{00}\beta^3y_j-288B^2s^i_0\beta^4s_0y_j+392\beta^3y^iy_js_0^k+96\beta^2y^iy_jB^2s_0|\beta^0\right. \\
& \quad \left.-832\beta^2y^iy_js_0^2-16\beta y^iy_jB^3r_{00|0}-240\beta^2y^iy_jBs_{0|0}-368\beta^2y^iy_jr_{k0}s^k_0+272\beta^2y^iy_js_0r_0+72\beta y^iy_jB^2r_{00|0}-120Bs^i\beta^5s_0b_j\right]
\end{aligned}$$

$$\begin{aligned}
& -108B^3s^i_0r_{00}\beta^3b_j + 24B^2s^i\beta^5s_0b_j - 8B^3s^ir_{00}\beta^4b_j - 720B^2s^i_k s^k_0\beta^5b_j + 504Bs^i_k s^k_0\beta^5b_j + 8B^2b^ir_0\beta^5s_j - 8B^2b^i\beta^5s_0s_j \\
& - 16Bb^ir_0\beta^5s_j + 40Bb^i\beta^5s_0s_j + 864B^2y^i\beta^3s_0s_{0j} - 72B^3y^ir_{00}\beta^2s_{0j} - 864By^i\beta^3s_0s_{0j} + 64\beta^5\delta^i_jB^3s^k_0s_k - 192\beta^5\delta^i_jB^2s^k_0s_k \\
& - 672\beta^5\delta^i_jBs^k_0s_k + 288\beta^4\delta^i_jB^2s^k_0r_{k0} + 28\beta^4\delta^i_jBs^k_0r_{k0} - 384\beta^4\delta^i_jBr_0s_0 - 2096b^ib_jB\beta^4s^2_0 - 162b^ib_jB\beta^4s_{0|0} - 22b^ib_js^k_0r_{k0}\beta^4 \\
& + 418b^ib_jr_0\beta^4s_0 + 32\beta^3\delta^i_jB^2r_{00}r_{00} + 32\beta^3\delta^i_jB^2r_{00}s_0 + 128\beta^3\delta^i_jBr_0r_{00} + 32\beta^3\delta^i_jBr_0s_0 + 8b^ib_jB^3\beta^4s_{0|0} + 136b^ib_jB^2\beta^4s^2_0 \\
& + 138b^ib_js^k_0\beta^5s_k - 120b^ib_jB^2\beta^4s_{0|0} + 18b^ib_jBr_{00}\beta^3 - 45b^ib_jr_0r_{00}\beta^3 - 693b^ib_jr_{00}\beta^3s_0 + 9b^ib_jBr_{00}\beta^2 - 54y_jb^iBr_{00}\beta^2 \\
& + 135b^iy_jr_{00}r_0\beta^2 - 16b^iB^2\beta^5s_0r_j + 32Bb^i\beta^5s_0r_j - 36Bb^ir_{00}\beta^4r_j + 9Bb^ir_{00}\beta^3r_{j0} + 80B^2y^ir_0\beta^3r_{j0} + 272By^ir_0\beta^3r_{j0} \\
& - 36By^ir_{00}\beta^2r_{j0} + 192By^i\beta^4s_0r_j - 16B^2y^ir_{00}\beta^3r_j - 64By^ir_{00}\beta^3r_j + 8B^3r^i_0\beta^4s_0b_j - 120B^2r^i_0\beta^4s_0b_j - 162Br^i_0\beta^4s_0b_j \\
& + 18Br^i_0r_{00}\beta^3b_j - 32B^3r^i_0\beta^3s_0y_j - 96B^2r^i_0\beta^3s_0y_j + 288Br^i_0\beta^3s_0y_j - 54Br^i_0r_{00}\beta^2y_j - 32b^iy_jB^3\beta^3s_{0|0} - 560b^iy_jB^2\beta^3s^2_0 \\
& - 496b^iy_js^k_0\beta^4s_k - 96b^iy_jB^2\beta^3s_{0|0} + 2368b^iy_jB\beta^3s^2_0 + 288b^iy_jB\beta^3s_{0|0} + 178b^iy_js^k_0r_{k0}\beta^3 - 304b^iy_jr_0s_0\beta^3 + 16\beta^4y^ib_jsks^k \\
& - 306b^iy_js^2_0\beta^2s_{0r00} + 45b^iy_jBr_{00}\beta + 69\beta^2y^ib_jBr_{00}|0 - 78\beta^2y^ib_jr_0r_{00}r_0 - 297\beta^2y^ib_js_0r_{00} + 69\beta^2y^ib_jBr_{00} + 192\beta^3y^ib_jB^2s_0 \\
& + \beta^3y^ib_jBs^2_0 - 24\beta^2y^ib_jB^3r_{00}|0 + \beta^3y^ib_jBs_{0|0} - 270\beta^3y^ib_jr_{k0}s^k_0 + 48\beta^3y^ib_js_0r_0 - \beta^2y^ib_jB^2r_{00}|0 + 84\beta^3y^ib_jB^2r_{00} \\
& + 36Bb^ir_0\beta^4r_{j0} + 96B^3s^i_k s^k_0\beta^5b_j + B^2s^i_0\beta^4s_0b_j - 3348Bs^i_0\beta^4s_0b_j - B^2y^is^k_j\beta^4r_{k0} - 144By^is^k_j\beta^4r_{k0} - 24B^2b^is^k_j\beta^5r_{k0} \\
& - 24B^2y^ir_{00}\beta^2r_{j0} - 138Bb^is^k_j\beta^5r_{k0} - 72B^2b^is^k_j\beta^6s_k + Bb^is^k_j\beta^6s_k - B^3b^is^k_j\beta^5r_{jk} + 32B^3s^ir_{00}\beta^3y_j + 336Bs^i\beta^4s_0y_j \\
& + 24B^2s^ir_{00}\beta^3y_j + B^2s^ir_{00}\beta^4b_j - 90Bs^ir_{00}\beta^4b_j + B^3s^i_0\beta^4s_0y_j + 32B^4s^i_k s^k_0\beta^4y_j - 512B^3s^i_k s^k_0\beta^4y_j + 480B^2s^i_k s^k_0\beta^4y_j \\
& + 448Bs^i_k s^k_0\beta^4y_j - 54Bb^ir_{00}\beta^4s_j + 18b^iy_jB^2r_{00}|0\beta^3 + 156\beta^3y^iy_jBr_{00}|0 - 152\beta^3y^iy_jr_{00}r_0 - 8\beta^3y^iy_js_0r_{00} + 162B^2s^i_0r_{00}\beta^3b_j \\
& + 18B^2r^i_0r_{00}\beta^3b_j + \beta^2y^iy^iB^2s^2_0 + B^2y^is^k_0\beta^4r_{jk} + 288By^is^k_0\beta^4r_{jk} + 333By^ir_{00}\beta^2s_0j - 32B^3y^is^k_j\beta^5s_k + 96B^2y^is^k_j\beta^5s_k \\
& + 144B^2y^ir_{00}\beta^2s_0j - 96By^ir_0\beta^4s_j - 96By^ir_0\beta^4s_0j + 8B^2y^ir_{00}\beta^3s_j + 8By^ir_{00}\beta^3s_j + 72B^3b^i\beta^4s_0s_0j + 112r^i_0\beta^4s_0b_j \\
& - 36r^i_0r_{00}\beta^3b_j + 62y^ir_0\beta^3r_{j0} - 3y^ir_{00}\beta^2r_{j0} - 8B^3b^i\beta^5s_{0|0} + 24B^2b^i\beta^5s_{0|0} + 138Bb^i\beta^5s_{0|0} - 46b^ir_0\beta^5s_j + 336By^is^k_j\beta^5s_k \\
& + 576b^i\beta^4s_0s_0j - 63b^ir_0\beta^3s_0j + 96B^3s^i_k s^k_0\beta^6y_j + y^i\beta^4s_0r_j - 10y^ir_{00}\beta^3r_j + By^i\beta^3r_{00}|j - 288B^2s^i_k s^k_0\beta^6y_j + b^iy_jB^2s^k_0\beta^4s_k \\
& - 54s^i_0r_{00}\beta^3b_j + 72B^2s^i\beta^6s_j + 7s^ir_{00}\beta^3y_j - 67s^ir_{00}\beta^4b_j - 144Bs^i\beta^6s_j - 72B^4s^i_0\beta^4s_0j + 1152B^3s^i_0\beta^4s_0j - 1404B^2s^i_0\beta^4s_0j \\
& - 360Bs^i_0\beta^4s_0j + 38\beta^4s^i_k s^k_0y_j + 120s^i_k s^k_0\beta^5b_j + 32\beta^5y^is^k_0s_k + 18\beta^4y^ir_{k0}s^k_j - 36\beta^4y^ir_{jk}s^k_0 - 80s^i\beta^4s_0y_j - 8B^3s^i\beta^5r_{j0} \\
& + 24B^2s^i\beta^5r_{j0} + 138Bs^i\beta^5r_{j0} - 288B^2y^i\beta^4s_{0|j} - 288By^i\beta^4s_{0|j} - 324y^i\beta^3s_0s_0j + 81y^ir_{00}\beta^2s_0j - 48y^ir_0\beta^4s_0j - 48y^i\beta^4s_0s_0j \\
& + 11y^ir_{00}\beta^3s_j + 144B^2y^i\beta^4s_{j|0} + 144By^i\beta^4s_{j|0} + 16b^is^k_0\beta^5r_{jk} + 130b^i\beta^5s_0s_j - 27b^ir_{00}\beta^4s_j + 90b^is^k_0\beta^6s_k + 128B^3s^i_0|\beta^4b_j \\
& - 360Bs^i_k s^k_0\beta^6y_j - 8B^3r^i_0\beta^5s_j + 24B^2r^i_0\beta^5s_j + 138Br^i_0\beta^5s_j + 258s^i\beta^5s_0b_j + 558s^i_0\beta^3s_0y_j - 153s^i_0r_{00}\beta^2y_j - 342s^i_0\beta^4s_0b_j \\
& - 12B^2s^i_0|\beta^4b_j - Bs^i_0|\beta^4b_j + 24B^4s^i_0|\beta^3y_j - 8B^4s^i_0|\beta^4b_j - B^2s^i_0|\beta^3y_j + Bs^i_0|\beta^3y_j - 8b^is^k_0\beta^5r_{k0} + 16B^3b^i\beta^5s_0 \\
& - 48B^2b^i\beta^5s_{0|j} - 276Bb^i\beta^5s_{0|j} + 92b^i\beta^5s_0r_j - 18b^ir_{00}\beta^4r_j + 54B^2b^i\beta^4r_{00}|j + 54Bb^i\beta^4r_{00}|j - 54B^2r^i_0\beta^4r_{j0} - 54Br^i_0\beta^4r_{j0} \\
& + 164r^i_0\beta^3s_0y_j - 27r^i_0r_{00}\beta^2y_j + 58\beta^3y^iy_jr_{00}|0 - 18y^iy_jB^2r_{00} - 24y^iy_jBr_{00} - 48B^3s^i_0|\beta^3y_j - 208\beta^2y^iy_jy^2_0 - 136\beta^2y^iy_js_0 \\
& + 18b^ir_0\beta^4r_{j0} - 9b^ir_0\beta^3r_{j0} + 54B^2r^i_0r_{00}\beta^4 + 54Br^i_0r_{00}\beta^4 + 16B^3r^i_0\beta^5s_0 - 48B^2r^i_0\beta^5s_0 - 276Br^i_0\beta^5s_0 - 54B^2b^i\beta^4r_{j0} \\
& - 54Bb^i\beta^4r_{j0}|0 - 64\beta^5\delta^i_js_k s^k_0 + \beta^4\delta^i_jBs^2_0|0 - 192\beta^4\delta^i_jBs^2_0 - 8B^3\delta^i_jB^3r_{00}|0 + b^jb^i\beta^4s^2_0 + 112b^jb^i\beta^4s_{0|0} - 36b^jb^i\beta^4r_{00}|0 \\
& + y_jb^i\beta^3s^2_0 + 164y_jb^i s_0|\beta^3 + \beta^4\delta^i_jBs_0|0 - 36\beta^4\delta^i_js_0r_0 - \beta^4\delta^i_js_0r_0 - 48\beta^3\delta^i_jB^2r_{00}|0 - 15\beta^3\delta^i_jBr_{00}|0 + 20\beta^3\delta^i_jr_{00}r_0 \\
& - 64\beta^3\delta^i_js_0r_{00} + \beta^2\delta^i_jBr^2_{00} - \beta^3y^ib_js_0|0 + \beta^2y^ib_jr_{00}|0 + 126b^ib_jr^2_{00}\beta^2 + 16y^iB^3\beta^3r_{00}|j + 96B^2y^i\beta^3r_{00}|j - 27b^iy_jr_{00}|\beta^2 \\
& + 36b^iy_jBr^2_{00} + 11B^3y^ib_js^2_0 - 16B^3y^i\beta^3r_{j0}|0 - 9B^2y^i\beta^3r_{j0}|0 - 30By^i\beta^3r_{j0}|0 - 2r^i_0r_{00}\beta^4 - 16r^i_0\beta^5s_0 - 27b^iy^i\beta^4r_{j0} \\
& + 6y^iy_jr^2_{00} + 27b^iy^i\beta^4r_{j0}|0 + 27r^i_0\beta^4r_{j0} - 9\beta^4\delta^i_js^2_0 - 18\beta^4\delta^i_js_0|0 + 17\beta^3\delta^i_jr^2_{00} - 34\beta^3y^ir_{00}|j + 34\beta^3y^ir_{j0} \\
& + 8b^i\beta^5s^i_0|0\beta^4b_j - 23s^i_0|\beta^4b_j - 90s^i\beta^6s_j - 18\beta^4y^is^i_0|0 + 36\beta^4y^is^i_0|0 + 8r^i_0\beta^5s_j - 45s^i_0\beta^4s_0j + 57s^i_0|\beta^3y_j + 8B^4s^i_0|\beta^5 \\
& - 32B^3s^i_0|\beta^5 - 276B^2s^i_0|\beta^5 + 64B^3s^i_0|\beta^5 + 552B^2s^i_0|\beta^5 + 64Bs^i_0|\beta^5 + 66s^i_k s^k_0\beta^6 + 89s^i_0|\beta^5 - 178s^i_0|\beta^5 \\
& + 384\beta^3y^iB^3s^k_0s_k - 96\beta^2y^iy_jB^2s^k_0r_{k0} - 352\beta^2y^iy_jB^2r_{0s0} - 840\beta^2y^iy_jBs^k_0r_{k0} - 64\beta^2y^iy_jBr_{0s0} + 64\beta^2y^iy_jB^2r_{0r0} \\
& - 164\beta^2y^iy_jBr_{0r0} + 316\beta^2y^iy_jBr_{00}s_0 - 128\beta^3y^iy_jB^3s^k_0s_k - 90b^ib_jBr_{0r0}\beta^3 + 52\beta^3y^ib_jB^2s^k_0r_{k0} - 96\beta^3y^ib_jB^2r_{0s0} \\
& - 384\beta^3y^ib_jBr_{0s0} + 96\beta^2y^ib_jB^2r_{0r0} - 600\beta^2y^ib_jB^2r_{00}s_0 + 72\beta^2y^ib_jBr_{0r0} - 1344\beta^2y^ib_jBr_{00}s_0 + 32\beta^4y^ib_jB^3s^k_0s_k \\
& + 96\beta^3y^ib_jB^3s^k_0r_{k0} - 720\beta^4y^ib_jBs^k_0s_k - 72b^iy_jB^2r_{00}\beta^2s_0 - 648b^iy_jB^2r_{00}\beta^2s_0 - 864\beta^3y^iy_jB^2s^k_0s_k - 32b^iy_jB^3s^k_0\beta^4s_k \\
& + 32b^iy_jB^3s^k_0r_{k0}\beta^3 - 48b^iy_jBs^k_0\beta^4s_k - 48b^iy_jB^2s^k_0r_{k0}\beta^3 + 32b^iy_jB^2r_{0}\beta^3s_0 + 272b^iy_jBr_{0}\beta^3s_0 - 342b^iy_jBs^k_0r_{k0}\beta^4 \\
& + 72b^iy_jBr_{00}\beta^3s_0 + 108B^2i^ib_jr_{00}\beta^3s_0 - 168b^iy_jB^2s^k_0\beta^5s_k - 8b^iy_jB^3s^k_0r_{k0}\beta^4 + 84b^iy_jBs^k_0\beta^5s_k + 48b^iy_jB^2s^k_0r_{k0}\beta^4 \\
& - 8b^iy_jB^2r_{0}\beta^4s_0 - 16b^iy^i\beta^5s_{0|j} + 224\beta^2y^iy_jB^3s^k_0r_{k0} + 16\beta^2y^iy_jB^2r_{00}s_0 - 192\beta^3y^ib_jBs^k_0r_{k0} - 32Bs^i_0|\beta^5 - 16B^4s^i_0|\beta^5 \\
& + 184b^iy_jBr_{0}\beta^4s_0 - 19\beta^4y^ib_jB^2s^k_0s_k \Big] - 144\hat{R}^i_j\beta^{10}(2B+1)(B+2)(B-1)
\end{aligned}$$

$$\begin{aligned}
t_{12} &:= -6y_j \beta^8 (32B^3 b^i s_0^2 + 8B^4 s_{0|0}^i + 16B^2 b^i s_0^2 + 16s_{0|0}^i B^3 - 344Bb^i s_0^2 - 96B^2 s_{0|0}^i - 136b^i s_0^2 - 104Bs_{0|0}^i - 13s_{0|0}^i) \\
t_{13} &:= +64By^i \beta^4 s_0 s_j - 8B^2 y^i r_{00} \beta^3 s_j - 16B^4 s_{k|0}^i s_k^k \beta^5 b_j + 32B^3 y^i s_j^k \beta^4 r_{k0} + 48B^2 y^i s_j^k \beta^4 r_{k0} - 84By^i s_j^k \beta^4 r_{k0} - 576B^3 s_{0|0}^i \beta^4 s_0 b_j \\
&\quad - 104Bb^i \beta^5 s_0 s_j + 12B^2 b^i r_{00} \beta^4 s_j - 234B^2 y^i r_{00} \beta^2 s_{0j} - 45By^i r_{00} \beta^2 s_{0j} - 192B^3 y^i \beta^3 s_0 s_{0j} + 1152B^2 y^i \beta^3 s_0 s_{0j} - 72B^3 y^i r_{00} \beta^2 s_{0j} \\
&\quad + 4By^i \beta^3 s_0 s_{0j} - 28B^2 y^i y_j s_0 r_0 + 768\beta^2 y^i y_j B^2 s_0^2 - 216\beta^3 y^i y_j s_k s_k^k - 432B^2 s_{0|0}^i r_{00} \beta^3 b_j - 16B^3 s^i \beta^6 s_j - 12B^2 b^i r_{00} \beta^4 r_{j0} \\
&\quad - 12Bb^i r_{00} \beta^4 r_{j0} - 12Bb^i \beta^4 s_0 r_{j0} + 9Bb^i r_{00} \beta^3 r_{j0} + 128B^3 s_{0|0}^i \beta^3 y_j + 300\beta y^i y_j B r_{00} s_0 + 96\beta y^i y_j B^2 r_{00} r_0 - 48\beta y^i y_j B^2 r_{00} s_0 \\
&\quad + 192\beta^3 y^i y_j B^3 s_{0|0}^i s_k - 62\beta^3 y^i y_j B^2 s_k^0 s_k + 96\beta^2 y^i y_j B^3 s_{0|0}^i r_{k0} - 816\beta^3 y^i y_j B s_k^0 s_k + 82y^i r_0 \beta^3 r_{j0} + 624\beta^2 y^i y_j B^2 s_{0|0}^i r_{k0} \\
&\quad + 552\beta^2 y^i y_j B s_k^0 r_{k0} - 15y^i r_{00} \beta^2 r_{j0} - 16B^2 b^i \beta^5 s_0 r_j - 112Bb^i \beta^5 s_0 r_j + 12B^2 b^i r_{00} \beta^4 r_j + 12Bb^i r_{00} \beta^4 r_j - 80B^2 y^i r_0 \beta^3 r_{j0} \\
&\quad - 64B^2 y^i \beta^4 s_0 r_j - By^i \beta^4 s_0 r_j + 16B^2 y^i r_{00} \beta^3 r_j - 8By^i r_{00} r_j + 16By^i r_0 \beta^3 r_{j0} + 6B^2 y^i r_{00} \beta^2 r_{j0} - 18By^i r_{00} \beta^2 r_{j0} - B^4 s_{0|0}^i \beta^5 \\
&\quad - 18y^i y_j r_{00}^2 + \beta^2 y^i b_j B^2 r_{00} s_0 - \beta^2 y^i b_j B r_{00} s_0 + \beta^4 y^i b_j B^3 s_{0|0}^i s_k + \beta^4 y^i b_j B^2 s_k^0 s_k - 28\beta^3 y^i b_j B^3 s_{0|0}^i r_{k0} \\
&\quad - 33\beta^4 b_j y^i B s_k^0 s_k + 256\beta^3 y^i b_j B^2 r_{00} s_0 + 58\beta^3 y^i b_j B s_k^0 r_{k0} - 32\beta^3 y^i b_j B r_{00} s_0 - 32\beta^2 y^i b_j B^2 r_{00} r_0 - 35b^i b_j r_{00} \beta^3 \\
&\quad - 54b^i b_j B r_{00}^2 \beta^2 + 288b^i y_j B^2 s_k^0 \beta^4 s_k + 72b^i y_j B s_k^0 \beta^3 s_k + 48b^i y_j B^2 s_k^0 r_{k0} \beta^3 - 204b^i y_j B s_k^0 r_{k0} \beta^3 + 33b^i y_j B r_{00} \beta^3 s_0 \\
&\quad + 387b^i b_j r_{00} \beta^3 s_0 + 72b^i y_j B^2 r_{00} \beta^2 s_0 - 306b^i y_j B r_{00} \beta^2 s_0 + 34\beta^3 y^i r_{00|j} - 34\beta^3 y^i r_{00|j} - 184\beta^4 \delta^i_j s_0^2 + 50\beta^4 \delta^i_j s_{0|0} \\
&\quad - 17\beta^3 \delta^i_j r_{00|0} + 60b^i y_j s_k^0 \beta^4 s_k - 192b^i y_j B^2 \beta^3 s_{0|0} + 560b^i y_j B \beta^3 s_0^2 - 156y^i b^i B \beta^3 s_{0|0} - 6b^i y_j s_k^0 r_{k0} \beta^3 - 224B^3 s_{0|0}^i \beta^5 \\
&\quad + 312b^i y_j r_{00} s_0 \beta^3 + 18b^i y_j B^2 r_{00|0} \beta^2 + 18b^i y_j B r_{00|0} \beta^2 - 45b^i y_j r_{00} r_0 \beta^2 - 360b^i y_j B^2 s_{0|0} r_0 + 18b^i y_j B^2 r_{00} \beta - 832b^i y_j B^2 \beta^3 s_0^2 \\
&\quad - 64\beta^3 y^i b_j B^3 s_{0|0} - 1088\beta^3 y^i b_j B^2 s_0^2 - 324\beta^4 y^i b_j s_k s_k^k - 96\beta^3 y^i b_j B^2 s_{0|0} + 640\beta^3 y^i b_j B s_0^2 - 288B^2 s^i \beta^4 s_0 y_j - 24B^4 s_{0|0} \beta^3 y_j \\
&\quad + 162\beta^2 y^i y_j s_0 r_{00} + 240\beta^2 y^i y_j B^2 s_{0|0} - 192\beta^2 y^i y_j B s_0^2 - 24\beta^2 y^i y_j B^3 r_{00|0} + 240\beta^2 y^i y_j B s_{0|0} + 132\beta^2 y^i y_j r_{k0} s_k^0 + 496B^4 s_{0|0}^i \beta^5 \\
&\quad - 14s_{0|0}^i \beta^3 y_j - 57s_{0|0}^i \beta^4 b_j - 60B^2 s_{0|0}^i \beta^5 - 248B^2 s_{0|0}^i \beta^5 - 124b^i \beta^5 s_{0|0} + 50\beta^4 y^i s_{0|0} - 100\beta^4 y^i s_{0|0} + 62r^i_0 \beta^5 s_j + 8B^4 s_{0|0}^i \beta^5 \\
&\quad + 62s^i \beta^5 r_{j0} - 72B^3 s_{0|0}^i r_{00} \beta^3 b_j - 32B^3 s_k^0 y^i \beta^5 s_k - 192B^2 s_k^0 y^i \beta^5 s_k + 216B^3 b^i \beta^4 s_0 s_{0j} - 900B^2 b^i \beta^4 s_0 s_{0j} \\
&\quad + 96B^4 s_k^0 s_k^0 \beta^4 y_j - 384B^3 s_k^0 s_k^0 \beta^4 y_j + 3024B^2 s_k^0 s_k^0 \beta^4 s_0 b_j - 432B^2 s_k^0 s_k^0 \beta^4 s_0 b_j + 351B^2 s_k^0 s_k^0 \beta^4 s_0 y_j + 864B^3 s_k^0 s_k^0 \beta^4 s_0 y_j \\
&\quad - 1296B^2 s_k^0 \beta^3 s_0 y_j - 72B^3 s_k^0 r_{00} \beta^2 y_j - 972B^2 s_k^0 \beta^3 s_0 y_j + 189B^2 s_k^0 r_{00} \beta^2 y_j + 24b^i b_j B^3 \beta^4 s_0 s_{0j} + 552b^i b_j B^2 \beta^4 s_0 s_{0j} \\
&\quad + 12b^i b_j B^2 \beta^4 s_{0|0} - 2376b^i b_j B \beta^4 s_{0|0} - 258b^i b_j B \beta^4 s_{0|0} - 240b^i b_j s_k^0 r_{k0} \beta^4 - 124r^i_j \beta^5 s_0 + 39r^i_j r_{00} \beta^4 \\
&\quad + 294b^i b_j r_{00} \beta^4 s_0 + 39b^i b_j \beta^4 r_{00|0} + 54b^i b_j B r_{00|0} \beta^3 - 39b^i b_j \beta^4 r_{j0|0} - 39r^i_0 \beta^4 r_{j0|0} + 376\beta^3 y^i b_j s_0^2 + 136\beta^3 y^i b_j s_{0|0} + 112B^3 s_{0|0}^i \beta^5 \\
&\quad - 41\beta^2 y^i b_j r_{00|0} + 27\beta^2 y^i b_j r_{00} - 16y^i B^3 \beta^3 r_{00|j} + 12B^2 y^i \beta^3 r_{00|j} + 78By^i \beta^3 r_{00|j} - 28y^i r_0 \beta^4 s_j + 16y^i \beta^4 s_0 s_j - 112s^i_k s_k^k \beta^6 \\
&\quad + 62b^i \beta^5 s_{j0} - 11y^i r_{00} \beta^3 s_j + 68\beta^5 y^i s_k s_k^k - 32B^3 y^i \beta^4 s_{j0} - 48B^2 y^i \beta^4 s_{j0} + 84By^i \beta^4 s_{j0} - 50\beta^4 y^i r_{k0} s_k^0 + 100\beta^4 y^i r_{jk} s_k^0 \\
&\quad + 96B^2 y^i \beta^4 s_{0|j} - 168By^i \beta^4 s_{0|j} - 96y^i \beta^3 s_0 s_{0j} + 27y^i r_{00} \beta^2 s_{0j} - 24B^4 s_{0|0}^i \beta^4 b_j + 48B^3 s_{0|0}^i \beta^4 b_j + 324B^2 s_{0|0}^i \beta^4 b_j \\
&\quad - 288\beta^2 y^i y_j s_0^2 + 60\beta^2 y^i y_j s_{0|0} - 18\beta^2 y^i y_j r_{00|0} - 16B^4 s_k^0 s_k^0 \beta^6 + 8\beta^2 y^i b_j B^3 r_{00|0} + 24\beta^3 y^i b_j B s_{0|0} \\
&\quad - 188\beta^3 y^i b_j s_0 r_0 - 66\beta^2 y^i b_j B^2 r_{00|0} + 18\beta^2 y^i b_j B^2 r_{00} - 117\beta^2 y^i b_j B r_{00|0} + 100\beta^2 y^i b_j r_{00} r_0 - 173\beta^2 y^i b_j s_0 r_{00} \\
&\quad - 102r^i_0 \beta^4 s_0 b_j + 27r^i_0 r_{00} \beta^3 b_j - 32\beta^4 \delta^i_j B^3 s_{0|0} + 128\beta^4 \delta^i_j B^2 s_0^2 - 136\beta^5 \delta^i_j s_k s_k^0 - 48\beta^4 \delta^i_j B^2 s_{0|0} + 128\beta^4 \delta^i_j B s_0^2 \\
&\quad + 84\beta^4 \delta^i_j B s_{0|0} - 6\beta^3 \delta^i_j B^2 r_{00|0} + 100\beta^4 \delta^i_j r_{k0} s_k^0 - 112\beta^4 \delta^i_j s_0 r_0 - 6\beta^2 \delta^i_j B^2 r_{00} - 39\beta^3 \delta^i_j B r_{00|0} + 52\beta^3 \delta^i_j r_{00} r_0 \\
&\quad - 30\beta^2 \delta^i_j B r_{00} + 16B^3 b^i s_k^0 \beta^5 s_k - 96\beta y_j B^2 r_{00|0} - 78\beta y_j B^2 r_{00|0} + 90\beta y_j y^i r_{00} r_0 + 16B^3 y^i \beta^3 r_{j0|0} - 12B^2 y^i \beta^3 r_{j0|0} \\
&\quad - 78By^i \beta^3 r_{j0|0} - 30b^i \beta^4 s_0 r_{j0} + 18b^i r_{00} \beta^3 r_{j0} - 1002b^i b_j \beta^4 s_0^2 - 102b^i b_j \beta^4 s_{0|0} + 27b^i b_j r_{00|0} \beta^3 - 54b^i b_j r_{00} \beta^2 s_0^2 \\
&\quad - 48B^2 s_{0|0}^i \beta^4 b_j - 84B^2 r^i_0 \beta^5 s_j + 30Br^i_0 \beta^5 s_j + 64\beta^5 \delta^i_j B^3 s_k s_k^0 + 384\beta^5 \delta^i_j B^2 s_k^0 s_k - 64\beta^4 \delta^i_j B^3 s_k^0 r_{k0} - 96\beta^5 \delta^i_j B s_k^0 s_k \\
&\quad + 76b^i y_j \beta^3 s_0^2 + 24b^i y_j s_{0|0} \beta^3 - 9b^i y_j r_{00|0} \beta^2 + 45b^i y_j r_{00}^2 - 5s_{0|0}^i \beta^5 + 110s_{0|0}^i \beta^5 + 16b^i b_j B^3 s_k^0 \beta^5 s_k - 4b^i b_j B^2 s_k^0 \beta^5 s_k \\
&\quad + 180b^i b_j B s_k^0 \beta^5 s_k + 132b^i b_j B^2 s_k^0 r_{k0} \beta^4 - 24b^i b_j B^2 r_{00} \beta^4 s_0 - 30b^i b_j B s_k^0 r_{k0} \beta^4 - 216b^i b_j B r_{00} \beta^4 s_0 + 68b^i b_j B r_{00} \beta^4 s_0 \\
&\quad + 72B^2 b^i r_{00} \beta^3 s_0 b_j + 48B^2 s_k^0 \beta^4 y_j + 352B^3 s_k^0 s_k^0 \beta^5 b_j - 744B^2 s_k^0 s_k^0 \beta^5 b_j - 152B^2 s_k^0 s_k^0 \beta^5 b_j + 60B^2 s_k^0 r_{00} \beta^4 b_j \\
&\quad + 48B^2 s_k^0 y^i \beta^5 s_k - 168B^2 b^i s_k^0 \beta^5 r_{jk} - 8By^i r_{00} \beta^3 s_j + 60B^2 s_k^0 \beta^5 r_{jk} + 18B^2 b^i r_{00} \beta^3 s_j + 8B^2 b^i r_{00} \beta^5 s_j - 8B^2 b^i \beta^5 s_0 s_{0j} \\
&\quad + 56B^2 b^i r_{00} \beta^5 s_j + 72B^2 b^i r_{00} \beta^3 s_0 j + 8B^2 b^i s_k^0 \beta^5 r_{k0} + 84B^2 b^i s_k^0 \beta^5 r_{k0} - 30B^2 b^i s_k^0 \beta^5 r_{k0} + 12B^2 b^i r_{00} \beta^4 s_j - 64B^3 s_k^0 y^i \beta^4 r_{jk} \\
&\quad - 468B^2 s_k^0 b_j - 120B^2 b^i s_k^0 \beta^6 s_k - 132B^2 b^i s_k^0 \beta^6 s_k - 16B^2 b^i s_k^0 \beta^5 r_{jk} + 96B^3 s_k^0 \beta^4 s_0 y_j + 144B^2 s_k^0 \beta^4 s_0 y_j + 120B^2 s_k^0 r_{00} \beta^3 y_j \\
&\quad + 16B^2 s_k^0 b_j - 24B^2 s_k^0 r_{00} \beta^4 b_j + 24B^2 r^i_0 \beta^4 s_0 b_j + 12B^2 r^i_0 \beta^4 s_0 b_j - 25B^2 r^i_0 \beta^4 s_0 b_j + 54B^2 r^i_0 r_{00} \beta^3 b_j - 96\beta^4 \delta^i_j B^2 s_k^0 r_{k0} \\
&\quad + 168\beta^4 \delta^i_j B s_k^0 r_{k0} + 128\beta^4 \delta^i_j B r_{00} s_0 - 32\beta^3 \delta^i_j B^2 r_{00} r_0 - 32\beta^3 \delta^i_j B^2 r_{00} s_0 + 16\beta^3 \delta^i_j B r_{00} r_0 + 112\beta^3 \delta^i_j B r_{00} s_0 \\
&\quad + 16B^3 r^i_j \beta^5 s_0 + 168B^2 r^i_j \beta^5 s_0 - 12B^3 r^i_j \beta^5 s_0 - 18B^2 r^i_j \beta^5 s_0 - 18B^2 r^i_j r_{00} \beta^4 + 72B^2 r^i_j r_{00} \beta^4 + 12B^3 b^i \beta^4 r_{j0|0} \\
&\quad - 72B^2 b^i \beta^4 r_{j0|0} + 12B^3 r^i_0 \beta^4 r_{j0|0} + 72B^2 r^i_0 r_{00} \beta^4 + 24r^i_0 \beta^3 s_0 y_j - 9r^i_0 r_{00} \beta^2 y_j - 12B^3 b^i \beta^4 r_{00|j} - 18B^2 b^i \beta^4 r_{00|j}
\end{aligned}$$

$$\begin{aligned}
& + 72Bb^i\beta^4r_{00|j} - 72Br_0^i\beta^4r_{j0} + 18B^2r_0^i\beta^4r_{j0} - 192B^2r_0^i\beta^3s_0y_j - 156Br_0^i\beta^3s_0y_j + 20b^i\beta^5s_0r_j - 24b^ir_{00}\beta^4r_j + 56y^i\beta^4s_0r_j \\
& - 6Bb^i\beta^5s_{0|j} - 10b^ir_0\beta^5s_j + 58b^i\beta^5s_0s_j - 51b^ir_{00}\beta^4s_j - 29Bs_k^is_j^k\beta^6 + 234b^i\beta^4s_0s_{0j} - 90b^ir_{00}\beta^3s_{0j} + 18B^2r_0^i r_{00}\beta^2y_j \\
& - 84B^2b^i\beta^5s_{j|0} + 30Bb^i\beta^5s_{j|0} + 160B^3s_k^is_j^k\beta^6 + 264B^2s_k^is_j^k\beta^6 - 156B^2s_k^is_{0|0}\beta^3y_j - 136Bs_k^is_{0|0}\beta^3y_j + 48s^i\beta^4s_0y_j \\
& + 154s^i\beta^5s_0b_j - 69s^ir_{00}\beta^4b_j + 120B^2s^i\beta^6s_j + 132Bs^i\beta^6s_j - 8B^3s^i\beta^5r_{j0} - 84B^2s^i\beta^5r_{j0} + 30Bs^i\beta^5r_{j0} + 16B^3b^i\beta^5s_{0|j} \\
& + 42\beta^4s_k^is_j^k\beta^6y_j - 216B^4s_k^is_0\beta^4s_{0j} + 1008B^3s_k^is_0\beta^4s_{0j} - 108B^2s_k^is_0\beta^4s_{0j} - 54s^i\beta^3s_0y_j - 9s^ir_{00}\beta^2y_j - 558s^i\beta^4s_0b_j \\
& - 96\beta^2y^iy_jB^2r_0s_0 - 12B^2b^i\beta^4s_0r_{j0} + 153s^i_0r_{00}\beta^3b_j + 16B^4s^i_0r_0\beta^3y_j + 74b^is_j^k\beta^6s_k - 62b^is_j^k\beta^5r_{k0} + 124b^is_k^k\beta^5r_{jk} \\
& + 168B^2b^i\beta^5s_{0|j} - 96B^2s_k^i_0y^i\beta^4r_{jk}\Big] + 6R^i_j\beta^8 \left(8B^4 + 16B^3 - 96B^2 - 104B - 13 \right) \\
t_{14} & := -8\beta^6y_j \left(16B^3b^i s_0^2 + 144B^2b^i s_0^2 + 36s^i_0B^3 + 42Bb^i s_0^2 + 54B^2s^i_0r_0 - 40b^i s_0^2 - 9s^i_0r_0 \right) \\
t_{15} & := 4\beta^2 \left[384B^3y^i\beta^3s_0s_{0j} - 336B^2s^i\beta^5s_0b_j - 144B^2y^i\beta^3s_0s_{0j} - 36B^3y^i r_{00}\beta^2s_{0j} - 432By^i\beta^3s_0s_{0j} + 72B^2y^i r_{00}\beta^2s_{0j} \right. \\
& + 80Bb^i\beta^5s_0s_j - 576B^3s^i_0\beta^3s_0y_j - 216B^2s^i_0\beta^3s_0y_j + 108B^3s^i_0r_{00}\beta^2y_j + 216Bs^i_0\beta^3s_0y_j + 16B^2b^i r_0\beta^5s_j - 16B^2b^i\beta^5s_0s_j \\
& - 32Bb^i r_0\beta^5s_j + 864B^3s^i_0\beta^4s_0b_j - 1296B^2s^i_0\beta^4s_0b_j - 240B^3b^i\beta^4s_0s_{0j} + 144B^2b^i\beta^4s_0s_{0j} - 16b^ib_jB^3\beta^4s_{0|0} - 32B^2b^i\beta^5s_0r_j \\
& - 36Bb^i r_{00}\beta^4r_j - 64\beta^5\delta^i_js_k^k + 64\beta^2y^iy_jB^3s_{0|0} - 640\beta^2y^iy_jB^2s_0^2 + 96B^2s^i\beta^4s_0y_j + 96\beta^2y^iy_jB^2s_{0|0} - 640\beta^2y^iy_jBs_0^2 \\
& - 16\beta y^iy_jB^3r_{00|0} + 48\beta^2y^iy_jBs_{0|0} + 16\beta^2y^iy_jr_{k0}s^k_0 - 64\beta^2y^iy_jr_{00}s_0r_0 + 18b^ib_jB^2r_{00|0}\beta^3 + 32B^3r^i_j\beta^5s_0 - 96B^2r^i_j\beta^5s_0 \\
& - 28Br^i_j\beta^5s_0 + 54B^2r^i_jr_{00}\beta^4 + 54Br^i_jr_{00}\beta^4 - 54B^2b^i\beta^4r_{j0|0} - 54Bb^i\beta^4r_{j0|0} + 54B^2b^i\beta^4r_{00|j} + 54Bb^i\beta^4r_{00|j} - 32B^3y^iy^is_k^k \\
& - 24\beta y^iy_jB^2r_{00|0} - 2\beta y^iy_jBr_{00|0} - 24Bs^i_0r_{00}\beta^3y_j + 16\beta y^iy_jr_{00r_0} + 64\beta y^iy_jr_{00r_0} + 2B^4s^i_k^s_j\beta^6 + 8\beta^2y^iy_jr_{00|0} - 2\beta y^iy_jr_{00|0} \\
& - 24y^iy_jB^2r_{00}^2 - 24y^iy_jBr_{00}^2 + 48B^2s^i\beta^6s_j - 192Bs^i\beta^6s_j - 16B^3s^i\beta^5r_{j0} + 48B^2s^i\beta^5r_{j0} + 114Bs^i\beta^5r_{j0} - 132y^i\beta^3s_0s_{0j} \\
& + 36\beta^4y^ir_{jks}^k - 24s^i\beta^4s_0y_j + 72B^2y^i\beta^4s_{j|0} + 72By^i\beta^4s_{j|0} + s^ir_{00}\beta^3y_j + 8Bs^i_0r_0\beta^3y_j - 16\beta^2y^iy_jr_{00}^2 + 64Bb^i\beta^5s_0r_j \\
& + 16B^4s^i_0r_0\beta^4b_j + 128B^3s^i_0r_0\beta^4b_j - 156B^2s^i_0r_0\beta^4b_j - 136Bs^i_0r_0\beta^4b_j - 84B^2s^i_0r_0\beta^3y_j - 16B^3b^i\beta^5s_{j|0} + 48B^2b^i\beta^5s_{j|0} \\
& + 114Bb^i\beta^5s_{j|0} - 228Bb^i\beta^5s_{0|j} - 16B^3r^i_0\beta^5s_j + 48B^2r^i_0\beta^5s_j + 114Br^i_0\beta^5s_j + 32B^3b^i\beta^5s_{0|j} - 96B^2b^i\beta^5s_{0|j} - 24y^ir_0\beta^4s_j \\
& + y^ir_{00}\beta^3s_j - 192\beta^4\delta^i_js^2_0 + 18\beta^4\delta^i_js_{0|0} - 5\beta^3\delta^i_jr_{00|0} + 10\beta^3y^ir_{00|j} - 10\beta^3y^ir_{j0|0} - 20\beta^2y^ib_jB^3r_{00|0} + 240\beta^3y^ib_jBs_{0|0} \\
& - 72B^3s^i_0r_{00}\beta^3b_j + 4B^2y^ir_{00}\beta^3s_j + 4B^2y^ir_{00}\beta^3s_j - 48By^i\beta^4s_0s_j + 336Bs^i\beta^5s_0b_j + 46Bb^i\beta^4s_0s_{0j} - 18B^2b^i\beta^5r_{00}\beta^3s_0 \\
& - 126Bb^i r_{00}\beta^3s_0 + 64B^3s^i\beta^5s_0b_j - 114Bb^i s_j^k\beta^5r_{k0} + 120Bs^k_jy^i\beta^5s_k + 64B^4s^i_k^s_0\beta^5b_j - 448B^3s^i_k^s_0\beta^5b_j + 96B^2s^i_k^s_0\beta^5b_j \\
& + 248Bs^i_k^s_0\beta^5b_j + 36B^2s^i_0\beta^4s_{0j} + 120Bs^i_0s_{0j} + 50b^is^k_j\beta^6s_k - 144B^2y^i\beta^4s_{0|j} - 144By^is_{0|j} + 96b^i\beta^4s_0s_{0j} - 18b^ir_{00}\beta^3s_{0j} \\
& + 98s^i\beta^5s_0b_j - 31s^ir_{00}\beta^4b_j + 98b^i\beta^5s_0s_j - 27b^ir_{00}\beta^4s_j + 64B^3s^i_k^s_j\beta^6 - 384B^2s^i_k^s_j\beta^6 - 200Bs^i_k^s_j\beta^6 + 32\beta^5y^is_k^s_j \\
& - 18\beta^4y^is^k_j - 16b^is^k_j\beta^5r_{k0} + 32b^is^k_0\beta^5r_{jk} - 38b^ir_0\beta^5s_j + 40s^i_k^s_0\beta^5b_j + 18\beta^4s^i_k^s_0y_j - 72B^2s^i_k^jy^i\beta^4r_{k0} - 72Bs^i_k^jy^i\beta^4r_{k0} \\
& - 48B^2b^is^k_j\beta^6s_k + 192Bb^is^k_j\beta^6s_k + 16B^3b^is^k_j\beta^5r_{k0} - 32B^3s^k_jy^i\beta^5s_k + 96B^2s^k_jy^i\beta^5s_k - 48B^2b^is^k_j\beta^5r_{k0} - 96B^4s^i_k^s_0\beta^4y_j \\
& + 144B^2s^i_k^s_0\beta^4y_j + 96Bs^i_k^s_0y_j - 32B^3b^is^k_0\beta^5r_{jk} + 96B^2b^is^k_0r_{jk} + 228Bs^i_0\beta^5r_{jk} + 72B^2s^i_0r_{00}\beta^4b_j + 378B^2s^i_0r_{00}\beta^3b_j \\
& - 32B^3b^is^k_j\beta^6s_k + 18B^2r^i_0r_{00}\beta^3b_j - 138Bs^i_0r_{00}\beta^4b_j + 16B^3s^i_0r_{00}\beta^4b_j + 144B^2b^4y^is^k_0r_{jk} + 144B\beta^4y^is^k_0r_{jk} - 96B^3s^i_k^s_0\beta^4s_{0j} \\
& + 32B^3s^i_0r_{00}\beta^3y_j + 24Bs^i_0\beta^4s_0y_j - 36B^2s^i_0r_{00}\beta^3y_j - 972Bs^i_0\beta^4s_0b_j + 189Bs^i_0r_{00}\beta^3b_j + 160B^4y^ib_jB^3s^i_0s_k - 576B^4y^ib_jB^2s^i_0s_k \\
& + 96B^3y^ib_jB^3s^i_0r_{k0} - 66y^ib_jBs^i_0s_k + 408\beta^3y^ib_jB^2s^i_0r_{k0} - 96\beta^3y^ib_js_0 + 336\beta^3y^ib_js_0^k r_{k0} - 480\beta^3y^ib_jBs_{0|0} + 80\beta^2y^ib_jB^2r_{00} \\
& - 148B^2y^ib_jB^2r_{00}s_0 + 164\beta^2y^ib_jBr_{00}s_0 - 16\beta^2y^ib_jBr_{00}s_0 + 192b^iy_jB^2s^i_0\beta^4s_k + 32b^iy_jB^2s^i_0r_{k0}\beta^3 - 168b^iy_jBs^i_0\beta^4s_k \\
& + 32b^iy_jB^2r_0\beta^3s_0 + 48b^iy_jBs^i_0r_{k0}\beta^3 + 104b^iy_jBr_0\beta^3s_0 - 108b^iy_jBr_0\beta^2s_0 + 108b^iy_jBr_0\beta^2s_0 + 96b^iy_jB^3s_k + 36Bb^ir_0\beta^4r_{j0} \\
& + 9Bb^ir_{00}\beta^3r_{j0} - 32B^3s_0y_j + 96Br^i_0\beta^3s_0y_j - 6y^iy_j - 18Br^i_0r_{00}\beta^2y_j + 64\beta^3\delta^i_jBr_0r_{00} + 208\beta^3\delta^i_jBr_0r_{00} + 64\beta^5\delta^i_jB^3s^i_0s_k \\
& - 192\beta^5\delta^i_jB^2s^i_0s_k - \delta^i_js^k_0s_k + 144\beta^4\delta^i_jB^2s^i_0r_{k0} + 144\delta^i_js^k_0r_{k0} - 192\beta^4\delta^i_jBr_0s_0 + 16\beta^3\delta^i_jB^2r_0r_{00} + 64\beta^3\delta^i_jB^2r_{00}s_0 \\
& - 848b^ib^2\beta^4s^i_0 + 10b^ib^is^k_0\beta^5s_k - 144b^ib^iB^2\beta^4s_{0|0} + 544b^ib^iB^2\beta^4s_0^2 - 42b^ib^iB^2\beta^4s_{0|0} - 22b^ib^iB^2s^i_0r_{k0}\beta^4 + 274b^ib^iB^2r_0\beta^4s_0 \\
& + 18b^ib^iB^2r_{00}\beta^3 - 5b^ib^i\beta^3 - 33b^ib^iB^2r_{00}\beta^3s_0 + 9b^ib^iB^2r_{00}\beta^2 + 46b^iy^ib^iB^2s^i_0r_{k0}\beta^3 - 16b^iy^ib^iB^2s^i_0r_{k0}\beta^3 - 18b^iy^ib^i\beta^2 + 45b^iy^ib^iB^2r_{00}\beta^2 \\
& + 240\beta^3y^ib^iB^2s_{0|0} + 240\beta^3y^ib^iB^2s_0 + 768\beta^3y^ib^iB^2s_0^2 - 184\beta^4y^ib^iB^2s^i_0s_k + 32B^3s^i_0\beta^6s_j - 112B^3s^i_0r_0\beta^3y_j - 8B^3y^ib^iB^2r_{00}s_0 \\
& - 48B^2y^ib^i\beta^3r_{j0|0} - 42B^2y^ib^i\beta^3r_{j0|0} + 18b^ir_0\beta^4r_{j0} + 16s^i_j\beta^5 - 32s^i_0\beta^5 + 78\beta^3y^ib^iB^2s^i_0s_k - 50s^i\beta^6s_j - 216\beta^3y^ib^iB^2r_{00}s_0 \\
& - 54b^iy^ib^i\beta^2s_0r_{00} + 15b^iy^ib^iBr^2_{00}\beta + 48y^ib^iB^2s_0r_{j0} - 14y^ir_0\beta^3r_{j0} + 58y^ir_0\beta^3r_{j0} - 9y^ir_0\beta^2r_{j0} + 40r^i_0\beta^4s_0b_j - 9r^i_0r_{00}\beta^3b_j \\
& - 72\beta^2y^ib^iB^2r_{00}|0 + 6\beta^2y^ib^iB^2r^2_{00} - 57\beta^2y^ib^iB^2r_{00}|0 - 12s^i_0\beta^4s_0j + 62\beta^2y^ib^iB^2r_{00}|0 + 29\beta^2y^ib^iB^2r_{00}|0 + 3\beta^2y^ib^iB^2r^2_{00} \\
& + 3b^ib^iB^2s^i_0\beta^5s_k + 16b^ib^iB^2s^i_0r_{k0} + 528b^ib^iB^2s^i_0\beta^5s_k + 16b^ib^iB^2s^i_0r_{k0}\beta^4s_0 - b^ib^iB^2s^i_0r_{k0}\beta^4 + 304b^ib^iB^2r_{00}\beta^4s_0 - 252b^ib^iB^2r_{00}\beta^3s_0 \\
& - 32b^iy^ib^iB^3\beta^3s_{0|0} + 576b^iy^ib^iB^2\beta^3s_0^2 - 36\beta^4y^ib^iB^2\beta^3s_0|0 - 120b^iy^ib^iB^2\beta^3s_0 + 648b^iy^ib^iB^2\beta^3s_0|0 + 72B^2b^iB^2r_{00}\beta^3s_0
\end{aligned}$$

$$\begin{aligned}
& -64b^i b_j B^3 s_0^k \beta^5 s_k - 32r_j^i \beta^5 s_0 - 256\beta^3 y^i y_j B^3 s_0^k s_k - 384\beta^3 y^i y_j B^2 s_0^k r_{k0} + 128\beta^2 y^i y_j B^3 s_0^k r_{k0} - 192\beta^3 y^i y_j B s_0^k s_k \\
& + 96\beta^2 y^i y_j B s_0^k r_{k0} - 256\beta^2 y^i y_j B r_{00} s_0 - 90b^i b_j B r_{00} r_{00} \beta^3 + 64\beta y^i y_j B^2 r_{00} r_{00} + 256\beta y^i y_j B^2 r_{00} s_0 + 64\beta y^i y_j B r_{00} r_{00} \\
& + 16s^i \beta^5 r_{j0} + 16b^i \beta^5 s_{j0} + 16r_0^i \beta^5 s_j + 16B^4 s_{j0}^i \beta^5 - 32B^4 s_{j0}^i \beta^5 + 128B^3 s_{j0}^i \beta^5 - 9\beta^2 \delta^i_j r_{00}^2 + 45B^2 s_{j0}^i \beta^5 \\
& + 2s^i k s_j^k \beta^6 - 32b^i \beta^5 s_{0j} + 128B s_{0j}^i \beta^5 + 10s^i_0 \beta^3 y_j - 14s^i_0 \beta^4 b_j - B^3 s_{j0}^i \beta^5 - 2B^2 s_{j0}^i \beta^5 - 64B s_{j0}^i \beta^5 + 8B^3 y^i \beta^3 r_{00} j \\
& + 48B^2 y^i \beta^3 r_{00j} + 42B y^i \beta^3 r_{00j} - 72\beta^3 y^i b_j s_0^2 + 60\beta^3 y^i b_j s_{0j} - 13\beta^2 y^i b_j r_{00j} + 108b^i y_j \beta^3 s_0^2 + 44b^i y_j s_{0j} \beta^3 - 9b^i y_j r_{00j} \beta^2 \\
& + 12b^i y_j \beta r_{00}^2 + 754b^i b_j \beta^4 s_0^2 + 40b^i b_j \beta^4 s_{0j} - 9b^i b_j r_{00j} \beta^3 + 45b^i b_j r_{00}^2 \beta^2 + 72\beta^4 \delta^i_j B^2 s_{0j} - 384\beta^4 \delta^i_j B s_0^2 - 4\beta^3 \delta^i_j B^3 r_{00j} \\
& + 36\beta^4 \delta^i_j r_{k0} s_0^k - 96\beta^4 \delta^i_j s_0 r_{0j} - 24\beta^3 \delta^i_j B^2 r_{00j} - 12\beta^2 \delta^i_j B^2 r_{00}^2 - 21\beta^3 \delta^i_j B r_{00j} + 28\beta^3 \delta^i_j r_{00j} r_{0j} + 18\beta^4 y^i s_{0j} \\
& + 88\beta^3 \delta^i_j s_{0j} r_{00j} - 24\beta^2 \delta^i_j B^2 r_{00}^2 + 90s^i_0 \beta^3 s_0 y_j - 27s^i_0 r_{00j} \beta^2 y_j - 54s^i_0 \beta^4 s_0 b_j - 9s^i_0 r_{00j} \beta^3 b_j + 240B^4 s^i_0 \beta^4 s_{0j} - 384B^3 s^i_0 \beta^4 s_{0j} \\
& + 96B y^i \beta^4 s_{0j} r_j - 8B^2 y^i r_{00j} \beta^3 r_j - 32B y^i r_{00j} \beta^3 s_j + 40B^2 y^i r_{00j} \beta^3 r_{j0} - 16B^3 r_0^i \beta^4 s_0 b_j - 144B^2 r_0^i \beta^4 s_0 b_j - 42B r_0^i \beta^4 s_0 b_j \\
& + 136B y^i r_0 \beta^3 r_{j0} - 12B^2 y^i r_{00j} \beta^2 r_{j0} - 24B y^i r_{00j} \beta^2 r_{j0} - 54B^2 r_0^i \beta^4 r_{j0} - 54B r_0^i \beta^4 r_{j0} + 44r_0^i \beta^3 s_0 y_j - 9r_0^i r_{00j} \beta^2 y_j \\
& + 76b^i \beta^5 s_0 r_j - 18b^i r_{00j} \beta^4 r_j - 54B b^i r_{00j} \beta^4 s_j - 48B y^i r_0 \beta^4 s_j + 192\beta^2 y^i y_j B^2 s_0^k r_{k0} \Big] + 72\hat{R}^i_j \beta^6 (2B + 1) (2B^2 + 2B - 1) \\
t_{16} & := \beta^4 y_j (2B + 1) (64B^2 b^i s_0^2 + 24s^i_0 \beta^3 - 272B b^i s_0^2 + 36B^2 s^i_0 \beta^3 - 224b^i s_0^2 - 90B s^i_0 \beta^3 - 51s^i_0 \beta^3) \\
t_{17} & := 2\beta \Big[-96B^2 s^i_k s^k_0 \beta^3 y_j - 32B s^i_k s^k_0 \beta^3 y_j + 72B^2 b^i r_{00j} \beta^2 s_{0j} - 36B b^i r_{00j} \beta^2 s_{0j} - 128\beta^2 y^i b_j B^3 s_{0j} + 896\beta^2 y^i b_j B^2 s_0^2 \\
& - 20b^i y_j s_0^k r_{k0} \beta^2 + 128b^i y_j r_{00j} \beta^2 + 12b^i y_j B^2 r_{00j} \beta^2 - 36b^i b_j B r_{00j} \beta^2 + 32b^i b_j B^3 \beta^3 s_{0j} - 1184b^i b_j B^2 \beta^3 s_0^2 + 204b^i b_j s_0^k \beta^4 s_k \\
& - 1328b^i b_j B \beta^3 s_0^2 - 180b^i b_j B \beta^3 s_{0j} - 124b^i b_j s_0^k r_{k0} \beta^3 - 96B^2 r_0^i \beta^2 s_0 y_j - 24B b^i r_0 \beta^3 r_{j0} - 24B b^i \beta^3 s_0 r_{j0} + 6B b^i r_{00j} \beta^2 r_{j0} \\
& - 24B^2 b^i \beta^3 s_0 r_{j0} + 432B^2 s^i_0 \beta^3 s_0 b_j - 432B s^i_0 \beta^3 s_0 b_j + 162B s^i_0 r_{00j} \beta^2 b_j + 576B^2 y^i \beta^2 s_0 s_{0j} - 72B^3 y^i r_{00j} \beta^2 s_0 \\
& - 108B^2 y^i r_{00j} \beta^2 s_{0j} - 54B y^i r_{00j} \beta^2 s_{0j} + 32B^2 b^i r_0 \beta^4 s_j - 32B^2 b^i \beta^4 s_0 s_j + 80B b^i r_0 \beta^4 s_j + 48B^2 y^i s_j^k \beta^3 r_{k0} + 24B y^i s_j^k \beta^3 r_{k0} \\
& - 384B^3 s_j^k s_0^k \beta^4 b_j - 432B^2 s_j^k s_0^k \beta^4 b_j + 32B^3 b^i s_j^k \beta^4 r_{k0} + 120B^2 b^i s_j^k \beta^4 r_{k0} - 12B b^i s_j^k \beta^4 r_{k0} - 288B^2 b^i s_j^k \beta^5 s_k - 72B b^i s_j^k \beta^5 s_k \\
& + 48B s^i_0 \beta^3 s_0 y_j - 32b^i y_j B^3 \beta^2 s_0^2 + 56b^i y_j s_0^k \beta^3 s_k - 96b^i y_j B^2 \beta^2 s_{0j} - 72b^i y_j B^2 s_{0j} + 192B^2 s^i_0 r_{00j} \beta^3 b_j + 36B s^i_0 r_{00j} \beta^3 b_j \\
& - 96B^2 y^i s_0^k \beta^3 r_{jk} - 48B y^i s_0^k \beta^3 r_{jk} - 288B^3 b^i \beta^3 s_0 s_{0j} - 216B^2 b^i \beta^3 s_0 s_{0j} + 32B^2 y^i r_0 \beta^3 s_j + 64B^2 y^i \beta^3 s_0 s_j + 32B y^i r_0 \beta^3 s_j \\
& + 64B y^i \beta^3 s_0 s_j - 8B^2 y^i r_{00j} \beta^2 s_j + 36B b_j y^i B^2 r_{00j} + 18B b_j y^i B r_{00j} - 24B b_j y^i r_{00j} r_{0j} - 78B b_j y^i s_0 r_{00j} + 56\beta^3 y^i b_j s_k s_k^k \\
& - 192B^2 y_j B^2 s_{0j} + 352B^2 b^i y_j B^2 \beta^2 s_0^2 + 688B^2 y_j B B^2 s_0^2 - 128B b^i \beta^4 s_0 s_j + 24B^2 b^i r_{00j} \beta^3 s_j + 54B s^i_0 r_{00j} \beta^3 y_j + 12B^2 y_j B r_{00j} \beta^3 \\
& - 30b^i y_j r_{00j} \beta^3 - 96y_j b^i \beta s_0 r_{00j} - 64B^3 y^i s_j^k \beta^4 s_k - 96B^2 y^i s_j^k \beta^4 s_k - 48B y^i s_j^k \beta^4 s_k - 64B^3 b^i s_0^k \beta^4 r_{jk} - 240B^2 b^i s_0^k \beta^4 r_{jk} \\
& + 128\beta^3 \delta^i_j B^3 s_0^k s_k - 72B r_0^i \beta^2 s_0 y_j + 12B^2 r_0^i r_{00j} \beta^2 y_j - 32B^2 r_0^i \beta^2 s_0 y_j + 268B^2 b_j r_0 \beta^3 s_0 + 36b^i b_j B r_{00j} \beta^2 \\
& - 90b^i b_j r_{00j} \beta^2 + 162b^i b_j r_{00j} \beta^2 s_0 + \beta^4 \delta^i_j B^2 s_0^k s_k - \beta^3 \delta^i_j B^3 s_0^k r_{k0} + 96\beta^4 \delta^i_j B s_0^k s_k - 96\beta^3 \delta^i_j B^2 s_0^k r_{k0} + \beta^3 \delta^i_j B^2 r_0 s_0 \\
& - 128\beta^2 \delta^i_j B r_{00j} s_0 - 64B^3 y^i s_0^k \beta^3 r_{jk} + 192B^4 s^i_k s^k_0 \beta^4 b_j + 288B y^i \beta^2 s_0 s_{0j} - 24B^2 b^i r_0 \beta^3 r_{j0} - 120b^i b_j B^2 \beta^3 s_0 s_0 \\
& + 128\beta^3 \delta^i_j B r_0 s_0 - 32\beta^2 \delta^i_j B^2 r_0 r_{00j} - 20\beta^2 y^i b_j r_{00j} s_0^k + 104\beta^2 y^i b_j s_0 r_{0j} - 64B^2 b^i \beta^4 s_0 r_{0j} - 160B b^i \beta^4 s_0 r_{0j} + 24B^2 b^i r_{00j} \beta^3 r_j \\
& - 64B^2 y^i \beta^3 s_0 r_j - 64B y^i \beta^3 s_0 r_j + 16B^2 y^i r_{00j} \beta^2 r_j + 16B y^i r_{00j} \beta^2 r_j - 120B^2 r_0^i \beta^3 s_0 b_j - 180B r_0^i \beta^3 s_0 b_j + 36B r_0^i r_{00j} \beta^2 b_j \\
& - 80B y^i r_0 \beta^2 r_{j0} + 12B^2 y^i r_{00j} \beta^2 r_{j0} + 12B y^i r_{00j} \beta^2 r_{j0} + 32B^3 r_0^i \beta^3 s_0 b_j - 288B^3 s^i_0 \beta^2 s_0 y_j - 432B^2 s^i_0 \beta^2 s_0 y_j + 72B^3 s^i_0 r_{00j} \beta^3 y_j \\
& - 216B s^i_0 \beta^2 s_0 y_j + 192B^3 s^i_0 \beta^4 s_0 b_j - 480B^2 s^i_0 \beta^4 s_0 b_j - 32B^3 s^i_0 r_{00j} \beta^3 b_j - 120B s^i_0 \beta^4 s_0 b_j - 64B^4 s^i_0 k s^k_0 \beta^3 y_j - 128B^3 s^i_0 k s^k_0 \beta^3 y_j \\
& + 24B y^i b_j B^3 r_{00j} s_0 - 96\beta^2 y^i b_j s_0 r_{00j} + 24B b^i r_{00j} \beta^3 s_j + 108B b^i \beta^3 s_0 s_{0j} - 216B^3 s^i_0 r_{00j} \beta^2 b_j - 64B^3 s^i_0 \beta^3 s_0 y_j + 32B^3 s^i_0 r_{00j} \beta^2 y_j \\
& - 128\beta^2 \delta^i_j B^2 r_{00j} s_0 - 32\beta^2 \delta^i_j B r_{00j} + 32B^3 y^i s_j^k \beta^3 r_{k0} + 24B b^i s_j^k \beta^4 r_{jk} + 384B^3 y^i \beta^2 s_0 s_{0j} - 48B s^i_0 k s^k_0 \beta^3 b_j + 24B^2 s^i_0 r_{00j} \beta^2 y_j \\
& + \beta^2 \delta^i_j r_{00j} + 16B^3 y^i \beta^2 r_{00j} + 24B^2 y^i \beta^2 r_{00j} + 12B y^i \beta^2 r_{00j} + 12b^i r_{00j} \beta^2 r_{00j} - 260b^i b_j \beta^3 s_0^2 - 56b^i b_j \beta^3 s_0 s_0 + 18b^i b_j r_{00j} \beta^2 \beta^2 \\
& + 16B^4 \delta^i_j s_0^k s_k - 48B^3 \delta^i_j B^2 s_0^2 + 320B^3 \delta^i_j B s_0^2 + 8\beta^2 \delta^i_j B^3 r_{00j} - 24B^3 \delta^i_j B s_0^2 - 8\beta^3 \delta^i_j r_{00j} s_0^k + 108B^2 s^i_0 r_{00j} \beta y_j \\
& + 32B^3 \delta^i_j s_0 r_{0j} + 12\beta^2 \delta^i_j B^2 r_{00j} + 12\beta^2 \delta^i_j B^2 r_{00j} - 8\beta^2 \delta^i_j r_{00j} r_{0j} - 32\beta^2 \delta^i_j s_0 r_{00j} + 12\beta^2 \delta^i_j B^2 r_{00j} - 32B^3 \delta^i_j B^3 s_0 s_0 \\
& - 16B^3 y^i \beta^2 r_{00j} + 24B^2 y^i \beta^2 r_{00j} - 12B y^i \beta^2 r_{00j} + 12y_j b^i B^2 r_{00j} - 120B^2 r^i_0 \beta^4 s_j + 12B^2 r^i_0 \beta^4 s_j + 288B^4 s^i_0 \beta^3 s_0 s_0 \\
& - 24B^3 r^i_j r_{00j} \beta^3 - 24B r^i_j \beta^4 s_0 - 36B^2 r^i_j r_{00j} \beta^3 + 36B r^i_j r_{00j} \beta^3 - 24B^3 b^i \beta^3 r_{00j} - 36B^2 b^i \beta^3 r_{00j} + 36B b^i \beta^3 r_{00j} \\
& + 64B^3 r^i_j \beta^4 s_0 + 24B^2 r^i_j \beta^4 s_0 + 24B^3 r^i_0 \beta^3 r_{0j} + 36B^2 r^i_0 \beta^3 r_{0j} - 36B r^i_0 \beta^3 r_{0j} + 8b^i \beta^4 s_0 r_{0j} - 16y^i \beta^3 s_0 r_{0j} + 4y^i r_{00j} \beta^2 r_j \\
& - 20y^i r_0 \beta^2 r_{0j} + 3y^i r_{00j} \beta^2 r_{0j} - 216B^2 s^i_0 \beta^3 s_0 s_0 + 144B s^i_0 \beta^3 s_0 s_0 + 12B^2 b^i \beta^4 s_{j0} - 120B^2 b^i \beta^4 s_{j0} + 8y^i r_0 \beta^3 s_j + 16y^i \beta^3 s_0 s_0 \\
& - 12b^i r_{00j} \beta^3 r_j + 48y^i \beta^2 s_0 s_0 - 9y^i r_{00j} \beta s_0 s_0 - 2y^i r_{00j} \beta^2 s_j - 4b^i r_0 \beta^4 s_j + 52b^i \beta^4 s_0 s_j - 30b^i r_{00j} \beta^3 s_j + 72b^i \beta^3 s_0 s_0 - 36b^i r_{00j} \beta^2 s_0 s_0 \\
& - 32B^3 b^i \beta^4 s_{j0} + 24B^3 b^i \beta^3 r_{j0} + 36B^2 b^i \beta^3 r_{j0} - 36B b^i \beta^3 r_{j0} - 56r^i_0 \beta^3 s_0 b_j + 18r^i_0 r_{00j} \beta^2 b_j - 16r^i_0 \beta^2 s_0 y_j + 3r^i_0 r_{00j} \beta y_j
\end{aligned}$$

$$\begin{aligned}
& -32 b^i s_j^k \beta^4 r_{k0} + 64 b^i s_0^k \beta^4 r_{jk} - 32 B^4 s_0^i \beta^3 b_j + 224 B^3 s_0^i \beta^3 b_j + 168 B^2 s_0^i \beta^3 b_j - 16 B s_0^i \beta^3 b_j + 4 \beta^3 y^i r_{k0} s_j^k \\
& -32 B^3 s^i \beta^4 r_{j0} - 120 B^2 s^i \beta^4 r_{j0} + 12 B s^i \beta^4 r_{j0} - 24 B b^i \beta^4 s_{0|j} - 180 s^i_0 \beta^3 s_0 b_j + 54 s^i_0 r_{00} \beta^2 b_j + 9 s^i_0 r_{00} \beta y_j - 8 \beta^4 y^i s_k s_j^k \\
& + 384 B^3 s_k^i s_j^k \beta^5 - 32 B^3 y^i \beta^3 s_{j|0} + 48 B^4 s_0^i \beta^3 b_j - 2 s^i r_{00} \beta^2 b_j + 64 B^3 y^i \beta^3 s_{0|j} + 96 B^2 y^i \beta^3 s_{0|j} + 48 B y^i \beta^3 s_{0|j} - 36 s^i_0 \beta^2 s_0 y_j \\
& - 4 \beta^3 s^i_k s_0^k y_j + 24 s^i_k s^k_0 \beta^4 b_j + 16 s^i_k s^k_0 \beta^3 s_0 y_j - 32 B^3 r^i_0 \beta^4 s_j + 24 b_j y^i B^2 r_{00}^2 + 24 b_j y^i B r_{00}^2 + 64 B^3 b^i \beta^4 s_{0|j} + 240 B^2 b^i \beta^4 s_{0|j} \\
& - 24 B y^i \beta^3 s_{j|0} + 84 s^i \beta^4 s_0 b_j - 34 s^i r_{00} \beta^3 b_j + 256 b^i y_j \beta^2 s_0 - 16 b^i y_j s_{0|0} \beta^2 + 3 b^i y_j r_{00} \beta + 224 \beta^2 y^i b_j s_0^2 - 16 \beta^2 y^i b_j s_{0|0} \\
& + 3 \beta^2 y^i b_j r_{00|0} + 12 b^i r_0 \beta^3 r_{j0} - 24 b^i \beta^3 s_0 r_{j0} - 72 B^2 s_0^i \beta^3 b_j - 48 B s_0^i \beta^3 b_j + 32 B^4 s_{j|0} \beta^4 + 48 B^2 s_0^i \beta^4 - 9 s^i_0 \beta^2 b_j \\
& + 256 B s_0^i \beta^4 + 160 B^3 s_0^i \beta^4 - 24 B^2 s_{j|0} \beta^4 - 128 B s_0^i \beta^4 - 4 \beta^3 y^i s_{j|0} + 8 \beta^3 y^i s_{0|j} - 36 s^i \beta^5 s_j + 32 s^i \beta^4 r_{j0} - 64 b^i \beta^4 s_{0|j} \\
& + 32 r^i_0 \beta^4 s_j + 32 b^i \beta^4 s_{j|0} - 60 s^i_k s_0^k \beta^5 + 72 s^i_0 \beta^3 s_{0j} - 20 s^i_0 \beta^3 b_j - 64 B^4 s_0^i \beta^4 - 320 B^3 s_0^i \beta^4 + 80 \beta^3 \delta^i_j s_0^2 - 4 \beta^3 \delta^i_j s_{0|0} \\
& + 3 \beta^2 \delta^i_j r_{00}^2 - 2 \beta^2 y^i r_{00|j} + 2 \beta^2 y^i r_{j0|0} - 64 r^i_0 \beta^4 s_0 + 24 r^i_0 r_{00} \beta^3 + 24 b^i \beta^3 r_{00|j} - 24 b^i \beta^3 r_{j0|0} - 24 r^i_0 \beta^3 r_{j0} + 6 b^i y_j r_{00}^2 \\
& + 6 y^i b_j r_{00}^2 + 288 B^2 s^i \beta^5 s_j - 40 s^i_0 \beta^4 + 80 s^i_0 \beta^4 + 672 b^3 b_j y^i B^2 s_0^k r_{k0} - 160 \beta^2 y^i b_j B^3 s_0^k r_{k0} + 336 \beta^3 y^i b_j B s_0^k r_{k0} \\
& + 416 B^2 y^i b_j B^2 r_{00} s_0 - 120 \beta^2 y^i b_j B s_0^k r_{k0} + 416 B^2 y^i b_j B r_{00} s_0 - 96 \beta y^i b_j B^2 r_{00} s_0 - 12 \beta y^i b_j B^2 r_{00} s_0 + 72 B s^i \beta^5 s_j + 36 b^i s_j^k \beta^5 s_k \\
& - 312 \beta y^i b_j B r_{00} s_0 + 48 \beta^3 y^i b_j B^3 s_0^k s_k + 88 y^i b_j B^2 s_0^k \beta^3 s_k + 32 b^i y_j B^3 s_0^k r_{k0} \beta^2 + 24 b^i y_j B s_0^k \beta^3 s_k - 48 b^i y_j B^2 s_0^k r_{k0} \beta^2 \\
& + 272 b^i y_j B r_{00} \beta^2 s_0 - 72 b^i y_j B^2 r_{00} \beta s_0 - 60 b^i y_j B r_{00} \beta - 228 b^i y_j B r_{00} \beta s_0 + 64 b^i y_j B^3 s_0^k \beta^3 s_k - 32 b^i b_j B^3 s_0^k r_{k0} \beta^3 \\
& + 24 b^i b_j B^2 s_0^k r_{k0} \beta^3 - 32 b^i b_j B^2 r_{00} \beta^3 s_0 - 10 b^i b_j B s_0^k r_{k0} \beta^3 - 128 b^i b_j B r_{00} \beta^3 s_0 + 360 b^i b_j B r_{00} \beta^2 s_0 + 21 B^2 b^i r_{00} \beta^2 s_0 b_j \\
& + 144 B^2 s^i_k s_j^k \beta^5 - 192 b^i b_j B^3 s_0^k \beta^4 s_k - 96 b^i b_j B^2 s_0^k \beta^4 s_k + 32 b^i y_j B^2 r_{00} \beta^2 s_0 + 896 \beta^2 y^i b_j B s_0^2 - 80 B^2 y^i r_{00} \beta^2 r_{j0} + 24 B b^i r_{00} \beta^3 r_{j0} \\
& - 144 B s_0^i s_j^k \beta^5 - 240 \beta^2 y^i b_j B^2 s_0^k r_{k0} \Big] - 3 \hat{R}_j^i \beta^4 (2B+1)^2 (4B^2 + 4B - 17)
\end{aligned}$$

$$t_{18} := 12 \beta^2 y_j (2B+1)^2 (4B b^i s_0^2 + 2B s^i_0 \beta^2 + s^i_0 \beta^2)$$

$$\begin{aligned}
t_{19} &:= 2(2B+1) \left(144 B^2 s^i_0 \beta^2 s_0 b_j + 144 B s^i_0 \beta^2 s_0 b_j - 36 B^2 s_0^i r_{00} \beta b_j - 8 B^2 b^i s_j^k \beta^3 r_{k0} + 28 B b^i s_j^k \beta^3 r_{k0} \right. \\
&\quad + 12 B^2 s^i_0 \beta y_j - 72 B^2 b^i \beta^2 s_0 s_{0j} - B b^i \beta^2 s_0 s_{0j} - s^i_0 r_{00} \beta b_j - 6 b^i b_j B r_{00|0} \beta + 30 b^i b_j r_{00} r_0 \beta + 78 b^i b_j \beta s_0 r_{00} + 16 b^i y_j B \beta s_0^2 \\
&\quad + 32 B^2 b^i s_j^k \beta^4 s_k - 40 B b^i s_j^k \beta^4 s_k + 16 B b^i \beta^3 s_0 r_j + 8 B b^i \beta^3 s_0 s_j + s^i_0 \beta y_j + 64 B^2 s^i \beta^3 s_0 b_j - 56 B s^i \beta^3 s_0 b_j - 24 B^2 s^i r_{00} \beta^2 b_j \\
&\quad + 24 B s^i r_{00} \beta^2 b_j + 24 B^2 r^i_0 \beta^2 s_0 b_j + 12 B r^i_0 \beta^2 s_0 b_j - 6 B r^i_0 r_{00} \beta b_j - 8 B b^i r_0 \beta^3 s_j - 48 B s^i_k s_0^k \beta^3 b_j - 28 b^i b_j \beta^3 s_k s_0^k + 32 r^i_j \beta^3 s_0 \\
&\quad + 24 b^i b_j B^2 \beta^2 s_{0|0} - 168 b^i b_j B \beta^2 s_0^2 + 12 b^i b_j B \beta^2 s_{0|0} + 36 b^i b_j \beta^2 r_{k0} s_0^k - 108 b^i b_j \beta^2 s_0 r_0 + 18 B b^i r_{00} \beta s_{0|0} + 64 B^3 s^i_k s_0^k \beta^3 b_j \\
&\quad + 16 B^2 b^i s_0^k \beta^3 r_{jk} - 56 B b^i s_0^k \beta^3 r_{jk} + 18 B b^i \beta^2 r_{j0|0} + 56 B r^i_j \beta^3 s_0 - 18 B r^i_j r_{00} \beta^2 + 9 b^i r_{00} \beta s_{0j} + 8 B^2 s^i \beta^3 r_{j0} + 8 B^3 s^i_0 \beta y_j \\
&\quad + 6 B s^i_0 \beta y_j + 8 B^2 r^i_0 \beta^3 s_j - 12 b^i r_{00} \beta^2 r_{j0} - 3 b^i r_{00} \beta r_{j0} - 28 B r^i_0 \beta^3 s_j - 18 B b^i \beta^2 r_{00|j} + 8 y_j b^i \beta s_0^2 - 32 b^i s_0^k \beta^3 r_{jk} - 28 B s^i \beta^3 r_{j0} \\
&\quad + 16 b^i s_j^k \beta^3 r_{k0} + 12 B^2 s^i_0 \beta^2 b_j + 30 B s^i_0 \beta^2 b_j + 36 s^i_0 \beta^2 s_0 b_j - 9 s^i_0 r_{00} \beta b_j + 20 b^i r_0 \beta^3 s_j - 44 b^i \beta^3 s_0 s_j - 9 r^i_j r_{00} \beta^2 \\
&\quad + 18 b^i r_{00} \beta^2 s_j + 8 B^2 b^i \beta^3 s_{j|0} - 40 b^i \beta^3 s_0 r_j + 12 b^i r_{00} \beta^2 r_j - 36 B^2 s^i_0 \beta^2 s_{0j} - 90 B s^i_0 \beta^2 s_{0j} - 44 s^i \beta^3 s_0 b_j + 18 s^i r_{00} \beta^2 b_j \\
&\quad + 48 B^2 s^i_k s_j^k \beta^4 + 72 B s^i_k s_j^k \beta^4 - 16 s^i_k s_0^k \beta^3 b_j - 28 b^i s_j^k \beta^4 s_k - 3 r^i_0 r_{00} \beta b_j - 22 b^i b_j \beta^2 s_0^2 - 3 b^i b_j r_{00|0} \beta - 28 B b^i \beta^3 s_{j|0} \\
&\quad - 16 B^2 b^i \beta^3 s_{0|j} + b^i \beta^3 s_{0|j} - 32 B^2 s^i \beta^4 s_j + 4 B s^i \beta^4 s_j - r^i_0 \beta^3 s_0 - 24 B^3 s^i_0 \beta^2 s_{0j} + 72 B^3 s^i_0 s_{0j} \beta^2 + 9 r^i_0 \beta^2 r_{j0} - 22 s^i_0 \beta^3 b_j \\
&\quad + 9 b^i \beta^2 r_{j0|0} + 18 B r^i_0 \beta^2 r_{j0} + 32 b^i \beta^3 s_{0|j} - 3 b_j b^i r_{00}^2 + 20 s^i_k s_j^k \beta^4 - 9 b^i \beta^2 r_{00|j} - 24 b^i b_j B^2 s_0^k r_{k0} \beta^2 - 88 b^i b_j B s_0^k \beta^3 s_k \\
&\quad + 42 B s^i_j \beta^3 - 16 s^i \beta^3 r_{j0} - 16 b^i \beta^3 s_{j|0} + 9 s^i_0 \beta^2 b_j + 16 B^3 s^i_0 \beta^3 - 72 B^2 s^i_0 \beta^3 - 84 B s^i_0 \beta^3 + 28 s^i \beta^4 s_j - 64 b^i b_j B^2 s_0^k \beta^3 s_k \\
&\quad + 11 s^i_j \beta^3 + 60 b^i b_j B s_0^k \beta^2 - 24 b^i b_j B r_{00} \beta^2 s_0 + 36 B b^i b_j r_{00} \beta s_0 \Big) - 12 \hat{R}_j^i \beta^2 (2B+1)^3
\end{aligned}$$

$$t_{20} := -y_j (2B+1)^3 \left(-8 b^i s_0^2 + (2B+1) s^i_0 \beta \right)$$

$$\begin{aligned}
t_{21} &:= 2(2B+1)^2 \left(-4 B^2 s^i_0 \beta b_j - 4 B s^i_0 \beta b_j - 2 B^2 s^i_k s_j^k + 2 B r^i_0 s_j - 4 B b^i s_0 | j - 4 B^2 s^i s_j + 2 B s^i r_{j0} + 2 B^i b_j s_0 | 0 + 2 B r^i_j r_{00} \right. \\
&\quad - 2 B r^i_0 r_{j0} + 2 B b^i r_{00|j} - 2 b^i r_{00} r_j + 2 r^i_0 s_0 b_j + 2 b^i r_0 r_{j0} + 2 b^i s_0 r_{j0} - 4 B^2 s^i_j | 0 \beta - 4 B s^i_0 \beta + 8 B^2 s^i_0 \beta s_j - 2 s^i_0 r_{00} b_j \\
&\quad - 4 b^i b_j B s_0^k r_{k0} + 8 B b^i s_0^k \beta r_{jk} + 8 B b^i s_j^k \beta^2 s_k + 8 B s^i \beta s_0 b_j + 8 B^2 s^i_k s_0^k \beta b_j + 8 B s^i_k s_0^k \beta s_k - 4 B b^i b_j s_0^k \beta s_k - 4 B b^i s_j^k \beta r_{k0} \\
&\quad + 12 B^2 s^i_0 s_{0j} + 12 B s^i_0 s_{0j} + 3 s^i_0 s_{0j} - \beta s^i_j | 0 + 2 B s^i_0 | j - b^i r_{j0} | 0 - r^i_0 r_{j0} + b^i r_{00} | j + r^i_0 r_{00} - s^i_0 | 0 b_j - 8 B b^i \beta s_0 | j - 8 B s^i \beta^2 s_j \\
&\quad + 4 B s^i \beta r_{j0} - 4 B s^i r_{00} b_j + 2 B s^i_k s_0 b_j + 4 B r^i_0 \beta s_j + 4 B b^i \beta s_j + 4 B^2 b^i s_k s_j^k - 2 B b^i r_{k0} s_j^k + 4 B b^i r_{jk} s_0^k - 12 B b^i s_0 s_{0j} \\
&\quad - 4 b^i r_0 \beta s_j + 4 b^i \beta s_0 s_j + 8 b^i \beta s_0 r_j + 4 B r^i_0 s_0 b_j + 4 B b^i b_j s_0 | 0 B - 2 b^i b_j s_0^k r_{k0} - 4 b^i b_j s_0 r_0 + 4 s^i \beta s_0 b_j - 6 b^i s_0 s_{0j} - 4 b^i b_j s_0^2 \\
&\quad + 8 B s^i_0 \beta + 2 B b^i s_j | 0 - 8 B^2 s^i_k s_j^k \beta^2 - 8 B s^i_k s_j^k \beta^2 - 8 b^i b_j B s_0^k \beta s_k \Big) + \hat{R}_j^i (2B+1)^4
\end{aligned}$$

6. Appendix 2

$$\begin{aligned}
t'_2 &:= +108(2B+1)s^i_{0|0}\beta^{18}y_j \\
t'_4 &:= -27\beta^{16}y_j(8B^2s^i_{0|0} + 24b^is^2_0 + 8Bs^i_{0|0} - 7s^i_{0|0}), \\
t'_6 &:= 24\beta^{14}y_j(4s^i_{0|0}B^3 + 42Bb^is^2_0 + 6B^2s^i_{0|0} - 24b^is^2_0 - 24Bs^i_{0|0} - 13s^i_{0|0}), \\
t'_8 &:= -2\beta^{12}y_j(8B^4s^i_{0|0} + 240B^2b^is^2_0 + 16B^3s^i_{0|0} - 696Bb^is^2_0 - 312B^2s^i_{0|0} \\
&\quad - 624b^is^2_0 - 320Bs^i_{0|0} + 41s^i_{0|0}) \\
t'_{10} &:= -16y_j\beta^{10}(-4B^3b^is^2_0 + 60B^2b^is^2_0 + 18s^i_{0|0}B^3 + 81Bb^is^2_0 + 27B^2s^i_{0|0} - 56b^is^2_0 \\
&\quad - 27Bs^i_{0|0} - 18s^i_{0|0}) \\
t'_{12} &:= 6y_j\beta^8(32B^3b^is^2_0 + 8B^4s^i_{0|0} + 16B^2b^is^2_0 + 16s^i_{0|0}B^3 - 344Bb^is^2_0 - 96B^2s^i_{0|0} \\
&\quad - 136b^is^2_0 - 104Bs^i_{0|0} - 13s^i_{0|0}) \\
t'_{14} &:= 8\beta^6y_j(16B^3b^is^2_0 + 144B^2b^is^2_0 + 36s^i_{0|0}B^3 + 42Bb^is^2_0 + 54B^2s^i_{0|0} - 40b^is^2_0 \\
&\quad - 9s^i_{0|0}) \\
t'_{16} &:= -\beta^4y_j(2B+1)(64B^2b^is^2_0 + 24s^i_{0|0}B^3 - 272Bb^is^2_0 + 36B^2s^i_{0|0} - 224b^is^2_0 \\
&\quad - 90Bs^i_{0|0} - 51s^i_{0|0}) \\
t'_{18} &:= -12\beta^2y_j(2B+1)^2(4Bb^is^2_0 + 2Bs^i_{0|0} + s^i_{0|0})
\end{aligned}$$

7. Appendix 3

$$\begin{aligned}
t''_2 &:= -6\beta^{14}(-243s^i_{0|0}\beta^4s_{0j} + 8By^i_{r00}\beta^2r_{j0} - 240\beta^2y^iy_jBs^k_0r_{k0} - 24\beta^2y^iy_jBr_{0r00} - 30\beta^2y^ib_jr_{00|0} + 64\beta y^ib_jr^2_{00} - 3b^ir_{00}\beta^3r_{j0} \\
&\quad - 6r^i_{0r00}\beta^2y_j - 162s^i_{0r00}\beta^2y_j - 27b^ir_{000}\beta^3s_{0j} + 18y^ir_{00}\beta^2s_{0j} - 72\beta^4y^ir_{jk}s^k_0 + 36\beta^4y^ir_{k0}s^k_j + 108\beta^4s^i_k s^k_0y_j + 81s^i_{0r00}\beta^3b_j \\
&\quad - 9r^i_{0r00}\beta^3b_j - 12Br^i_{0r00}\beta^2y_j + 18\beta^3\delta^i_jBr_{00|0} - 24\beta^3\delta^i_jr_{00r0} - 16\beta^2\delta^i_jBr^2_{00} + 96\beta^2y^iy_jr_{k0}s^k_0 + 16\beta y^iy_jB^2r_{00|0} \\
&\quad - 8\beta y^iy_jBr_{00|0} + 48\beta y^iy_jr_{00r0} - 9r^i_jr_{00}\beta^4 + \beta^3\delta^i_jr_{00|0} - \beta^2\delta^i_jr^2_{00} - 36\beta^3y^ir_{00|j} + \beta^3y^ir_{j0|0} - 72\beta^4\delta^i_jr_{k0}s^k_0 + 12y^ir_{00}\beta^3r_j \\
&\quad + 10y^ir_{00}\beta^2r_{j0} - 36By^i\beta^3r_{00|j} + 36By^i\beta^3r_{j0|0} + 36\beta^3y^ib_jr_{k0}s^k_0 - 6B\beta^2y^ib_jr_{00|0} + 36\beta^2y^ib_jr_{00r0} - 22\beta y^ib_jBr^2_{00} \\
&\quad + 90By^ir_{00}\beta^2s_{0j} + s^i_{0r00}\beta^2y_j + y^iy_jr^2_{00} + 10b^iy_jBr^2_{00\beta} - 9b^ib_jr_{00|0}\beta^3 + b^ib_jr^2_{00\beta^2} + 36b^iy_js^k_0r_{k0}\beta^3 - 12b^iy_jBr_{00|0}\beta^2 \\
&\quad + 30b^iy_jr_{00r0}\beta^2 - 6b^iy_jr_{00|0}\beta^2 + 8b^iy_j\beta^2r^2_{00} - 12y^iy_jB^2r^2_{00} + 16y^iy_jBr^2_{00} - 44\beta y^iy_jr_{00|0} - 9b^i\beta^4r_{00|j} + 9b^i\beta^4r_{j0|0} \\
&\quad + 9r^i_{0r0}\beta^4r_{j0} - 36y^ir_{00}\beta^3r_{j0}) + 108\hat{R}^i_j\beta^{18}(2B+1) \\
t''_4 &:= 2\beta^{12}(18b^ir_{00}\beta^4r_j - 12b^iy_jB^2r_{00|0}\beta^2 - 36b^ib_jBr_{00|0}\beta^3 + 612\beta^3y^ib_jr_{k0}s^k_0 + 12\beta^2y^ib_jB^2r_{00|0} - 36\beta y^ib_jB^2r^2_{00} \\
&\quad - 114\beta^2y^ib_jBr_{00|0} + 264\beta^2y^ib_jr_{00r0} + 16\beta y^iy_jB^3r_{00|0} + 1128\beta^2y^iy_jr_{k0}s^k_0 - 300\beta y^iy_jBr_{00|0} + 260\beta y^iy_jr_{00r0} \\
&\quad + 72b^iy_js^k_0r_{k0}\beta^3 + 60b^iy_jBr_{00r0}\beta^2 + 36y^iy_jB^2r^2_{00} + 654y^iy_jBr^2_{00} - 148\beta y^iy_jr_{00|0} - 64\beta y^iy_jB^2r_{0r00} - 18r^i_{0r00}\beta^3b_j \\
&\quad - 204y^ir_0\beta^3r_{j0} + 18y^ir_{00}\beta^2r_{j0} - 672\beta^2y^iy_jB^2s^k_0r_{k0} - 6Bb^ir_{00}\beta^3r_{j0} - 72\beta^3y^ib_jBs^k_0r_{k0} + 24\beta^2y^ib_jBr_{0r00} + 588y^iy_jr^2_{00} \\
&\quad + 324B^2s^i_{0r00}\beta^2y_j - 1134Bs^i_{0r00}\beta^2y_j - 108Bb^ir_{00}\beta^3s_{0j} + 216By^is^k_j\beta^4r_{k0} + 60y^ir_{00}\beta^3r_j + 252B^2y^ir_{00}\beta^2s_{0j} \\
&\quad - 18By^ir_{00}\beta^2s_{0j} - 432By^is^k_0\beta^4r_{jk} + 864Bs^i_k s^k_0\beta^4y_j + 486Bs^i_{0r00}\beta^3b_j - 12b^iy_jBr_{00|0}\beta^2 + 30b^iy_jr_{00r0}\beta^2 - 12b^iy_jBr^2_{00\beta} \\
&\quad + 108\beta^4y^ir_{k0}s^k_j - 114\beta^2y^ib_jr_{00|0} + 462\beta y^ib_jr^2_{00} + 162s^i_k s^k_0\beta^5b_j - 486s^i_{0r00}\beta^3b_j + 90\beta^3s^i_jBr_{00|0} + 108b^ir_{00}\beta^3s_{0j} \\
&\quad - 216\beta^4\delta^i_jr_{k0}s^k_0 + 36\beta^3\delta^i_jB^2r_{00|0} - 12\beta^2\delta^i_jB^2r^2_{00} - 120\beta^3\delta^i_jr_{00r0} - 108\beta^2\delta^i_jBr^2_{00} + 78b^iy_jr_{00|0}\beta^2 - 222b^iy_j\beta r^2_{00} \\
&\quad + 162s^i_k s^k_0\beta^6 + 1215s^i_{0|j}\beta^4s_{0j} - 432Bs^i_{0|j}\beta^5 + 90b^ib_jr_{0r00}\beta^3 + 36b^ib_jBr^2_{00\beta^2} - 40\beta^4s^i_k s^k_0y_j - 12Br^i_{0r00}\beta^2y_j
\end{aligned}$$

$$\begin{aligned}
& + 624 \beta^2 y^i y_j B s_0^k r_{k0} + 36 \beta^3 y^i r_{00|j} - 36 \beta^3 y^i r_{j0|0} + 27 b^i \beta^4 r_{j0|0} + 27 r^i \beta^4 r_{j0} - 18 \beta^3 \delta^i_j r_{00|0} - 60 \beta^2 \delta^i_j r_{00}^2 - 27 r^i_j r_{00} \beta^4 \\
& - 432 \beta^4 \delta^i_j B s_0^k r_{k0} + 54 b^i s^k_j \beta^5 r_{k0} - 12 B^2 r^i_0 r_{00} \beta^2 y_j - 96 \beta^3 \delta^i_j B r_{00} r_{00} - 18 b^i b_j r_{00|0} \beta^3 + 36 b^i b_j r_{00}^2 \beta^2 - 18 b^i r_{00} \beta^4 r_{j0} \\
& - 12 b^i r_{00} \beta^3 r_{j0} - 72 B^2 y^i \beta^3 r_{00|j} - 180 B y^i \beta^3 r_{00|j} + 72 B^2 y^i \beta^3 r_{j0|0} + 180 B y^i \beta^3 r_{j0|0} - 36 B r^i_0 r_{00} \beta^3 b_j - 192 B y^i r_{00} \beta^3 r_{j0} \\
& + 12 B^2 y^i r_{00} \beta^2 r_{j0} + 60 B y^i r_{00} \beta^2 r_{j0} - 194 B s^i_0 \beta^4 s_{0j} - 108 b^i s^k_0 \beta^5 r_{jk} - 234 y^i r_{00} \beta^2 s_{0j} - 216 \beta^4 y^i r_{jks}^k - 162 s^i_0 r_{00} \beta^2 y_j \\
& - 27 \hat{R}^i_j \beta^{16} (8B^2 + 8B - 7) \\
t''_6 := & 4 \beta^{10} \left[- 54 B r^i_0 r_{00} \beta^2 y_j - 39 r^i_0 r_{00} \beta^3 b_j - 50 y^i r_{00} \beta^3 r_{j0} + 2 y^i r_{00} \beta^2 r_{j0} + 186 b^i y_j s^k_0 r_{k0} \beta^3 - 90 b^i r_{00} \beta^3 s_{0j} + 22 y^i r_{00} \beta^3 r_j \right. \\
& + 45 b^i y_j B r^2_{00} \beta + 117 \beta^2 y^i b_j B r_{00|0} - 86 \beta^2 y^i b_j r_{00r0} - 351 \beta y^i b_j B r^2_{00} - 4 \beta^2 y^i b_j B^3 r_{00|0} - 294 \beta^3 y^i b_j r_{k0} s^k_0 + 24 \beta^2 y^i b_j B^2 r_{00|0} \\
& - 18 \beta y^i b_j B^2 r^2_{00} + 12 B b^i r_{00} \beta^4 r_{j0} + 864 B s^i_k s^k_0 \beta^4 y_j - 66 b^i b_j B \beta^3 r_{00|0} + 6 b^i b_j B^2 r_{00|0} \beta^3 + 156 \beta y^i y_j B r_{00|0} - 96 \beta y^i y_j r_{00r0} \\
& - 162 B^2 s^i_0 r_{00} \beta^3 b_j + 96 \beta^2 y^i y_j B^3 s^k_0 r_{k0} - 288 \beta^2 y^i y_j B^2 s^k_0 r_{k0} - 1296 \beta^2 y^i y_j B s^k_0 r_{k0} - 264 \beta y^i y_j B r_{00} r_{00} - 30 b^i b_j B r_{00} r_{00} \beta^3 \\
& - 72 B^2 y^i s^k_j \beta^4 r_{k0} - 72 B y^i s^k_j \beta^4 r_{k0} - 54 B b^i s^k_j \beta^5 r_{k0} - 432 B^2 s^i_k s^k_0 \beta^4 y_j + 108 s^i_{j|0} \beta^5 - 216 s^i_{j|0} \beta^5 - 216 B s^i_k s^k_j \beta^6 \\
& - 369 s^i_0 r_{00} \beta^2 y_j - 36 s^i_0 r_{00} \beta^3 b_j + 972 B^2 s^i_0 \beta^4 s_{0j} - 1944 B s^i_0 \beta^4 s_{0j} + 54 \beta^4 s^i_k s^k_0 y_j + 216 s^i_k s^k_0 \beta^5 b_j + 90 \beta^4 y^i r_{k0} s^k_j \\
& - 180 \beta^4 y^i r_{jks}^k + 81 y^i r_{00} \beta^2 s_{0j} - 27 B s^i_0 r_{00} \beta^2 y_j + 144 B y^i s^k_0 \beta^4 r_{jk} + 369 B y^i r_{00} \beta^2 s_{0j} + 216 B y^i s^k_j \beta^5 s_k + 72 B^2 y^i r_{00} \beta^2 s_{0j} \\
& - 90 B b^i r_{00} \beta^3 s_{0j} + 567 B s^i_0 r_{00} \beta^3 b_j - 36 B^2 s^i_0 r_{00} \beta^2 y_j - 432 B^2 s^i_0 r_{00} \beta^2 y_j + 108 B b^i s^k_0 \beta^5 r_{jk} - 6 b^i r_{00} \beta^4 r_j \\
& - 18 B^2 r^i_0 \beta^4 r_{j0} - 18 B r^i_0 \beta^4 r_{j0} - 27 r^i_0 r_{00} \beta^2 y_j - 162 y^i y_j r^2_{00} + 12 b^3 y^i b_j B^2 s^k_0 r_{k0} \\
& - 528 \beta^3 y^i b_j B s^k_0 r_{k0} + 16 \beta^2 y^i b_j B^2 r_{00} - 92 \beta^2 y^i b_j B r_{00} - 36 r^i_j r_{00} \beta^4 - 36 b^i \beta^4 r_{00|j} + 36 b^i \beta^4 r_{j0|0} + 49 \beta^3 \delta^i_j r_{00|0} \\
& - 27 \beta^2 \delta^i_j r^2_{00} - 98 \beta^3 y^i r_{00|j} + 98 \beta^3 y^i r_{j0|0} + 54 s^i_k s^k_j \beta^6 - 240 \beta^2 y^i y_j r_{k0} s^k_0 + 120 \beta y^i y_j B^2 r_{00|0} - 6 \beta y^i y_j r_{00|0} \\
& - 108 y^i y_j B^2 r^2_{00} - 360 y^i y_j B r^2_{00} - 216 B s^i_k s^k_0 \beta^5 b_j + 36 r^i_0 \beta^4 r_{j0} - 36 B^3 y^i r_{00} \beta^2 s_{0j} + 144 \beta^4 \delta^i_j B^2 s^k_0 r_{k0} + 144 \beta^4 \delta^i_j B s^k_0 r_{k0} \\
& - 18 b^i b_j s^k_0 r_{k0} \beta^4 + 16 \beta^3 \delta^i_j B^2 r_{00} + 64 \beta^3 \delta^i_j B r_{00} + 3 b^i b_j B r^2_{00} \beta^2 - 54 b^i y_j B r_{00|0} \beta^2 + 135 b^i y_j r_{00} r_0 \beta^2 + 24 b^i y_j B^2 s^k_0 r_{k0} \beta^3 \\
& - 48 b^i y_j B s^k_0 r_{k0} \beta^3 - 90 b^i b_j B s^k_0 r_{k0} \beta^4 - 15 b^i b_j r_{00} r_0 \beta^3 - 12 b^i r_{00} \beta^3 r_{j0} + 18 B^2 r^i_j r_{00} \beta^4 + 18 B r^i_j r_{00} \beta^4 - 12 B b^i r_{00} \beta^4 r_j \\
& - 18 B^2 b^i \beta^4 r_{00|0} - 18 B b^i \beta^4 r_{00|0} + 3 B b^i r_{00} \beta^3 r_{j0} + 40 B^2 y^i r_{00} \beta^3 r_{j0} + 136 B y^i r_{00} \beta^3 r_{j0} - 12 B^2 y^i r_{00} \beta^2 r_{j0} - 8 B^2 y^i r_{00} \beta^3 r_j \\
& - 32 B y^i r_{00} \beta^3 r_j + 6 B r^i_0 r_{00} \beta^3 b_j - 4 \beta^3 \delta^i_j B^3 r_{00|0} - 8 B^3 y^i \beta^3 r_{j0|0} - 39 b^i b_j r_{00|0} \beta^3 - 180 \beta^4 \delta^i_j r_{k0} s^k_0 - 24 \beta^3 \delta^i_j B^2 r_{00|0} \\
& + 12 \beta^2 \delta^i_j B^2 r^2_{00} + 33 \beta^3 \delta^i_j B r_{00|0} - 44 \beta^3 \delta^i_j r_{00r0} + 24 \beta^2 \delta^i_j B r^2_{00} + 25 \beta^2 y^i b_j r_{00|0} - 48 B^2 y^i \beta^3 r_{j0|0} - 288 \beta y^i b_j r^2_{00} \\
& + 123 b^i b_j r^2_{00} \beta^2 + 8 B^3 y^i \beta^3 r_{00|j} + 48 B^2 y^i \beta^3 r_{00|j} - 27 b^i y_j r_{00|0} \beta^2 + 36 b^i y_j B r^2_{00} + 6 b^i b_j B r_{00|0} \beta^3 \Big] \\
& + 24 \hat{R}^i_j \beta^{14} (2B^2 + 2B - 13) \\
t''_8 := & - 4 \beta^8 \left[- 36 B^2 b^i r_{00} \beta^3 s_{0j} - 90 B b^i r_{00} \beta^3 s_{0j} - 27 B s^i_0 r_{00} \beta^3 b_j + 108 B^3 s^i_0 r_{00} \beta^2 y_j - 162 B^2 s^i_0 r_{00} \beta^2 y_j - 729 B s^i_0 r_{00} \beta^2 y_j \right. \\
& + 72 B^2 b^i s^k_0 \beta^5 r_{jk} - 36 B b^i s^k_0 \beta^5 r_{jk} + 32 b^i y_j B^3 r_{00|0} + 324 \beta^2 y^i y_j r_{k0} s^k_0 + 96 \beta y^i y_j B^2 r_{00|0} - 36 B^3 s^i_0 r_{00} \beta^3 b_j \\
& - 216 B^2 s^i_k s^k_0 \beta^5 b_j + 648 B s^i_k s^k_0 \beta^5 b_j + 36 B^3 y^i r_{00} \beta^2 s_{0j} + 32 \beta^4 \delta^i_j B^3 s^k_0 r_{k0} + 48 \beta^4 \delta^i_j B^2 s^k_0 r_{k0} - 408 \beta^4 \delta^i_j B s^k_0 r_{k0} \\
& + 258 b^i b_j s^k_0 r_{k0} \beta^4 + 16 \beta^3 \delta^i_j B^2 r_{00} - 80 \beta^3 \delta^i_j B r_{00} r_{00} - 54 b^i b_j B r_{00|0} \beta^3 + 135 b^i b_j r_{00} r_{00} \beta^3 + 54 b^i b_j B r^2_{00} \beta^2 - 18 b^i y_j B r_{00|0} \beta^2 \\
& + 45 b^i y_j r_{00} r_0 \beta^2 - 4 B^2 b^i r_{00} \beta^4 r_j - 4 B b^i r_{00} \beta^4 r_j - 9 B b^i r_{00} \beta^3 r_{j0} + 40 B^2 y^i r_{00} \beta^3 r_{j0} - 152 B y^i r_{00} \beta^3 r_{j0} + 6 B^2 y^i r_{00} \beta^2 r_{j0} \\
& + 54 B y^i r_{00} \beta^2 r_{j0} - 8 B^2 y^i r_{00} \beta^3 r_j + 40 B y^i r_{00} \beta^3 r_j - 54 B r^i_0 r_{00} \beta^3 b_j - 18 B^2 r^i_0 r_{00} \beta^2 y_j - 18 B r^i_0 r_{00} \beta^2 y_j + 14 b^i y_j s^k_0 r_{k0} \beta^3 \\
& - 18 b^i y_j B^2 r_{00|0} \beta^2 - 18 b^i y_j B r^2_{00} \beta + 69 \beta^2 y^i b_j B r_{00|0} + 20 \beta^2 y^i b_j r_{00r0} - 396 \beta y^i b_j B r^2_{00} + 4 \beta^2 y^i b_j B^3 r_{00|0} + 78 \beta^3 y^i b_j r_{k0} s^k_0 \\
& - 120 \beta y^i b_j B^2 r^2_{00} + 4 B^2 b^i r_{00} \beta^4 r_{j0} + 4 B b^i r_{00} \beta^4 r_{j0} - 16 B^3 y^i s^k_j \beta^4 r_{k0} - 24 B^2 y^i s^k_j \beta^4 r_{k0} + 204 B y^i s^k_j \beta^4 r_{k0} - 36 B^2 b^i s^k_j \beta^5 r_{k0} \\
& - 192 B^3 s^i_k s^k_0 \beta^4 y_j + 1008 B^2 s^i_k s^k_0 \beta^4 y_j - 144 B s^i_k s^k_0 \beta^4 y_j - 54 \beta y^i y_j B r_{00|0} + 58 \beta y^i y_j r_{00r0} + 432 B^2 s^i_0 r_{00} \beta^3 b_j \\
& + 32 B^3 y^i s^k_0 \beta^4 r_{jk} + 48 B^2 y^i s^k_0 \beta^4 r_{jk} - 408 B y^i s^k_0 \beta^4 r_{jk} + 9 B y^i r_{00} \beta^2 s_{0j} + 306 B^2 y^i r_{00} \beta^2 s_{0j} - 27 r^i_0 r_{00} \beta^3 b_j + 54 s^i_0 r_{00} \beta^2 y_j \\
& - 369 s^i_0 r_{00} \beta^3 b_j + 432 B^3 s^i_0 \beta^4 s_{0j} - 2268 B^2 s^i_0 \beta^4 s_{0j} + 648 B s^i_0 \beta^4 s_{0j} - 186 \beta^4 s^i_k s^k_0 \beta^5 b_j + 106 \beta^4 y^i r_{k0} s^k_j \right]
\end{aligned}$$

$$\begin{aligned}
& -4B^3b^i\beta^4r_{j0|0} + 58y^ir_{00}\beta^3r_j - 174By^i\beta^3r_{00|j} - 216B^2s^i_k s^k_j \beta^6 + 216Bs^i_k s^k_j \beta^6 - 108y^ir_{00}\beta^2s_{0j} + 162s^i_k s^k_j \beta^6 \\
& + 90\beta^2y^ib_j B^2r_{00|0} + 26b^ir_{00}\beta^4r_j + 4B^3b^i\beta^4r_{00|j} + 6B^2b^i\beta^4r_{00|j} - 78Bb^i\beta^4r_{00|j} - 4B^3r^i_0\beta^4r_{j0} - 6B^2r^i_0\beta^4r_{j0} - 96s^i_0|j\beta^5 \\
& + 78Br^i_0\beta^4r_{j0} + 36r^i_0r_{00}\beta^2y_j - 74\beta y^iy_j r_{00|0} - 60y^iy_j B^2r_{00}^2 - 57y^iy_j Br_{00}^2 + 4B^3r^i_j r_{00}\beta^4 - 26b^ir_0\beta^4r_{j0} - 18b^ir_{00}\beta^3r_{j0} \\
& + 6B^2r^i_j r_{00}\beta^4 - 78Br^i_j r_{00}\beta^4 - 4\beta^3d^i_j B^3r_{00|0} - 194y^ir_0\beta^3r_{j0} - 27b^ib_j r_{00|0}\beta^3 - 212\beta^4d^i_j r_{k0}s^k_0 + 30y^ir_{00}\beta^2r_{j0} \\
& + 30\beta^3d^i_j B^2r_{00|0} + 6\beta^2d^i_j B^2r_{00}^2 + 87\beta^3d^i_j Br_{00|0} - 116\beta^3d^i_j r_{00}r_0 - 18\beta^2d^i_j Br_{00}^2 - 28\beta^2y^ib_j r_{00|0} - 159\beta y^ib_j r_{00}^2 \\
& + 54b^ib_j r_{00}^2\beta^2 + 8B^3y^i\beta^3r_{00|j} - 60B^2y^i\beta^3r_{00|j} + 36b^iy_j r_{00|0}\beta^2 - 117b^iy_j B^2r_{00}^2 - 6B^2b^i\beta^4r_{j0|0} - 8B^3y^i\beta^3r_{j0|0} \\
& + 60B^2y^i\beta^3r_{j0|0} + 174By^i\beta^3r_{j0|0} - 18y^iy_j r_{00}^2 - 40b^i\beta^4r_{00|j} - 40r^i_0r_{00}\beta^4 + 78Bb^i\beta^4r_{j0|0} + 40b^i\beta^4r_{j0|0} \\
& + 40r^i_0\beta^4r_{j0} + 22\beta^3d^i_j r_{00|0} - 33\beta^2d^i_j r_{00}^2 - 128\beta y^iy_j B^2r_{00}r_0 + 216s^i_0\beta^4s_{0j} - 684\beta^3y^ib_j Bs^k_0r_{k0} - 96\beta^2y^iy_j B^3s^k_0r_{k0} \\
& - 960\beta^2y^iy_j B^2s^k_0r_{k0} - 240\beta^2y^iy_j Bs^k_0r_{k0} - 200\beta y^iy_j Br_0r_{00} - 360\beta^3y^ib_j B^2s^k_0r_{k0} - 16\beta^2y^ib_j B^2r_0r_{00} - 184\beta^2y^ib_j Br_0r_{00} \\
& + 90b^iy_j Br_0r_{00}\beta^2 - 72b^iy_j B^2s^k_0r_{k0}\beta^3 + 204b^iy_j Bs^k_0r_{k0}\beta^3 - 30b^ib^iy_j Bs^k_0r_{k0}\beta^4 + 126b^ir_0r_{00}\beta^3s_{0j} - 12b^ib_j B^2s^k_0r_{k0}\beta^4 \\
& + 48\beta^3y^ib_j B^3s^k_0r_{k0} + 16b^iy_j B^3s^k_0r_{k0}\beta^3 \Big] - 2\hat{R}^i_j\beta^{12} \left(8B^4 + 16B^3 - 312B^2 - 320B + 41 \right) \\
t''_{10} := & -4\beta^6 \left[+ 18B^2b^ir_{00}\beta^3s_{0j} - 198Bb^ir_{00}\beta^3s_{0j} + 729Bs^i_0r_{00}\beta^3b_j + 72B^3s^i_0r_{00}\beta^2y_j + 432B^2s^i_0r_{00}\beta^2y_j - 351Bs^i_0r_{00}\beta^2y_j \right. \\
& + 276Bb^is^k_0\beta^5r_{jk} + 8B^3b^is^k_j\beta^5r_{k0} - 16\beta y^iy_j B^3r_{00|0} - 368\beta^2y^iy_j r_{k0}s^k_0 + 72\beta y^iy_j B^2r_{00|0} - 108B^3s^i_0r_{00}\beta^3b_j \\
& + 288\beta^4d^i_j B^2s^k_0r_{k0} + 28B\beta^4d^i_j s^k_0r_{k0} - 22b^ib_j s^k_0r_{k0}\beta^4 + 32\beta^3d^i_j B^2r_0r_{00} + 128\beta^3d^i_j Br_0r_{00} + 18b^ib_j Br_0r_{00}\beta^3 \\
& - 45b^ib_j r_{00}r_{00}\beta^3 + 9b^ib_j Br_0r_{00}^2\beta^2 - 54y^ib_j Br_{00|0}\beta^2 + 135b^iy_j r_{00}r_0\beta^2 - 36Bb^ir_0\beta^4r_j + 9Bb^ir_0\beta^3r_{j0} + 80B^2y^ir_0\beta^3r_{j0} \\
& + 272By^ir_0\beta^3r_{j0} - 36By^ir_{00}\beta^2r_{j0} - 16B^2y^ir_{00}\beta^3r_j - 64By^ir_{00}\beta^3r_j + 18Br^i_0r_{00}\beta^3b_j - 54Br^i_0r_{00}\beta^2y_j + 178b^iy_j s^k_0r_{k0}\beta^3 \\
& + 45b^iy_j Br_0r_{00}\beta^2 + 69\beta^2y^ib_j Br_{00|0} - 78\beta^2y^ib_j r_{00}r_0 + 69\beta y^ib_j Br_0r_{00}^2 - 24\beta^2y^ib_j B^3r_{00|0} - 270\beta^3y^ib_j r_{k0}s^k_0 - 48\beta^2y^ib_j B^2r_{00|0} \\
& + 84\beta y^ib_j B^2r_{00}^2 + 36Bb^ir_0\beta^4r_{j0} + 96B^3s^i_k s^k_0\beta^5b_j - 144B^2y^is^k_j\beta^4r_{k0} - 144By^is^k_j\beta^4r_{k0} - 24B^2y^ir_{00}\beta^2r_{j0} - 138Bb^is^k_j\beta^5r_{k0} \\
& + 32B^4s^i_k s^k_0\beta^4y_j - 512B^3s^i_k s^k_0\beta^4y_j + 480B^2s^i_k s^k_0\beta^4y_j + 448Bs^i_k s^k_0\beta^4y_j + 18b^ib_j B^2r_{00|0}\beta^3 + 156\beta y^ib_j Br_{00|0} \\
& + 162B^2s^i_0r_{00}\beta^3b_j + 18B^2r^i_0r_{00}\beta^3b_j + 288B^2y^is^k_0\beta^4r_{jk} + 288By^is^k_0\beta^4r_{jk} + 333By^ir_{00}\beta^2s_{0j} + 144B^2y^ir_{00}\beta^2s_{0j} \\
& + 48b^ib_j B^2s^k_0r_{k0}\beta^4 - 36r^i_0r_{00}\beta^3b_j + 62y^ir_0\beta^3r_{j0} - 3y^ir_{00}\beta^2r_{j0} - 63b^ir_{00}\beta^3s_{0j} + 96B^3s^i_k s^k_0\beta^6 - 10y^ir_{00}\beta^3r_j + 30By^i\beta^3r_{00|j} \\
& - 288B^2s^i_k s^k_0\beta^6 - 54s^i_0r_{00}\beta^3b_j - 72B^4s^i_0\beta^4s_{0j} + 1152B^3s^i_0\beta^4s_{0j} - 1404B^2s^i_0\beta^4s_{0j} - 360Bs^i_0\beta^4s_{0j} + 38\beta^4s^i_k s^k_0y_j \\
& + 120s^i_k s^k_0\beta^5b_j + 18\beta^4y^ir_{jk}s^k_j - 36\beta^4y^ir_{jk}s^k_0 - 720B^2s^i_k s^k_0\beta^5b_j + 504Bs^i_k s^k_0\beta^5b_j - 360Bs^i_k s^k_0\beta^6 - 153s^i_0r_{00}\beta^2y_j \\
& - 8b^is^k_j\beta^5r_{k0} - 18b^ir_0\beta^4r_j + 54B^2b^i\beta^4r_{00|j} + 54Bb^i\beta^4r_{00|j} - 54B^2r^i_0\beta^4r_{j0} - 54Br^i_0\beta^4r_{j0} - 27r^i_0r_{00}\beta^2y_j \\
& + 58\beta^3y^iy_j r_{00|0} - 18y^iy_j B^2r_{00}^2 - 24y^iy_j Br_{00}^2 + 18b^ir_0\beta^4r_{j0} - 9b^ir_0r_{00}\beta^3r_{j0} + 54B^2r^i_j r_{00}\beta^4 + 54Br^i_j r_{00}\beta^4 - 54B^2b^i\beta^4r_{j0|0} \\
& - 8\beta^3d^i_j B^3r_{00|0} - 36b^ib^ir_{00|0}\beta^3 - 36\beta^4d^i_j r_{k0}s^k_0 - 48\beta^3d^i_j B^2r_{00|0} - 15\beta^3d^i_j Br_{00|0} + 20\beta^3d^i_j r_{00}r_0 + 24\beta^2d^i_j Br_{00}^2 \\
& + 126b^ib_j r_{00}^2\beta^2 + 16y^iB^3\beta^3r_{00|j} + 96B^2y^i\beta^3r_{00|j} - 27b^iy_j r_{00|0}\beta^2 + 36b^iy_j Br_{00}^2 - 16B^3y^i\beta^3r_{j0|0} - 9B^2y^i\beta^3r_{j0|0} \\
& - 30By^i\beta^3r_{j0|0} - 2r^i_j r_{00}\beta^4 + 6y^iy_j r_{00}^2 + 27b^iy^ir_{j0|0} + 27r^i_0\beta^4r_{j0} + 17\beta^3d^i_j r_{00|0} + 21\beta^2s^i_j r_{00}^2 - 34\beta^3y^ir_{00|j} + 34\beta^3y^ir_{j0|0} \\
& - 840\beta^2y^iy_j Bs^k_0r_{k0} + 64\beta y^iy_j B^2r_0r_{00} - 164\beta y^iy_j Br_0r_{00} - 90b^ib_j Br_0r_{00}\beta^3 + 52\beta^3y^ib_j B^2s^k_0r_{k0} + 96\beta^2y^ib_j B^2r_0r_{00} \\
& + 72b^2y^ib_j Br_0r_{00} + 96\beta^3y^ib_j B^3s^k_0r_{k0} - 648b^iy_j Br_0r_{00}\beta^2r_0 + 32b^iy_j B^3s^k_0r_{k0}\beta^3 - 48b^iy_j B^2s^k_0r_{k0}\beta^3 - 342b^ib_j Bs^k_0r_{k0}\beta^4 \\
& \left. + 224\beta^2y^iy_j B^3s^k_0r_{k0} - 192\beta^3y^ib_j Bs^k_0r_{k0} - 45s^i_0\beta^4s_{0j} \right] - 144\hat{R}^i_j\beta^{10} (2B+1)(B+2)(B-1) \\
t''_{12} := & -16B^4s^i_k s^k_0\beta^5b_j + 32B^3y^is^k_j\beta^4r_{k0} + 48B^2y^is^k_j\beta^4r_{k0} - 84By^is^k_j\beta^4r_{k0} - 234B^2y^ir_{00}\beta^2s_{0j} - 45By^ir_{00}\beta^2s_{0j} \\
& - 72B^3y^ir_{00}\beta^2s_{0j} - 432B^2s^i_0r_{00}\beta^3b_j - 16B^3s^i_k\beta^6s_j - 12B^2b^ir_0\beta^4r_{j0} - 12Bb^ir_0\beta^4r_{j0} + 9Bb^ir_{00}\beta^3r_{j0} + 96\beta y^ib_j B^2r_0r_{00} \\
& + 96\beta^2y^iy_j B^3s^k_0r_{k0} + 82y^ir_0\beta^3r_{j0} + 624\beta^2y^iy_j B^2s^k_0r_{k0} + 552\beta^2y^iy_j Bs^k_0r_{k0} - 15y^ir_{00}\beta^2r_{j0} + 12B^2b^ir_{00}\beta^4r_j + 12Bb^ir_{00}\beta^4r_j \\
& - 80B^2y^ir_0\beta^3r_{j0} + 16B^2y^ir_{00}\beta^3r_j - 8By^ir_{00}\beta^3r_j + 16By^ir_0\beta^3r_{j0} + 6B^2y^ir_{00}\beta^2r_{j0} - 18By^ir_{00}\beta^2r_{j0} - 18y^iy_j r_{00}^2 \\
& + 148\beta^2y^ib_j Br_0r_{00} - 128\beta^3y^ib_j B^3s^k_0r_{k0} + 20\beta^3b^iy^iB^2s^k_0r_{k0} + 588\beta^3y^ib_j Bs^k_0r_{k0} - 32\beta^2y^ib_j B^2r_0r_{00} - 135b^ib_j r_{00}r_{00}\beta^3 \\
& - 54b^ib_j B^2r_{00}\beta^2 + 48b^iy_j B^2s^k_0r_{k0}\beta^3 - 204B^ib^iy_j s^k_0r_{k0}\beta^3 - 90B^ib^iy_j r_{00}r_{00}\beta^2 + 34\beta^3y^ir_{00|j} - 34\beta^3y^ir_{j0|0} - 17\beta^3d^i_j r_{00|0} \\
& - 18\beta^2d^i_j r_{00}^2 + 18b^iy_j B^2r_{00|0}\beta^2 + 18b^iy_j Br_{00|0}\beta^2 - 45b^iy_j r_{00}r_0\beta^2 + 18b^iy_j Br_{00}^2\beta - 24\beta y^iy_j B^3r_{00|0} + 45s^i_0\beta^4s_{0j}
\end{aligned}$$

$$\begin{aligned}
& -72 B^3 s^i_0 r_{00} \beta^3 b_j + 96 B^4 s^i_k s^k_0 \beta^4 y_j - 384 B^3 s^i_k s^k_0 \beta^4 y_j + 351 B s^i_0 r_{00} \beta^3 b_j - 72 B^3 s^i_0 r_{00} \beta^2 y_j + 378 B^2 s^i_0 r_{00} \beta^2 y_j \\
& -240 b^i b_j s^k_0 r_{k0} \beta^4 + 39 r^i_j r_{00} \beta^4 + 39 b^i \beta^4 r_{00|j} + 54 b^i b_j B r_{00|0} \beta^3 - 39 b^i \beta^4 r_{j0|0} + 112 B^3 s^i_j |_0 \beta^5 - 41 \beta^2 y^i b_j r_{00|0} \\
& +27 \beta y^i b_j r^2_{00} - 16 y^i B^3 \beta^3 r_{00|j} + 12 B^2 y^i \beta^3 r_{00|j} + 78 B y^i \beta^3 r_{00|j} - 50 \beta^4 y^i r_{k0} s^k_j + 100 \beta^4 y^i r_{jk} s^k_0 + 96 B^2 y^i \beta^4 - 54 b^i b_j r^2_{00} \beta^2 \\
& +27 y^i r_{00} \beta^2 s_{0j} - 18 y^i y_j B^2 r^2_{00} - 45 y^i y_j B r^2_{00} - 18 \beta y^i y_j r_{00|0} - 16 B^4 s^i_k s^k_j \beta^6 + 8 \beta^2 y^i b_j B^3 r_{00|0} + 218 \beta^3 b_j y^i r_{k0} s^k_0 \\
& -66 \beta^2 y^i b_j B^2 r_{00|0} + 18 \beta y^i b_j B^2 r^2_{00} - 117 B \beta^2 y^i b_j r_{00|0} + 100 \beta^2 y^i b_j r_{00|r} + 72 \beta y^i b_j B r^2_{00} + 27 r^i_0 r_{00} \beta^3 b_j + 8 \beta^3 \delta^i_j B^3 r_{00|0} \\
& -6 \beta^3 \delta^i_j B^2 r_{00|0} + 100 \beta^4 \delta^i_j r_{k0} s^k_0 - 6 \beta^2 \delta^i_j B^2 r^2_{00} - 39 \beta^3 \delta^i_j B r_{00|0} + 52 \beta^3 \delta^i_j r_{00|r} - 96 \beta y^i y^j B^2 r_{00|0} - 78 \beta y^i y^j B r_{00|0} \\
& +90 \beta y^i y^j r_{00|r} + 16 B^3 y^i \beta^3 r_{j0|0} - 12 B^2 y^i \beta^3 r_{j0|0} - 78 B y^i \beta^3 r_{j0|0} + 24 b^i r_0 \beta^4 r_{j0} + 18 b^i r_{00} \beta^3 r_{j0} + 27 b^i b_j r_{00|0} \beta^3 \\
& -9 b^i y_j r_{00|0} \beta^2 + 45 b^i y_j \beta r^2_{00} - 24 b^i b_j B^3 s^k_0 r_{k0} \beta^4 + 132 b^i b_j B^2 s^k_0 r_{k0} \beta^4 - 288 B^2 s^i_k s^k_0 \beta^4 y_j + 48 B s^i_k s^k_0 \beta^4 y_j + 352 B^3 s^i_k s^k_0 \beta^5 b_j \\
& -744 B^2 s^i_k s^k_0 \beta^5 b_j - 168 B^2 b^i s^k_0 \beta^5 r_{jk} + 60 B b^i s^k_0 \beta^5 r_{jk} + 18 B b^i r_{00} \beta^3 s_{0j} + 72 B^2 b^i r_{00} \beta^3 s_{0j} + 8 B^3 b^i s^k_j \beta^5 r_{k0} + 84 B^2 b^i s^k_j \beta^5 r_{k0} \\
& -30 B b^i s^k_j \beta^5 r_{k0} - 64 B^3 s^k_0 y^i \beta^4 r_{jk} - 16 B^3 b^i s^k_0 \beta^5 r_{jk} + 54 B r^i_0 r_{00} \beta^3 b_j - 96 \beta^4 \delta^i_j B^2 s^k_0 r_{k0} + 168 \beta^4 \delta^i_j B s_0 r_{k0} - 32 \beta^3 \delta^i_j B^2 r_{00|r} \\
& +16 \beta^3 \delta^i_j B r_{00|r} + 18 B r^i_0 r_{00} \beta^2 y_j - 12 B^3 r^i_j r_{00} \beta^4 - 18 B^2 r^i_j r_{00} \beta^4 + 72 B r^i_j r_{00} \beta^4 + 12 B^3 b^i \beta^4 r_{j0|0} + 18 B^2 b^i \beta^4 r_{j0|0} \\
& -72 B b^i \beta^4 r_{j0|0} + 12 B^3 r^i_0 \beta^4 r_{j0} + 72 B r^i_j r_{00} \beta^4 - 9 r^i_0 r_{00} \beta^2 y_j - 12 B^3 b^i \beta^4 r_{00|j} - 18 B^2 b^i \beta^4 r_{00|j} + 72 B b^i \beta^4 r_{00|j} \\
& -72 B r^i_0 \beta^4 r_{j0} + 18 B^2 r^i_0 \beta^4 r_{j0} - 24 b^i r_{00} \beta^4 r_j - 26 y^i r_{00} \beta^3 r_j - 296 B s^i_k s^k_0 \beta^6 - 90 b^i r_{00} \beta^3 s_{0j} + 18 B^2 r^i_0 r_{00} \beta^2 y_j + 160 B^3 s^i_k s^k_j \beta^6 \\
& +264 B^2 s^i_k s^k_0 \beta^6 + 74 s^i_k s^k_0 \beta^5 b_j + 42 \beta^4 s^i_k s^k_0 y_j - 216 B^4 s^i_0 \beta^4 s_{0j} + 1008 B^3 s^i_0 \beta^4 s_{0j} - 108 B^2 s^i_0 \beta^4 s_{0j} - 9 s^i_0 r_{00} \beta^2 y_j \\
& +153 s^i_0 r_{00} \beta^3 b_j - 62 b^i s^k_j \beta^5 r_{k0} + 124 b^i s^k_0 \beta^5 r_{jk} - 96 B^2 s^k_0 y^i \beta^4 r_{jk} \Big] + 6 \hat{R}^i_j \beta^8 (8 B^4 + 16 B^3 - 96 B^2 - 104 B - 13)
\end{aligned}$$

$$\begin{aligned}
t''_{14} := & 4 \beta^2 \Big[-36 B^3 y^i r_{00} \beta^2 s_{0j} + 72 B^2 y^i r_{00} \beta^2 s_{0j} + 108 B^3 s^i_0 r_{00} \beta^2 y_j - 36 B b^i r_{00} \beta^4 r_j - 16 \beta y^i y_j B^3 r_{00|0} + 16 \beta^2 y^i y_j r_{k0} s^k_0 \\
& +18 b^i b_j B^2 r_{00|0} \beta^3 + 54 B^2 r^i_j r_{00} \beta^4 + 54 B r^i_j r_{00} \beta^4 - 54 B^2 b^i \beta^4 r_{j0|0} - 54 B b^i \beta^4 r_{j0|0} + 54 B^2 b^i \beta^4 r_{00|j} + 54 B b^i \beta^4 r_{00|j} \\
& -18 b^i r_{00} \beta^4 r_j - 24 \beta y^i y_j B^2 r_{00|0} - 2 \beta y^i y_j B r_{00|0} + 16 \beta y^i y_j r_{00|r} - 2 \beta y^i y_j r_{00|0} - 24 y^i y_j B^2 r^2_{00} - 24 y^i y_j B r^2_{00} + 27 y^i r_{00} \beta^2 s_{0j} \\
& +36 \beta^4 y^i r_{jk} s^k_0 - 5 \beta^3 \delta^i_j r_{00|0} + 10 \beta^3 y^i r_{00|j} - 10 \beta^3 y^i r_{j0|0} - 20 \beta^2 y^i b_j B^3 r_{00|0} - 81 B s^i_0 r_{00} \beta^2 y_j - 72 B^3 s^i_0 r_{00} \beta^3 b_j \\
& -9 r^i_0 r_{00} \beta^2 y_j - 18 B^2 b^i r_{00} \beta^3 s_{0j} - 126 B b^i r_{00} \beta^3 s_{0j} - 114 B b^i s^k_j \beta^5 r_{k0} + 64 B^4 s^i_k s^k_0 \beta^5 b_j - 448 B^3 s^i_k s^k_0 \beta^5 b_j + 96 B^2 s^i_k s^k_0 \beta^5 b_j \\
& +248 B s^i_k s^k_0 \beta^5 b_j + 36 B^2 s^i_0 \beta^4 s_{0j} + 120 B s^i_0 \beta^4 s_{0j} - 18 b^i r_{00} \beta^3 s_{0j} + 64 B^3 s^i_k s^k_j \beta^6 - 384 B^2 s^i_k s^k_j \beta^6 - 200 B s^i_k s^k_j \beta^6 \\
& -18 \beta^4 y^i r_{k0} s^k_j - 16 b^i s^k_j \beta^5 r_{k0} + 32 b^i s^k_0 \beta^5 r_{jk} + 40 s^i_k s^k_0 \beta^5 b_j + 18 \beta^4 s^i_k s^k_0 y_j - 72 B s^k_j y^i \beta^4 r_{k0} + 16 B^3 b^i s^k_j \beta^5 r_{k0} \\
& -48 B^2 b^i s^k_j \beta^5 r_{k0} + 144 B^2 s^i_k s^k_0 \beta^4 y_j + 96 B s^i_k s^k_0 \beta^4 y_j - 32 B^3 b^i s^k_0 \beta^5 r_{jk} + 96 B^2 b^i s^k_0 \beta^5 r_{jk} + 228 B b^i s^k_0 \beta^5 r_{jk} \\
& +48 b^i y_j B s^k_0 r_{k0} \beta^3 + 378 B^2 s^i_0 r_{00} \beta^3 b_j + 18 B^2 r^i_0 r_{00} \beta^3 b_j + 144 B^2 \beta^4 y^i s^k_0 r_{jk} + 144 B \beta^4 y^i s^k_0 r_{jk} + 189 B s^i_0 r_{00} \beta^3 b_j \\
& +96 \beta^3 y^i b_j B^3 s^k_0 r_{k0} + 408 \beta^3 y^i b_j B^2 s^k_0 r_{k0} + 336 \beta^3 y^i b_j s^k_0 r_{k0} + 80 \beta^2 y^i b_j B^2 r_{00|r} + 164 \beta^2 y^i b_j B r_{00|r} + 32 b^i y_j B^3 s^k_0 r_{k0} \beta^3 \\
& +36 B b^i r_0 \beta^4 r_{j0} + 9 B b^i r_{00} \beta^3 r_{j0} - 6 y^i y_j r^2_{00} - 18 B r^i_0 r_{00} \beta^2 y_j + 64 \beta^3 \delta^i_j B r_{00|r} + 144 \beta^4 \delta^i_j B^2 s^k_0 r_{k0} + 144 \beta^4 \delta^i_j B s^k_0 r_{k0} \\
& -22 b^i b_j s^k_0 r_{k0} \beta^4 + 18 b^i b_j B r_{00|0} \beta^3 - 5 b^i b_j r_{00|r} \beta^3 + 9 b^i b_j B r^2_{00} \beta^2 + 46 b^i y_j s^k_0 r_{k0} \beta^3 - 18 b^i y_j B r_{00|0} \beta^2 + 45 b^i y_j r_{00|r} \beta^2 \\
& -8 B^3 y^i \beta^3 r_{j0|0} - 48 B^2 y^i \beta^3 r_{j0|0} - 42 B y^i \beta^3 r_{j0|0} + 18 b^i r_0 \beta^4 r_{j0} + 78 \beta^3 y^i b_j r_{k0} s^k_0 + 16 \beta^3 \delta^i_j B^2 r_{00|r} + 15 b^i y_j B^2 r^2_{00} \\
& -14 y^i r_{00} \beta^3 r_j + 58 y^i r_0 \beta^3 r_{j0} - 9 y^i r_{00} \beta^2 r_{j0} - 9 r^i_0 r_{00} \beta^3 b_j - 72 B^2 y^i b_j B^2 r_{00|0} + 6 \beta y^i b_j B^2 r^2_{00} - 57 \beta^2 y^i b_j B r_{00|0} \\
& -12 s^i_0 \beta^4 s_{0j} + 62 \beta^2 y^i b_j r_{00|r} + 3 \beta y^i b_j r^2_{00} + 16 b^i b_j B^3 s^k_0 r_{k0} \beta^4 - 318 b^i b_j B s^k_0 r_{k0} \beta^4 + 128 B^2 y^i y_j B^3 s^k_0 r_{k0} + 192 B^2 y^i y_j B^2 s^k_0 r_{k0} \\
& +96 \beta^2 y^i y_j B s^k_0 r_{k0} - 90 b^i b_j B r_{00|r} \beta^3 + 64 \beta y^i y_j B^2 r_{00|r} + 64 \beta y^i y_j B r_{00|r} - 9 \beta^2 \delta^i_j r^2_{00} + 8 B^3 y^i \beta^3 r_{00|j} + 48 B^2 y^i \beta^3 r_{00|j} \\
& +42 B y^i \beta^3 r_{00|j} - 13 \beta^2 y^i b_j r_{00|r} - 9 b^i y_j r_{00|r} \beta^2 + 12 b^i y_j \beta r^2_{00} - 9 b^i b_j r_{00|r} \beta^3 + 45 b^i b_j r^2_{00} \beta^2 - 4 \beta^3 \delta^i_j B^3 r_{00|j} \\
& +36 \beta^4 \delta^i_j r_{k0} s^k_0 - 24 \beta^3 \delta^i_j B^2 r_{00|0} - 12 \beta^2 \delta^i_j B^2 r^2_{00} - 21 \beta^3 \delta^i_j B r_{00|r} + 28 \beta^3 \delta^i_j r_{00|r} - 24 \beta^2 \delta^i_j B r^2_{00} - 27 s^i_0 r_{00} \beta^2 y_j \\
& -9 s^i_0 r_{00} \beta^3 b_j + 240 B^4 s^i_0 \beta^4 s_{0j} - 384 B^3 s^i_0 \beta^4 s_{0j} - 8 B^2 y^i r_{00} \beta^3 r_j - 32 B y^i r_{00} \beta^3 r_j + 40 B^2 y^i r_0 \beta^3 r_{j0} + 18 B r^i_0 r_{00} \beta^3 b_j \\
& +136 B y^i r_0 \beta^3 r_{j0} - 12 B^2 y^i r_{00} \beta^2 r_{j0} - 24 B y^i r_{00} \beta^2 r_{j0} - 54 B^2 r^i_0 \beta^4 r_{j0} - 54 B r^i_0 \beta^4 r_{j0} \Big] + 72 \hat{R}^i_j \beta^6 (2 B + 1) (2 B^2 + 2 B - 1)
\end{aligned}$$

$$\begin{aligned}
t''_{16} := & 2 \beta \Big[-96 B^2 s^i_k s^k_0 \beta^3 y_j - 32 B s^i_k s^k_0 \beta^3 y_j + 72 B^2 b^i r_{00} \beta^2 s_{0j} - 36 B b^i r_{00} \beta^2 s_{0j} - 20 b^i y_j s^k_0 r_{k0} \beta^2 + 12 b^i y_j B^2 r_{00|0} \beta \\
& -36 b^i b_j B r^2_{00} \beta - 124 b^i b_j s^k_0 r_{k0} \beta^3 - 24 B b^i r_0 \beta^3 r_{j0} + 6 B b^i r_{00} \beta^2 r_{j0} + 162 B s^i_0 r_{00} \beta^2 b_j - 72 B^3 y^i r_{00} \beta s_{0j} - 108 B^2 y^i r_{00} \beta s_{0j} \\
& -54 B y^i r_{00} \beta s_{0j} + B^2 y^i s^k_j \beta^3 r_{k0} + B y^i s^k_j \beta^3 r_{k0} - B^3 s^i_k s^k_0 \beta^4 b_j - 432 B^2 s^i_k s^k_0 \beta^4 b_j + 32 B^3 b^i s^k_j \beta^4 r_{k0} + 120 B^2 b^i s^k_j \beta^4 r_{k0}
\end{aligned}$$

$$\begin{aligned}
& -12Bb^i s_j^k \beta^4 r_{k0} - 96B^2 y^i s_0^k \beta^3 r_{jk} - 48By^i s_0^k \beta^3 r_{jk} + 36\beta b_j y^i B^2 r_{00|0} + 18\beta b_j y^i B r_{00|0} - 24\beta b_j y^i r_{00r0} + 54Bs_0^i r_{00\beta} y_j \\
& + 12b^i y_j Br_{00|0} \beta - 30b^i y_j r_{00r0} \beta - 64B^3 b^i s_0^k \beta^4 r_{jk} - 240B^2 b^i s_0^k \beta^4 r_{jk} + 12B^2 r_0^i r_{00\beta} y_j + 12Br_0^i r_{00\beta} y_j + 36b^i b_j Br_{00|0} \beta^2 \\
& - 90b^i b_j r_{0r00} \beta^2 - \delta_j^i B^3 s_0^k r_{k0} - \beta^3 \delta_j^i B^2 s_0^k r_{k0} - \beta^3 \delta_j^i B s_0^k r_{k0} - 64B^3 y^i s_0^k \beta^3 r_{jk} + 192B^4 s_0^k s_0^k \beta^4 b_j - 24B^2 b^i r_0 \beta^3 r_{j0} \\
& - 32\beta^2 \delta_j^i B^2 r_{0r00} - 20\beta^2 y^i b_j r_{k0} s_0^k + 24B^2 b^i r_{00} \beta^3 r_j + 16B^2 y^i r_{00} \beta^2 r_j + 16By^i r_{00} \beta^2 r_j + 36Br_0^i r_{00} \beta^2 b_j - 80By^i r_0 \beta^2 r_{j0} \\
& + B^2 y^i r_{00\beta} r_{j0} + 12By^i \beta r_{j0} + B^3 s_0^i r_{00\beta} y_j - 64B^4 s_0^i s_0^k \beta^3 y_j - 128B^3 s_0^i s_0^k \beta^3 y_j + 24\beta y^i b_j B^3 r_{00|0} - 216B^3 s_0^i r_{00\beta} \beta^2 b_j \\
& - 32\beta^2 \delta_j^i Br_{0r00} + 32B^3 y^i s_0^k \beta^3 r_{k0} + 24B^i s_0^k \beta^4 r_{jk} - 48Bs_0^i s_0^k \beta^4 b_j + \beta^2 \delta_j^i r_{00|0} + 16B^3 y^i \beta^2 r_{j0|0} + 24B^2 y^i \beta^2 r_{j0|0} \\
& + 12By^i \beta^2 r_{j0|0} + 12b^i r_{00} \beta^2 r_{j0} + 18b^i r_{00|0} \beta^2 - 36b^i b^i r_{00\beta} + 8\beta^2 \delta_j^i B^3 r_{00|0} - 8\beta^3 \delta_j^i r_{k0} s_0^k + 108B^2 s_0^i r_{00\beta} y_j \\
& + 12\beta^2 \delta_j^i B^2 r_{00|0} + 12\beta \delta_j^i B^2 r_{00} + 6\beta^2 \delta_j^i Br_{00|0} - 8\beta^2 \delta_j^i r_{00} r_0 + 12\beta \delta_j^i Br_{00} - 16B^3 y^i \beta^2 r_{00|j} - 24B^2 y^i \beta^2 r_{00|j} \\
& - 4\beta^3 s_0^i s_0^k y_j - 12By^i r_{00|j} + 12y_j b^i r_{00} + B^4 s_0^i \beta^3 s_{0j} - 288B^3 s_0^i \beta^3 s_{0j} - 24B^3 r_j^i r_{00} \beta^3 - 36B^2 r_j^i r_{00} \beta^3 + 36Br^i_j r_{00} \beta^3 \\
& - 24B^3 b^i \beta^3 r_{00|j} - 36B^2 b^i \beta^3 r_{00|j} + b^i \beta^3 r_{00|j} + B^3 r_0^i \beta^3 r_{j0} + B^2 r_0^i \beta^3 r_{j0} - Br_0^i \beta^3 r_{j0} + y^i r_{00rj} \\
& - 20y^i r_0 \beta^2 r_{j0} + 3y^i r_{00\beta} r_{j0} - 216B^2 s_0^i \beta^3 s_{0j} + 144Bs_0^i \beta^3 s_{0j} - 12b^i r_{00} \beta^3 r_j - 9y^i r_{00} \beta s_{0j} - 36b^i r_{00} \beta^2 s_{0j} + 24s_0^i s_0^k \beta^4 b_j \\
& + 24B^3 b^i \beta^3 r_{j0|0} + 36B^2 b^i \beta^3 r_{j0|0} - 36Bb^i \beta^3 r_{j0|0} + 18r_0^i r_{00} \beta^2 b_j + 3r_0^i r_{00\beta} y_j - 32b^i s_0^k \beta^4 r_{k0} + 64b^i s_0^k \beta^4 r_{jk} \\
& + 4\beta^3 y^i r_{k0} s_0^k - 8\beta^3 y^i r_{jk} s_0^k + 54s_0^i r_{00} \beta^2 b_j + 9s_0^i r_{00\beta} y_j + 384B^3 s_0^i s_0^k \beta^5 + 24b_j y^i B^2 r_{00} + 24b_j y^i Br_{00} + 3b^i y_j r_{00|0} \beta \\
& + 3\beta y^i b_j r_{00|0} + 12b^i r_0 \beta^3 r_{j0} - 60s_0^i s_0^k \beta^5 + 72s_0^i \beta^3 s_{0j} + 3\beta \delta_j^i r_{00}^2 - 2\beta^2 y^i r_{00|j} + 2\beta^2 y^i r_{j0|0} + 24r^i_j r_{00} \beta^3 + 24b^i \beta^3 r_{00|j} \\
& - 24b^i \beta^3 r_{j0|0} - 24r_0^i \beta^3 r_{j0} + 6b^i y_j + 6y^i b_j r_{00}^2 - y^i b_j B^3 s_0^k r_{k0} - 120\beta^2 y^i b_j Bs_0^k r_{k0} - 96\beta y^i b_j B^2 r_{0r00} - 96\beta y^i b_j Br_{0r00} \\
& + 32b^i y_j B^3 s_0^k r_{k0} \beta^2 - 48b^i y_j B^2 s_0^k r_{k0} \beta^2 - 72b^i y_j Bs_0^k r_{k0} \beta^2 - 60b^i y_j Br_{0r00} \beta - 32b^i b_j B^3 s_0^k r_{k0} \beta^3 + 24b^i b_j B^2 s_0^k r_{k0} \beta^3 \\
& - 10b^i b_j Bs_0^k r_{k0} \beta^3 + 144B^2 s_0^i s_0^k \beta^5 - 80B^2 y^i r_0 \beta^2 r_{j0} + 24Bb^i r_{00} \beta^3 r_j - 144Bs_0^i s_0^k \beta^5 - 240\beta^2 y^i b_j B^2 s_0^k r_{k0} \\
& - 3\hat{R}_j^i \beta^4 (2B+1)^2 (4B^2 + 4B - 17) \\
t''_{18} & := 2(2B+1) \left(-36B^2 s_0^i r_{00} \beta b_j - 8B^2 b^i s_0^k \beta^3 r_{k0} + 28Bb^i s_0^k \beta^3 r_{k0} - 36Bs_0^i r_{00} \beta b_j - 6b^i b_j Br_{00|0} \beta + 30b^i b_j r_{00r0} \beta \right. \\
& \quad - 6Br_0^i r_{00} \beta b_j - 48Bs_0^i s_0^k \beta^3 b_j + 36b^i b_j \beta^2 r_{k0} s_0^k + 18Bb^i r_{00} \beta s_{0j} + 64B^3 s_0^i s_0^k \beta^3 b_j + 16B^2 b^i s_0^k \beta^3 r_{jk} - 56Bb^i s_0^k \beta^3 r_{jk} \\
& \quad + 18Bb^i \beta^2 r_{j0|0} - 18Br_0^i r_{00} \beta^2 + 9b^i r_{00} \beta s_{0j} - 18Bb^i \beta^2 r_{00|j} - 32b^i s_0^k \beta^3 r_{jk} - 27s_0^i \beta^2 s_{0j} + 16b^i s_0^k \beta^3 r_{k0} - 9s_0^i r_{00} \beta b_j \\
& \quad - 9r_0^i r_{00} \beta^2 + 12b^i r_{00} \beta^2 r_j - 36B^2 s_0^i \beta^2 s_{0j} - 90Bs_0^i \beta^2 s_{0j} - 32B^3 s_0^i s_0^k \beta^4 + 48B^2 s_0^i s_0^k \beta^4 + 72Bs_0^i s_0^k \beta^4 - 16s_0^i s_0^k \beta^3 b_j \\
& \quad - 3r_0^i r_{00} \beta b_j - 3b^i b_j r_{00|0} \beta + 72B^3 s_0^i s_{0j} \beta^2 + 9r_0^i \beta^2 r_{j0} + 9b^i \beta^2 r_{j0|0} + 18Br_0^i \beta^2 r_{j0} - 3b_j b^i r_{00}^2 + 20s_0^i s_0^k \beta^4 - 9b^i \beta^2 r_{00|j} \\
& \quad \left. - 24b^i b_j B^2 s_0^k r_{k0} \beta^2 + 60b^i b_j Bs_0^k r_{k0} \beta^2 \right) - 12\hat{R}_j^i \beta^2 (2B+1)^3 \\
t''_{20} & := 2(2B+1)^2 \left(-2\beta^2 s_0^i s_0^k \beta^4 + 2Br_0^i r_{00} - 2Bb^i r_{j0|0} - 2Br_0^i r_{j0} + 2Bb^i r_{00|j} - 2b^i r_{00} r_j + 2b^i r_0 r_{j0} - 4b^i b_j Bs_0^k r_{k0} \right. \\
& \quad + 8Bb^i s_0^k \beta r_{jk} + 8B^2 s_0^i s_0^k \beta b_j + 8Bs_0^i s_0^k \beta b_j + 12B^2 s_0^i s_{0j} + 12Bs_0^i s_{0j} + 3s_0^i s_{0j} - b^i r_{j0|0} - r_0^i r_{j0} + b^i r_{00|j} + r^i_j r_{00} \\
& \quad \left. + 2\beta s_0^i s_0^k b_j - 2\beta b^i r_{k0} s_0^k + 4\beta b^i r_{jk} s_0^k - 2b^i b_j s_0^k r_{k0} - 8B^2 s_0^i s_0^k \beta^2 - 8Bs_0^i s_0^k \beta^2 \right) + \hat{R}_j^i (2B+1)^4
\end{aligned}$$

8. Appendix 4

$$\begin{aligned}
d_1 &:= 0 \\
d_2 &:= 6\beta^{14} \left(18\beta^3 s_{|0} + 79Br_{00|0} \beta + 250s_0 r_{00} \beta + 28B^2 r_{00|0} \beta - 6rr_{00} \beta^2 + 180s_0 r_0 \beta^2 - 82r_{00} r_0 \beta - 18Br_{00|0} \beta^2 - 17Bs_{0|0} \beta^2 \right. \\
& \quad - 54s_0^m s_{0m} \beta^4 - 108s_0^m s_{0m} \beta^2 + 212Bs_0 r_{00} \beta - 68Br_{00} r_0 \beta + 73nr_0^2 - 21nr_{00|0} \beta + 72ns_0|0 \beta^2 + 288\beta^3 s_0^m s_m + 432Bs_0^m s_{0m} \beta^2 \\
& \quad + 18Br_m^m r_{00} \beta^2 + 18Bb^m r_{00|m} \beta^2 - 276Bs_0^m r_{0m} \beta^2 + 18r_m^m r_{00} \beta^2 + 18b^m r_{00|m} \beta^2 - 156s_0^m r_{0m} \beta^2 - 36\beta^3 r_m^m s_0 - 36\beta^3 b^m s_{0|m} \\
& \quad + 36\beta^3 r_0^m s_m + 36\beta^3 s_0^m r_m + 4B^2 nr_0^2 + 52Bnr_0^2 - 240ns_0 r_{00} \beta + 64nr_{00} r_0 \beta - 12nB^2 r_{00|0} \beta - 48Bnr_{00|0} \beta - 96s_0 nr_0 \beta^2 \\
& \quad - 96Bs_0 nr_0 \beta + 32Bnr_{00} r_0 \beta + 144Bs_0^m nr_{0m} \beta^2 + 144ns_0^m r_{0m} \beta^2 - 144ns_0^m s_m \beta^3 + 138s_0^2 \beta^2 + 6r_0^2 \beta^2 - 18r_{0|0} \beta^2 - 114s_0|0 \beta^2 \\
& \quad \left. + 72Bns_{0|0} \beta^2 + 72s_0^m \beta^3 + 10r_{00|0} \beta + 12B^4 r_{00}^2 - 8B^3 r_{00}^2 - 12B^2 r_{00}^2 - 165Br_{00}^2 + 2r_{00}^2 + 14Bs_0^m s_{0|m} \beta^3 \right) + 27\widehat{\text{Ric}}\beta^{16} (8B+7) \\
d_3 &:= -1296(n-1)s_0^2 \beta^{15}
\end{aligned}$$

$$\begin{aligned}
d_4 &:= -6\beta^{12} \left(-96Bs_0nr_{00}\beta + 32Bnr_{00}r_0\beta + 144Bs_0^mnr_{0m}\beta^2 + 212Bs_0r_{00}\beta - 165Br_{00}^2 - 68Br_{00}r_0\beta + 432Bs_0^ms_{0m}\beta^2 \right. \\
&\quad - 294Bs_0^mr_{0m}\beta^2 + 18Br_m^mr_{00}\beta^2 - 18Br_0^mr_{0m}\beta^2 + 18Bb^mr_{00|m}\beta^2 - 18Bb^mr_{0m|0}\beta^2 - 96ns_0r_0\beta^2 + 72Bs_0|0n\beta^2 - 12B^2nr_{00|0}\beta \\
&\quad - 48Bnr_{00|0}\beta - 240s_0nr_{00}\beta + 64nr_{00}r_0\beta + 144s_0^mn_{0m}\beta^2 - 144s_0^ms_m\beta^3 + 79Br_{00|0}\beta - 82r_{00}r_0\beta + 250s_0r_{00}\beta + 28B^2r_{00|0}\beta \\
&\quad + 180s_0r_0\beta^2 - 174Bs_0|0\beta^2 - 6rr_{00}\beta^2 + 144Bs_0^m\beta^3 - 54s_i^ms_i^m\beta^4 + 12B^4r_{00}^2 - 108s_0^ms_{0m}\beta^2 + 306s_0^ms_m\beta^3 + 18r_m^mr_{00}\beta^2 \\
&\quad - 18r_m^mr_{0m}\beta^2 + 18b^mr_{00|m}\beta^2 - 8B^3r_{00}^2 - 18b^mr_{0m|0}\beta^2 - 174s_0^mr_{0m}\beta^2 - 36\beta^3r_m^ms_0 + 54\beta^3r_m^ms_m + 36\beta^3s_0^mr_m - 36\beta^3b^ms_{0|m} \\
&\quad + 18\beta^3b^ms_{0|m} + 4B^2nr_{00}^2 + 52Bnr_{00}^2 + 72s_0|0n\beta^2 - 21nr_{00|0}\beta + 2r_{00}^2 + 138s_0^2\beta^2 + 6r_0^2\beta^2 - 114s_0|0\beta^2 + 72s_0^m\beta^3 + 10r_{00|0}\beta \\
&\quad \left. + 73nr_{00}^2 - 12B^2r_{00}^2 \right) - 216\widehat{\text{Ric}}\beta^{14}B(B+2) \\
d_5 &:= 432(n-1)(8B-5)s_0^2\beta^{13} \\
d_6 &:= -2\beta^{10} \left(-54\beta^3s_0|0 + 15Br_{00|0}\beta - 334s_0r_{00}\beta - 108\beta^3Bs_0|0 - 20B^3r_{00|0}\beta - 144B^2r_{00|0}\beta - 24Br_0^2\beta^2 + 30rr_{00}\beta^2 \right. \\
&\quad - 540s_0r_0\beta^2 - 10r_{00}r_0\beta + 216s_0r\beta^3 + 36B^2r_{0|0}\beta^2 + 90Br_{0|0}\beta^2 - 1296Bs_0^2\beta^2 + 372B^2s_0|0\beta^2 + 570Bs_0|0\beta^2 - 432B^2s_0^m\beta^3 \\
&\quad - 43s_0^m\beta^3 + 54s_i^ms_i^m\beta^4 + 216s^ms_m + s_0^ms_{0m}\beta^2 - 160B^2s_0r_{00}\beta + 80B^2r_{00}r_0\beta - 712Bs_0r_{00}\beta + 344Br_{00}r_0\beta - 864Bs_0r_0\beta^2 \\
&\quad - r_{00}^2 - 35nr_{00|0}\beta - 36ns_0|0\beta^2 - s_0^2\beta^2 - 288\beta^3s_0^ms_m + 24Brr_{00}\beta^2 + s_i^ms_i^m\beta^4 - 1296B^2s_0^ms_{0m}\beta^2 + 1296Bs_0^ms_{0m}\beta^2 \\
&\quad - 36B^2r_m^mr_{00} - 36B^2b^mr_{00|m}\beta^2 - r_m^mr_{00}\beta^2 - 90Bb^mr_{00|m}\beta^2 + s_0^mr_{0m}\beta^2 - 333Br_{00}^2 - 216\beta^3Bs_0^mr_m + 216\beta^3Br_m^mr_0 \\
&\quad + 216B^2s_0|0m - \beta^3Br_0^ms_m - B\beta^3s_0^ms_m + 18r_m^mr_0\beta^2 + 18b^mr_{00|m}\beta^2 - 300s_0^mr_{0m}\beta^2 + 108\beta^3r_m^ms_0 + 108\beta^3b^ms_{0|m} \\
&\quad - 108\beta^3r_0^ms_m - \beta^3s_0^mr_m + s_0^m\beta^2r_{0m} - 36nB^2r_{00}^2 - 204Bnr_{00}^2 + 736ns_0r_{00}\beta - 104nr_{00}r_0\beta + 480nBs_0^2\beta^2 - 144nB^2s_0|0\beta^2 \\
&\quad + 8r_{00|0}\beta + 84nB^2nr_{00|0}\beta + 78Bnr_{00|0}\beta + s_0nr_0\beta^2 - 360nBs_0|0\beta^2 - 288B^2s_0^mr_{0m}\beta^2 - 32nB^2nr_{00}r_0\beta + 640nBs_0r_{00}\beta \\
&\quad - r_{00}r_0\beta - 294r_{00}^2 + 384nBs_0r_0\beta^2 + s_0r_{00}\beta - 720Bs_0^mr_{0m}\beta^2 + 864nBs_0^ms_m\beta^3 - 72ns_0^mr_{0m}\beta^2 + 432ns_0^ms_m\beta^3 \\
&\quad + 1350s_0^2\beta^2 - 30r_0^2\beta^2 - 18r_{00|0}\beta^2 - 186s_0|0\beta^2 + 216s_0^m\beta^3 + 122r_{00|0}\beta + 12B^5r_{00}^2 - 30B^4r_{00}^2 + 12B^3r_{00}^2 + 264B^2r_{00}^2 \\
&\quad \left. + 24\widehat{\text{Ric}}\beta^{12}(B-1)(4B^2+19B+13) \right) \\
d_7 &:= -432(n-1)(8B^2-16B-1)s_0^2\beta^{11} \\
d_8 &:= 2\beta^8 \left(90\beta^3s_0|0 + 279Br_{00|0}\beta - 566s_0r_{00}\beta - 72\beta^3B^2s_0|0 - 72\beta^3Bs_0|0 - 28B^3r_{00|0}\beta + 60B^2r_{00|0}\beta - 8B^2r_0^2\beta^2 - 32Br_0^2\beta^2 \right. \\
&\quad + 564s_0r_0\beta^2 - 242r_{00}r_0\beta + 144s_0r\beta^3 + 8B^3r_{0|0}\beta^2 + 48B^2r_{0|0}\beta^2 - 66Br_{0|0}\beta^2 - 1208B^2s_0^2\beta^2 + 2512Bs_0^2\beta^2 + 88B^3s_0|0\beta^2 \\
&\quad - 606Bs_0|0\beta^2 - 192B^3s_0^m\beta^3 - 288B^2s_0^m\beta^3 + 720Bs_0^m\beta^3 - 270s_i^ms_i^m\beta^4 + 36s_0^ms_{0m}\beta^2 - 80B^2s_0r_{00}\beta + 112B^2r_{00}r_0\beta \\
&\quad - r_{00}r_0\beta - s_0r_0\beta^2 - 624Bs_0r_0\beta^2 - 99nr_{00}^2 - r_{00|0}\beta + 176ns_0|0\beta^2 - 928ns_0^2\beta^2 + 960\beta^3s_0^ms_m + 288s_0r\beta^3 + 8B^2rr_{00}\beta^2 \\
&\quad + 32Brr_{00}\beta^2 + 432B^2s_i^ms_i^m\beta^4 + 432Bs^ms_m\beta^4 - 576B^3s_0^ms_{0m}\beta^2 + 1728B^2s_0^ms_{0m}\beta^2 + 432Bs_0^ms_{0m}\beta^2 - 48B^2r_m^mr_{00}\beta^2 \\
&\quad - 48B^2b^mr_{00|m}\beta^2 + 66Br_m^mr_{00}\beta^2 + 66Bb^mr_{00|m}\beta^2 - 948Bs_0^mr_{0m}\beta^2 - 8B^3r_m^mr_{00}\beta^2 - 8B^3b^mr_{00|m}\beta^2 + 144B^3s_0^mr_{0m}\beta^2 \\
&\quad - 144\beta^3Bs_0^mr_m + 144\beta^3B^2r_m^ms_0 + 144\beta^3B^2b^mr_{0|m} - 144\beta^3B^2r_0^ms_m - 1344\beta^3B^2s_0^ms_m - 144\beta^3B^2s_0^mr_m + 144\beta^3Br_m^ms_0 + 90r_{00}^2 \\
&\quad + 144\beta^3Bb^ms_0|m - 144\beta^3Br_0^ms_m - 48\beta^3Bs_0^ms_m + 98r_m^mr_0\beta^2 + 98b^mr_{00|m}\beta^2 - 448s_0^mr_{0m}\beta^2 - 60B^3r_{00}^2 - 12B^2r_{00}^2 - 513Br_{00}^2 \\
&\quad - 180\beta^3b^ms_{0|m} + 180\beta^3r_0^ms_m + 180\beta^3s_0^mr_m + 384B^2s_0^2\beta^2r_{0m} - 48B^2nr_{00}^2 - 168Br_{00}^2 + 608ns_0r_{00}\beta + 128nr_{00}r_0\beta + 78B^4r_{00}^2 \\
&\quad - 1600nBs_0^2\beta^2 + 512nB^2s_0^2\beta^2 - 32nB^3s_0|0\beta^2 - 192B^2ns_0|0\beta^2 + 16B^3nr_{00|0}\beta + 24B^2nr_{00|0}\beta - 96Bnr_{00|0}\beta - 64ns_0|r_0\beta^2 \\
&\quad + 48Bns_0|0\beta^2 - 384B^2s_0^mr_{0m}\beta^2 - 64B^2nr_{00}r_0\beta + 704nBs_0r_{00}\beta - 64Bnr_{00}r_0\beta + 94r_{00|0}\beta + 128nB^2s_0r_0\beta^2 + 408s_0^m\beta^3 \\
&\quad + 512nBs_0r_0\beta^2 + 128nB^2s_0r_{00}\beta + 96nBs_0^mr_{0m}\beta^2 + 76nB^2s_0^ms_m\beta^3 + 576nBs_0^ms_m\beta^3 - 64nB^3s_0^mr_{0m}\beta^2 + 352ns_0^mr_{0m}\beta^2 \\
&\quad + 1306s_0^2\beta^2 + 22r_0^2\beta^2 - 98r_{00|0}\beta^2 - 418s_0|0\beta^2 - 22r_{00}\beta^2 + 288B^2s_0|0\beta^2 - 180\beta^3r_m^ms_0 - 20Bs_0r_{00}\beta + 12B^5r_{00}^2 - 288ns_0^ms_m\beta^3 \\
&\quad \left. - 2\widehat{\text{Ric}}\beta^{10}(8B^4+64B^3-132B^2-392B-115) \right) \\
d_9 &:= 48(n-1)(32B^3-168B^2+24B+31)s_0^2\beta^9 \\
d_{10} &:= 2\beta^6 \left(-282r_{00}^2 + 840B^2s_0^m\beta^2r_{0m} - 686s_0^2\beta^2 - 58r_0^2\beta^2 + 44r_{0|0}\beta^2 - 106\beta^3s_0|0 - 52s_0^m\beta^3 + 100s_0|0\beta^2 + 24\beta^3B^2s_0|0 \right. \\
&\quad - 204\beta^3Bs_0|0 - 24B^3r_{00|0}\beta - 192B^2r_{00|0}\beta - 129Br_{00|0}\beta + 690s_0r_{00}\beta + 102r_{00}r_0\beta - 1832B^2s_0^2\beta^2 - 1280Bs_0^2\beta^2 + 8B^2r_0^2\beta^2 \\
&\quad + 540B^2s_0|0\beta^2 + 750Bs_0|0\beta^2 - 40Br_0^2\beta^2 + 58rr_{00}\beta^2 - 564s_0r_0\beta^2 + 360s^ms_m\beta^4 + 138s_i^ms_i^m\beta^4 + 24s_0^ms_{0m}\beta^2 + 408s_0r\beta^3 \\
&\quad + 60B^2r_{0|0}\beta^2 + 174Br_{0|0}\beta^2 + 64B^3s_0^m\beta^3 - 816B^2s_0^m\beta^3 - 848Bs_0^m\beta^3 - 44r_m^mr_{00}\beta^2 - 44b^mr_{00|m}\beta^2 + 24s_0^mr_{0m}\beta^2 \right)
\end{aligned}$$

$$\begin{aligned}
& -212\beta^3 r_0^m s_m - 212\beta^3 s_0^m r_m + 288B^3 s_0^2 \beta^2 - 140ns_0|_0 \beta^2 + 672ns_0^2 \beta^2 + 45nr_{00|0} \beta + 16\beta^3 B^3 s_{|0} + 57nr_{00}^2 + 32B^4 s_{0|m}^m \beta^3 \\
& + 64nB^3 s_0^m r_{0m} \beta^2 + 212\beta^3 b^m s_{0|m} - 192nB^2 s_0^m r_{0m} \beta^2 - 96nB^2 s_{0|0} \beta^2 - 32\beta^3 B^3 r_m^m s_0 + 32\beta^3 B^3 r_0^m s_m - 96B^3 s_0^m s_{0m} \beta^2 \\
& - 672nBs_0^m r_{0m} \beta^2 + 120nB^2 r_{00}^2 + 48nB^2 r_{00}^2 + 126nBr_{00|0} \beta - 480ns_0 r_{00} \beta - 168nr_{00} r_0 \beta + 1024nB^2 s_0^2 \beta^2 + 1024nBs_0^2 \beta^2 \\
& - 128nB^3 s_0^2 \beta^2 - 96B^2 s_0 r_0 \beta^3 + 448ns_0 r_0 \beta^2 + 72nB^2 nr_{00|0} \beta - 336Bs_{0|0} \beta^2 - 32\beta^3 B^3 b^m s_{0|m} + 416ns_0^m s_m \beta^3 - 280ns_0^m r_{0m} \beta^2 \\
& + 96B^2 r_{00} r_0 \beta + 624Bs_0 r_{00} \beta + 432Br_{00} r_0 \beta - r_{00} \beta^2 + 40Br_{00} \beta^2 + s_0 r_0 \beta^2 - s_{0|r} \beta^2 - 96Bs_0 r \beta^3 + 96B^4 s_0^m s_{0m} \beta^2 \\
& + 144B^2 s_0^m s_{0m} \beta^2 + 48Bs_0^m s_{0m} \beta^2 - 60B^4 r_{00}^2 + 320B^3 s_0^m s_m \beta^3 + 8B^3 r_m^m r_{00} \beta^2 + 8B^3 b^m r_{00|m} \beta^2 - 192B^3 s_m^i s_i^m \beta^4 + 24B^3 r_{00}^2 \\
& + 144Bg^m s_m \beta^4 + 720Bs_0^i s_i^m \beta^4 + 192B^2 r_{00}^2 + 48\beta^3 B^2 s_0^m r_m - 60B^2 r_m^m r_{00} \beta^2 - 60B^2 b^m r_{00|m} \beta^2 - 174Br_m^m r_{00} \beta^2 - 321Br_{00}^2 \\
& + 804Bs_0^m r_{0m} \beta^2 - 48\beta^3 B^2 r_m^m s_0 - 48\beta^3 B^2 s_{0|m} + 48\beta^3 B^2 r_0^m s_m - 384\beta^3 B^2 s_0^m s_m - 48B^3 s_0^m r_{0m} \beta^2 + 408\beta^3 Br_m^m s_0 \\
& - 408\beta^3 Br_0^m s_m - 2208\beta^3 Bs_0^m s_m - 408\beta^3 Bs_0^m r_m - 6r_{00|0} \beta + 24B^5 r_{00}^2 - 174B^2 b^m r_{00|m} \beta^2 - 288B^2 s^m s_m \beta^4 + 144B^2 s_m^i s_i^m \beta^4 \\
& - 40B^3 s_{0|0} \beta^2 - 8B^3 r_{0|0} \beta^2 + 212\beta^3 r_m^m s_0 - 320\beta^3 s_0^m s_m + 32\beta^3 B^3 s_0^m r_m + 408\beta^3 Bb^m s_{0|m} \\
& + 2\widehat{\text{Ric}}\beta^8 (8B^4 - 80B^3 - 348B^2 - 176B + 29)
\end{aligned}$$

$$d_{11} := -16(2B+1) (8B^3 - 132B^2 + 186B + 19) (n-1) s_0^2 \beta^7$$

$$d_{12} := -2\beta^4 (-102r_{00}^2 - 102s_0^2 \beta^2 - 10r_0^2 \beta^2 + 18\beta^3 s_{0|0} + 168s_0^m s_m \beta^3 - 34r_{0|0} \beta^2 - 66s_{0|0} \beta^2 - 144\beta^3 B^2 s_{|0} - 144\beta^3 Bs_{|0} \\
- 40B^3 r_{00|0} \beta - 36B^2 r_{00|0} \beta + 39Br_{00|0} \beta + 202s_0 r_{00} \beta - 2r_{00} r_0 \beta - 272B^2 s_0^2 \beta^2 - 608Bs_0^2 \beta^2 - 16B^2 r_0^2 \beta^2 + 144B^3 s_{0|0} \beta^2 \\
+ 384B^2 s_{0|0} \beta^2 - 30Bs_{0|0} \beta^2 - 64Br_0^2 \beta^2 + 10rr_{00} \beta^2 + 20s_0 r_0 \beta^2 + 64s^m s_m \beta^4 - 86s_m^i s_i^m \beta^4 + 84s_0^m s_{0m} \beta^2 + 288s_0 r \beta^3 \\
+ 16B^3 r_{0|0} \beta^2 + r_{0|0} \beta^2 + r_{0|0} \beta^2 + s_{0|m} \beta^3 + 34r_m^m r_{00} \beta^2 + 34b^m r_{00|m} \beta^2 + 4s_0^m r_{0m} \beta^2 - 36\beta^3 r_m^m s_0 - 36\beta^3 b^m s_{0|m} \\
+ 36\beta^3 r_0^m s_m + 36\beta^3 s_0^m r_m + 384B^2 s_0^m \beta^2 r_{0m} + 35ns_0^2 \beta^2 + 11nr_{00|0} \beta + 21nr_{00}^2 + 96\beta^3 s_0^m s_m + 352B^2 s_0 r_{00} \beta + 160B^2 r_{00} r_0 \beta \\
+ 868Bs_0 r_{00} \beta + 76Br_{00} r_0 \beta + 16B^2 r_{00} \beta^2 + 64Br_{00} \beta^2 - 544B^2 s_0 r_0 \beta^2 - 736Bs_0 r_0 \beta^2 + 576Bs_0 r \beta^3 - 192B^4 s_0^m s_{0m} \beta^2 \\
+ 384B^3 s_0^m s_{0m} \beta^2 + s_0^m s_{0m} \beta^2 - s_0^m s_m \beta^3 - r_m^m r_{00} \beta^2 - 512B^3 s_0^2 \beta^2 - 40ns_{0|0} \beta^2 - 384B^3 s_{0|m} \beta^3 - 576B^2 s_{0|m} \beta^3 \\
- 16B^3 b^m r_{00|m} \beta^2 - 32B^4 s_m^i s_i^m \beta^4 - 64nB^3 s_0^m r_{0m} \beta^2 - 256nB^2 s_0 r_{00} \beta - 64B^3 s_m^m s_m \beta^4 + 128B^3 s_m^i s_i^m \beta^4 + 192B^2 s^m s_m \beta^4 \\
+ 672B^2 s_m^i s_i^m \beta^4 + s^m s_m \beta^4 + 128Bs_0^i s_i^m \beta^4 - \beta^3 B^2 s_0^m r_m - r_m^m r_{00} \beta^2 - 48B^3 r_{00}^2 - 30Br_m^m r_{00} \beta^2 - 30Bb^m r_{00|m} \beta^2 \\
- 180Bs_0^m r_{0m} \beta^2 + 288\beta^3 B^2 r_m^m s_0 + 288\beta^3 B^2 b^m s_{0|m} - 288\beta^3 B^2 r_0^m s_m - 1536\beta^3 B^2 s_0^m s_m - 160Bnr_{00} r_0 \beta + 128nB^2 s_0 r_0 \beta^2 \\
+ 512nBs_0 r_{00} \beta^2 - 384nB^2 s_0^m r_{0m} \beta^2 - 336nBs_0^m r_{0m} \beta^2 + 576nB^2 s_0^m s_m \beta^3 + 576nBs_0^m s_m n \beta^3 + 224ns_0 r_0 \beta^2 + 36B^2 r_{00}^2 + 60nBr_{00}^2 \\
- 64nB^2 r_{00} r_0 \beta - 544nBs_0 r_{00} \beta - 208ns_0 r_{00} \beta + 48nBr_{00|0} \beta - 96\beta^3 Bs_0^m s_m - 288\beta^3 Bs_0^m r_m - 80s_0^m r_{0m} \beta^2 + 144ns_0^m s_m \beta^3 \\
+ 224B^3 s_0^m r_{0m} \beta^2 + 288\beta^3 Br_m^m s_0 + 288\beta^3 Bb^m s_{0|m} - 288\beta^3 Br_m^m s_m + 10r_{00|0} \beta + 24B^5 r_{00}^2 - 84B^2 r_{00}^2 - 405Br_{00}^2 + 48B^4 r_{00}^2 \\
- 64nr_{00} r_0 \beta + 128nB^2 s_0^2 \beta^2 + 704nBs_0^2 \beta^2 + 16nB^3 r_{00|0} \beta + 60B^2 nr_{00|0} \beta - 192nB^2 s_{0|0} \beta^2 - 32nB^3 s_{0|0} \beta^2 - 168Bns_0|0 \beta^2 \\
+ 256nB^3 s_0^2 \beta^2 - 96B^2 b^m r_{00|m} \beta^2) + 8\widehat{\text{Ric}}\beta^6 (2B+1)(B-1)(2B^2 + 17B + 17)$$

$$d_{13} := -48(n-1) (2B+1)^2 (4B^2 - 20B + 7) s_0^2 \beta^5$$

$$d_{14} := -2\beta^2 (6r_{00}^2 + 264B^2 s_0^m \beta^2 r_{0m} - 12nB^2 r_{00}^2 - 12Bnr_{00}^2 + 32ns_0 r_{00} \beta + 8nr_{00} r_0 \beta - 128nB^2 s_0^2 \beta^2 - 224nBs_0^2 \beta^2 + 32\beta^3 B^3 s_{|0} \\
- 64ns_0^2 \beta^2 + 24nBs_0|0 \beta^2 - 32ns_0 r_0 \beta^2 + 128nB^3 s_0^2 \beta^2 - 192B^2 s_0 r \beta^3 + 42s_{0|0} \beta^2 - 50\beta^3 s_{|0} - 104s_0^m s_m \beta^3 + 18s_0^2 \beta^2 - 26r_0^2 \beta^2 \\
+ 48\beta^3 B^2 s_{|0} - s_{|0} - 4B^3 r_{00|0} \beta - 4B^2 r_{00|0} \beta - 27Br_{00|0} \beta - 26s_0 r_0 \beta - 14r_{00} r_0 \beta + 144B^2 s_0^2 \beta^2 + 48Bs_0^2 \beta^2 + 16B^2 r_0^2 \beta^2 \\
- 48B^3 s_{0|0} \beta^2 + 156B^2 s_{0|0} \beta^2 + 174Bs_0|0 \beta^2 - 8Br_0^2 \beta^2 + 26rr_{00} \beta^2 - 20s_0 r_0 \beta^2 + 136s^m s_m \beta^4 + 82s_m^i s_i^m \beta^4 - 68s_0^m s_{0m} \beta^2 \\
- 16B^3 r_{0|0} \beta^2 + 12B^2 r_{0|0} \beta^2 + 78Br_{0|0} \beta^2 + 128B^3 s_0^m s_m \beta^3 - 336B^2 s_0^m s_m \beta^3 - 400Bs_0^m s_m \beta^3 - 34r_m^m r_{00} \beta^2 - 34b^m r_{00|m} \beta^2 \\
+ 100\beta^3 b^m s_{0|m} - r_m^m s_m - 100\beta^3 s_0^m r_m - s_0^m s_m \beta^3 + 128nB^2 s_0 r_{00} \beta + 32nB^2 r_{00} r_0 \beta + 128nBs_0 r_{00} \beta + 32Bnr_{00} r_0 \beta \\
- 192nB^2 s_0^m s_m \beta^3 - 96nBs_0^m s_m \beta^3 + 64nB^3 s_0^m r_{0m} \beta^2 + 96nB^2 s_0^m r_{0m} \beta^2 + 48nBs_0^m r_{0m} \beta^2 - 8nB^3 r_{00|0} \beta - 12nB^2 r_{00|0} \beta \\
+ 48nB^2 s_{0|0} \beta^2 - 6Bnr_{00|0} \beta - 64\beta^3 B^3 b^m s_{0|m} - 16ns_0^m s_m \beta^3 + 8ns_0^m r_{0m} \beta^2 + 64\beta^3 B^3 s_0^m r_m + 64\beta^3 B^3 r_0^m s_m - 64\beta^3 B^3 r_m^m s_0 \\
- s_0 r_0 \beta + 16B^2 r_{00} r_0 \beta - s_0 r_0 \beta + 88Br_{00} r_0 \beta - 16B^2 rr_{00} \beta^2 + 8Br_{00} \beta^2 + 160B^2 s_0 r_0 \beta^2 - 320Bs_0 r_0 \beta^2 - 192Bs_0 r \beta^3 \\
- 128B^4 s_0^m s_{0m} \beta^2 + 128B^3 s_0^m s_{0m} \beta^2 - 48B^2 s_0^m s_{0m} \beta^2 - 208Bs_0^m s_{0m} \beta^2 + 320B^3 s_0^m s_m \beta^3 + 16B^3 r_m^m r_{00} \beta^2 + 16B^3 b^m r_{00|m} \beta^2 \\
- s^m s_m \beta^4 - s_m^i s_i^m \beta^4 - s^m s_m \beta^4 + 96B^2 s_m^i s_i^m \beta^4 + 96Bs_0^m s_m \beta^4 + 272Bs_0^i s_i^m \beta^4 + 96\beta^3 B^2 s_0^m r_m - 12B^2 r_m^m r_{00} \beta^2 \\
- s_m^i s_i^m \beta^4 - 128nB^2 s_0 r_0 \beta^2 - s_0 r_0 \beta^2 - b^m r_{00|m} \beta^2 - 78Br_m^m r_{00} \beta^2 - 3nr_{00}^2 + 4ns_0|0 \beta^2 - 192B^3 s_0^2 \beta^2 - nr_{00|0} \beta^2 \\
- 78Bb^m r_{00|m} \beta^2 + 36Bs_0^m r_{0m} \beta^2 - 96\beta^3 B^2 r_m^m s_0 - 96\beta^3 B^2 b^m s_{0|m} + 96\beta^3 B^2 r_0^m s_m - 384\beta^3 B^2 s_0^m s_m - 32B^3 s_0^m r_{0m} \beta^2 \\
+ 168\beta^3 Br_m^m s_0 + 168\beta^3 Bb^m s_{0|m} - 168\beta^3 Br_0^m s_m - 336\beta^3 Bs_0^m s_m - 168\beta^3 Bs_0^m r_m - 2r_{00|0} \beta + 12B^5 r_{00}^2 - 30B^2 r_{00}^2 + 12B^3 r_{00}^2 \\
+ 96B^2 r_{00}^2 + 57Br_{00}^2 + 32nB^3 s_{0|0} \beta^2 + 168s_0 r \beta^3 + 34r_{0|0} \beta^2 - 52s_0^m r_{0m} \beta^2) - 8\widehat{\text{Ric}}\beta^4 (2B+1)^2 (B+2) (B-4)$$

$$\begin{aligned}
d_{15} &:= -48(n-1)(2B-3)(2B+1)^3 s_0^2 \beta^3 \\
d_{16} &:= 2(2B+1) \left(40B^2 s_0^m \beta^2 r_{0m} + 30s_0^2 \beta^2 - 14r_0^2 \beta^2 + 10r_{0|0} \beta^2 - 18\beta^3 s_{|0} - 24s_0^m \beta^3 + 10s_{0|0} \beta^2 - 36\beta^3 B s_{|0} - 6B^2 r_{00|0} \beta \right. \\
&\quad \left. - 18s_0 r_{00} \beta - 6r_{00} r_0 \beta - 108B s_0^2 \beta^2 + 28B^2 s_{0|0} \beta^2 + 34Bs_{0|0} \beta^2 - 4Br_0^2 \beta^2 + 14rr_{00} \beta^2 - 4s_0 r_0 \beta^2 + 64s^m s_m \beta^4 + 22s_i^i s_i^m \beta^4 \right. \\
&\quad \left. - 20s_0^m s_{0m} \beta^2 + 144s_0 r \beta^3 + 4B^2 r_{0|0} \beta^2 + 22Br_{0|0} \beta^2 - 96B s_{0|m}^m \beta^3 - 96Bs_0^m s_m \beta^3 - 10r_{0|m}^m r_{00} \beta^2 - 10b^m r_{00|m} \beta^2 - 20s_0^m r_{0m} \beta^2 \right. \\
&\quad \left. - 36\beta^3 r_0^m s_m - 36\beta^3 s_0^m r_m + 24Bs_0 r_{00} \beta + 24Br_{00} r_0 \beta + 4Br_{00} r_0 \beta^2 - 104Bs_0 r_0 \beta^2 + 32B^3 s_0^m s_{0m} \beta^2 - 48B^2 s_0^m s_{0m} \beta^2 \right. \\
&\quad \left. - 16B^3 s_i^i s_i^m \beta^4 - 32B^2 s^m s_m \beta^4 + 72B^2 s_i^i s_i^m \beta^4 + 112Bs^m s_m \beta^4 + 84Bs_i^i s_i^m \beta^4 - 4B^2 r_m^m r_{00} \beta^2 - 4B^2 b^m r_{00|m} \beta^2 - 22Br_m^m r_{00} \beta^2 \right. \\
&\quad \left. - 22Bb^m r_{00|m} \beta^2 - 20Bs_0^m r_{0m} \beta^2 + 36\beta^3 r_m^m s_0 + 36\beta^3 b^m s_{0|m} - 96\beta^3 B^2 s_0^m s_m + 72\beta^3 Br_m^m s_0 + 72\beta^3 Bb^m s_{0|m} - 72\beta^3 Br_0^m s_m \right. \\
&\quad \left. - 48\beta^3 B s_0^m s_m - 72\beta^3 B s_0^m r_m + 6B^4 r_{00}^2 - 6B^2 r_{00}^2 - 3Br_{00}^2 - 3Br_{00|0} \beta - 72Bs_0^m s_{0m} \beta^2 \right) - \widehat{\text{Ric}} \beta^2 (2B+1)^3 (2B+13) \\
d_{17} &:= -16(n-1)(2B+1)^4 s_0^2 \beta \\
d_{18} &:= 2(2B+1)^2 \left((2B+1) \left[-8\beta^2 s^m s_m - 4\beta s_0 r_m^m + 4\beta r_m s_0^m + 4\beta s_m r_0^m - 4\beta b^m s_{0|m} - 2\beta^2 s_i^i s_i^m + 2\beta s_{0|m}^m + 2s_0^m s_{0m} \right. \right. \\
&\quad \left. \left. + 2s_0^m r_{0m} + 2\beta s_{|0} + r_{00|m} b^m - s_{0|0} - r_{0|0} \right] + 2(4B+3)s_0^2 - 24\beta r s_0 + 4r_0 s_0 - 2rr_{00} + 2r_0^2 \right) + \widehat{\text{Ric}} (2B+1)^4
\end{aligned}$$

9. Appendix 5

$$\begin{aligned}
d'_2 &:= -432(n-1)(8B-5)s_0^2 \beta^{13} \\
d'_4 &:= 432(n-1)(8B^2-16B-1)s_0^2 \beta^{11} \\
d'_6 &:= -48(n-1)(32B^3-168B^2+24B+31)s_0^2 \beta^9 \\
d'_8 &:= 16(n-1)(2B+1)(8B^3-132B^2+186B+19)s_0^2 \beta^7 \\
d'_{10} &:= 48(n-1)(2B+1)^2(4B^2-20B+7)s_0^2 \beta^5 \\
d'_{12} &:= 48(n-1)(2B-3)(2B+1)^3 s_0^2 \beta^3
\end{aligned}$$

10. Appendix 6

$$\begin{aligned}
d''_2 &:= 6\beta^{14} \left(18\beta^3 s_{|0} + 79Br_{00|0} \beta + 28B^2 r_{00|0} \beta - 6rr_{00} \beta^2 - 82r_{00} r_0 \beta - 18Br_{0|0} \beta^2 + 14Bs_0^m s_m^i \beta^3 - 54s_i^m s_m^i \beta^4 - 108s_0^m s_{0m} \beta^2 \right. \\
&\quad \left. - 68Br_{00} r_0 \beta + 73nr_{00}^2 - 21nr_{00|0} \beta + 288\beta^3 s_0^m s_m + 432Bs_0^m s_{0m} \beta^2 + 18Br_m^m r_{00} \beta^2 + 18Bb^m r_{00|m} \beta^2 - 276Bs_0^m r_{0m} \beta^2 \right. \\
&\quad \left. + 18r_m^m r_{00} \beta^2 + 18b^m r_{00|m} \beta^2 - 156s_0^m r_{0m} \beta^2 + 36\beta^3 s_0^m r_m + 4B^2 nr_{00}^2 + 52Bnr_{00}^2 + r_{00} r_0 \beta - 12nB^2 r_{00|0} \beta - 48Bnr_{00|0} \beta \right. \\
&\quad \left. + 32nBr_{00} r_0 \beta + s_0^m r_{0m} \beta^2 + s_0^m r_{0m} \beta^2 - 144ns_0^m s_m \beta^3 + 6r_0^2 \beta^2 - 18r_{0|0} \beta^2 + 72s_0^m \beta^3 + 10r_{00|0} \beta + 12B^4 r_{00}^2 - 8B^3 r_{00}^2 \right. \\
&\quad \left. - 12B^2 r_{00}^2 - 165Br_{00}^2 + 2r_{00}^2 \right) + 27\widehat{\text{Ric}} \beta^{16} (8B+7) \\
d''_4 &:= -6\beta^{12} \left(+ 32Bnr_{00} r_0 \beta + 144ns_0^m r_{0m} \beta^2 - 165Br_{00}^2 - 68Br_{00} r_0 \beta + 432Bs_0^m s_{0m} \beta^2 - 294Bs_0^m r_{0m} \beta^2 + 18Br_m^m r_{00} \beta^2 \right. \\
&\quad \left. - 18Br_m^m r_{0m} \beta^2 + 18Bb^m r_{00|m} \beta^2 - 18Bb^m r_{0m|0} \beta^2 - 12nB^2 r_{00|0} \beta - 48nBr_{00|0} \beta + 64nr_{00} r_0 \beta + 144ns_0^m r_{0m} \beta^2 + 79Br_{00|0} \beta \right. \\
&\quad \left. + 28B^2 r_{00|0} \beta - 6rr_{00} \beta^2 + 144Bs_0^m s_m^i \beta^3 - 54s_i^m s_m^i \beta^4 + 12B^4 r_{00}^2 - 108s_0^m s_{0m} \beta^2 + 18r_m^m r_{00} \beta^2 - 18r_0^m r_{0m} \beta^2 + 18b^m r_{00|m} \beta^2 \right. \\
&\quad \left. - 8B^3 r_{00}^2 - 18b^m r_{0m|0} \beta^2 - 174s_0^m r_{0m} \beta^2 + 54\beta^3 r_0^m s_m + 36\beta^3 s_0^m r_m + 4B^2 nr_{00}^2 + 52Bnr_{00}^2 - 21nr_{00|0} \beta + 2r_{00}^2 + 72s_0^m \beta^3 \right. \\
&\quad \left. + 10r_{00|0} \beta + 73nr_{00}^2 - 12B^2 r_{00}^2 \right) - 216\widehat{\text{Ric}} \beta^{14} B (B+2) \\
d''_6 &:= -2\beta^{10} \left(- 54\beta^3 s_{|0} + 15Br_{00|0} \beta - 108\beta^3 Bs_{|0} - 20B^2 r_{00|0} \beta - 144B^2 r_{00|0} \beta - 24Br_0^2 \beta^2 + 30rr_{00} \beta^2 - 10r_{00} r_0 \beta \right. \\
&\quad \left. + r_{0|0} \beta^2 + 90Br_0|0 \beta^2 - s_0^m s_m^i \beta^3 - 432Bs_0^m s_m^i \beta^3 + 54s_i^m s_m^i \beta^4 + 324s_0^m s_{0m} \beta^2 + 80B^2 r_{00} r_0 \beta + 344Br_{00} r_0 \beta - 165nr_0^2 \right. \\
&\quad \left. - 35nr_{00|0} \beta + 24Br_{00} \beta^2 + 432Bs_i^i s_m^i \beta^4 - 1296B^2 s_0^m s_{0m} \beta^2 + 1296Bs_0^m s_{0m} \beta^2 - 36B^2 r_m^m r_{00} \beta^2 - 36B^2 b^m r_{00|m} \beta^2 \right. \\
&\quad \left. - 90Br_m^m r_{00} \beta^2 - 90Bb^m r_{00|m} \beta^2 + 780Bs_0^m r_{0m} \beta^2 - 333Br_{00}^2 - 216\beta^3 Bs_0^m r_m + 18r_m^m r_{00} \beta^2 + 18b^m r_{00|m} \beta^2 - 300s_0^m r_{0m} \beta^2 \right)
\end{aligned}$$

$$\begin{aligned}
& +600 B^2 s_0^m \beta^2 r_{0m} - 36 B^2 n r_{00}^2 - 204 B n r_{00}^2 - 104 n r_{00} r_0 \beta + 8 n B^3 r_{00|0} \beta + 84 n B^2 r_{00|0} \beta + 78 B n r_{00|0} \beta - 288 n B^2 s_0^m r_{0m} \beta^2 \\
& - 32 n B^2 r_{00} r_0 \beta - 224 B n r_{00} r_0 \beta - 294 r_{00}^2 - 720 n B s_0^m r_{0m} \beta^2 + 864 n B s_0^m s_m \beta^3 - 72 n s_0^m r_{0m} \beta^2 - 30 r_0^2 \beta^2 - 18 r_{0|0} \beta^2 + 216 s_0^m |m \beta^3 \\
& + 122 r_{00|0} \beta + 12 B^5 r_{00}^2 - 30 B^4 r_{00}^2 + 12 B^3 r_{00}^2 + 264 B^2 r_{00}^2) + 24 \widehat{\text{Ric}} \beta^{12} (B - 1) (4 B^2 + 19 B + 13) \\
d''_8 & := 2 \beta^8 (90 \beta^3 s_{|0} + 279 B r_{00|0} \beta - 72 \beta^3 B s_{|0} - 72 B^3 s_{|0} - 28 B^3 r_{00|0} \beta + 60 B^2 r_{00|0} \beta - 8 B^2 r_0^2 \beta^2 - 32 B r_0^2 \beta^2 - 22 r r_{00} \beta^2 \\
& + 8 B^3 r_{0|0} \beta^2 + r_{0|0} \beta^2 - 66 B r_{0|0} \beta^2 - 192 B^3 s_0^m |m \beta^3 - 288 B^2 s_0^m |m \beta^3 + 720 B s_0^m |m \beta^3 - 27 s_i^m s_m \beta^4 + 36 s_0^m s_{0m} \beta^2 + 112 B^2 r_{00} r_0 \beta \\
& - 140 B r_{00} r_0 \beta - 99 n r_{00}^2 - 79 n r_{00|0} \beta + 8 B^2 r r_{00} \beta^2 + 32 B r_{00} \beta^2 + 432 B^2 s_i^m s_m \beta^4 - 576 B^3 s_0^m s_{0m} \beta^2 + 1728 B^2 s_0^m s_{0m} \beta^2 \\
& + 432 B s_0^m s_{0m} \beta^2 - 48 B^2 r_0^2 m r_0 \beta^2 - 48 B^2 b^m r_{00|m} \beta^2 + 66 B r_0^2 m r_0 \beta^2 + 66 B b^m r_{00|m} \beta^2 - 948 B s_0^m r_{0m} \beta^2 - 8 B^3 r_m^2 r_{00} \beta^2 \\
& - 8 B^3 b^m r_{00|m} \beta^2 + 144 B^3 s_0^m r_{0m} \beta^2 - 144 \beta^3 B^2 s_0^m r_m + 90 r_0^2 + 98 r_m^2 r_{00} \beta^2 + 98 b^m r_{00|m} \beta^2 - 444 s_0^m r_{0m} \beta^2 \\
& - 60 B^3 r_{00}^2 - 12 B^2 r_{00}^2 - r_{00}^2 + 180 \beta^3 s_0^m r_m + 384 B^2 s_0^m \beta^2 r_{0m} - r_{00}^2 - 168 B n r_{00}^2 + 128 n r_{00} r_0 \beta + 78 B^4 r_{00}^2 + 16 n B^3 r_{00|0} \beta \\
& + 24 B^2 n r_{00|0} \beta - 96 B n r_{00|0} \beta - 384 B^2 s_0^m n r_{0m} \beta^2 - 64 B^2 n r_{00} r_0 \beta - 64 B n r_{00} r_0 \beta + 94 r_{00|0} \beta + 408 s_0^m |m \beta^3 + 12 B^5 r_{00}^2 \\
& + 96 n B s_0^m r_{0m} \beta^2 - 64 n B^3 s_0^m r_{0m} \beta^2 + 352 n s_0^m r_{0m} \beta^2 + 22 r_0^2 \beta^2 - 98 r_{0|0} \beta^2) - 2 \widehat{\text{Ric}} \beta^{10} (8 B^4 + 64 B^3 - 132 B^2 - 392 B - 115) \\
d''_{10} & := 2 \beta^6 (- 282 r_{00}^2 + 840 B^2 s_0^m \beta^2 r_{0m} - 58 r_0^2 \beta^2 + 44 r_{0|0} \beta^2 - 106 \beta^3 s_{|0} - 52 s_0^m |m \beta^3 + 24 \beta^3 B^2 s_{|0} - 204 \beta^3 B s_{|0} - 24 B^3 r_{00|0} \beta \\
& - 192 B^2 r_{00|0} \beta - 129 B r_{00|0} \beta + 69 s_0^m r_{00} \beta + 102 r_{00} r_0 \beta + 8 B^2 r_0^2 \beta^2 - 40 B r_0^2 \beta^2 + 58 r_{00} \beta^2 + 138 s_i^m s_m \beta^4 + 24 s_0^m s_{0m} \beta^2 - 8 B^3 r_{0|0} \beta^2 \\
& + 60 B^2 r_{0|0} \beta^2 + 174 B r_{0|0} \beta^2 + 64 B^3 s_0^m |m \beta^3 - 816 B^2 s_0^m |m \beta^3 - 848 B s_0^m |m \beta^3 - 44 r_m^2 r_{00} \beta^2 - 44 b^m r_{00|m} \beta^2 + 24 s_0^m r_{0m} \beta^2 \\
& - 212 \beta^3 s_0^m r_m + 45 n r_{00|0} \beta + 16 \beta^3 B^3 s_{|0} + 57 n r_{00}^2 + 32 B^4 s_0^m |m \beta^3 + 64 n B^3 s_0^m r_{0m} \beta^2 + 212 \beta^3 b^m s_0^m |m - 192 n B^2 s_0^m r_{0m} \beta^2 \\
& - 96 B^3 s_0^m s_{0m} \beta^2 + 32 \beta^3 B^3 s_0^m r_m - 672 n B s_0^m r_{0m} \beta^2 + 120 n B r_{00}^2 + 48 n B^2 r_{00}^2 + 126 n B r_{00|0} \beta - 168 n r_{00} r_0 \beta + 72 n B^2 n r_{00|0} \beta \\
& - 280 n s_0^m r_{0m} \beta^2 + 96 B^2 r_{00} r_0 \beta + 432 B r_{00} r_0 \beta - 8 B^2 r r_{00} \beta^2 + 40 B r r_{00} \beta^2 + 96 B^4 s_0^m s_{0m} \beta^2 + 48 B s_0^m s_{0m} \beta^2 - 60 B^4 r_{00}^2 \\
& + 804 B s_0^m r_{0m} \beta^2 - 48 B^3 s_0^m r_{0m} \beta^2 + 320 B^3 s_0^m s_m \beta^3 + 8 B^3 r_m^2 r_{00} \beta^2 + 8 B^3 b^m r_{00|m} \beta^2 - 192 B^3 s_i^m s_m \beta^4 + 24 B^3 r_{00}^2 \\
& + 144 B^3 s_m \beta^4 + 720 B s_i^m s_m \beta^4 + 192 B^2 r_{00}^2 + 48 \beta^3 B^2 s_0^m r_m - 60 B^2 r_m^2 r_{00} \beta^2 - 60 B^2 b^m r_{00|m} \beta^2 - 174 B r_m^2 r_{00} \beta^2 - 321 B r_{00}^2 \\
& - 408 \beta^3 B s_0^m r_m - 6 r_{00|0} \beta + 24 B^5 r_{00}^2 - 174 B b^m r_{00|m} \beta^2 + 144 B^2 s_i^m s_m \beta^4) + 2 \widehat{\text{Ric}} \beta^8 (8 B^4 - 80 B^3 - 348 B^2 - 176 B + 29) \\
d''_{12} & := - 2 \beta^4 (- 102 r_{00}^2 - 10 r_0^2 \beta^2 + 18 \beta^3 s_{|0} + 168 s_0^m |m \beta^3 - 34 r_{0|0} \beta^2 - 144 \beta^3 B^2 s_{|0} - 144 \beta^3 B s_{|0} - 40 B^3 r_{00|0} \beta \\
& - 36 B^2 r_{00|0} \beta + 39 B r_{00|0} \beta - 2 r_{00} r_0 \beta - 16 B^2 r_0^2 \beta^2 - 64 B r_0^2 \beta^2 + 10 r r_{00} \beta^2 - 86 s_i^m s_m \beta^4 + 84 s_0^m s_{0m} \beta^2 + 16 B^3 r_{0|0} \beta^2 \\
& + 96 B^2 r_{0|0} \beta^2 + 30 B r_{0|0} \beta^2 + 144 B s_0^m |m \beta^3 + 34 r_m^2 r_{00} \beta^2 + 34 b^m r_{00|m} \beta^2 + 4 s_0^m r_{0m} \beta^2 - 36 \beta^3 b^m s_{0|m} + 36 s_0^3 s_m r_m \\
& + 384 B^2 s_0^m \beta^2 r_{0m} + 11 n r_{00|0} \beta + 21 n r_{00}^2 + 160 B^2 r_{00} r_0 \beta + 76 B r_{00} r_0 \beta + 16 B^2 r r_{00} \beta^2 + 64 B r r_{00} \beta^2 - 192 B^4 s_0^m s_{0m} \beta^2 \\
& + 384 B^3 s_0^m s_{0m} \beta^2 + 48 B s_0^m s_{0m} \beta^2 - 16 B^3 r_m^2 r_{00} \beta^2 - 384 B^2 s_0^m |m \beta^3 - 576 B^2 s_0^m |m \beta^3 - 16 B^3 b^m r_{00|m} \beta^2 - 32 B^4 s_i^m s_m \beta^4 \\
& - 64 n B^3 s_0^m r_{0m} \beta^2 + 128 B^3 s_i^m s_m \beta^4 + 672 B^2 s_i^m s_m \beta^4 + 128 B s_i^m s_m \beta^4 - 288 \beta^3 B^2 s_0^m r_m - 96 B^2 r_m^2 r_{00} \beta^2 - 48 B^3 r_{00}^2 \\
& - 30 B r_m^2 r_{00} \beta^2 - 30 B b^m r_{00|m} \beta^2 - 180 B s_0^m r_{0m} \beta^2 - 160 B n r_{00} r_0 \beta - 384 n B^2 s_0^m r_{0m} \beta^2 - 336 n B s_0^m r_{0m} \beta^2 + 36 B^2 r_{00}^2 + 60 n B r_{00}^2 \\
& - 64 n B^2 r_{00} r_0 \beta - 288 \beta^3 B s_0^m r_m - 80 n s_0^m r_{0m} \beta^2 + 144 n s_0^m s_m \beta^3 + 224 B^3 s_0^m r_{0m} \beta^2 + 10 r_{00|0} \beta + 24 B^5 r_{00}^2 - 84 B^2 r_{00}^2 \\
& - 405 B r_{00}^2 + 48 B^4 r_{00}^2 - 64 n r_{00} r_0 \beta + 16 n B^3 r_{00|0} \beta + 60 n B^2 r_{00|0} \beta - 96 B^2 b^m r_{00|m} \beta^2) + 8 \widehat{\text{Ric}} \beta^6 (2 B + 1) (B - 1) \\
d''_{14} & := - 2 \beta^2 (6 r_{00}^2 + 264 B^2 s_0^m \beta^2 r_{0m} - 12 n B^2 r_{00}^2 - 12 B n r_{00}^2 + 32 \beta^3 B^3 s_{|0} - 50 \beta^3 s_{|0} - 104 s_0^m |m \beta^3 - 26 r_0^2 \beta^2 + 34 r_{0|0} \beta^2 + 48 \beta^3 B^2 s_{|0} \\
& - 84 \beta^3 B s_{|0} - 4 B^3 r_{00|0} \beta - 48 B^2 r_{00|0} \beta - 14 r_{00} r_0 \beta - 8 B r_0^2 \beta^2 + 26 B r r_{00} \beta^2 + 82 s_i^m s_m \beta^4 - 68 s_0^m s_{0m} \beta^2 - 16 B^3 r_{0|0} \beta^2 + 12 B^2 r_{0|0} \beta^2 \\
& + 78 B r_{0|0} \beta^2 + 128 B^3 s_0^m |m \beta^3 - 336 B^2 s_0^m |m \beta^3 - 400 B s_0^m |m \beta^3 - 34 r_m^2 r_{00} \beta^2 - 34 b^m r_{00|m} \beta^2 - 52 s_0^m r_{0m} \beta^2 + 100 \beta^3 b^m s_{0|m} \\
& - 10 \beta^3 s_0^m r_m - s_0^m s_m \beta^3 + 32 n B^2 r_{00} r_0 \beta + 32 B n r_{00} r_0 \beta + 64 n B^3 s_0^m r_{0m} \beta^2 + 96 n B^2 s_0^m r_{0m} \beta^2 + 48 n B s_0^m r_{0m} \beta^2 - 8 n B^3 r_{00|0} \beta \\
& - 12 n B^2 r_{00|0} \beta - 6 B n r_{00|0} \beta - B^3 b^m s_{0|m} + 8 n s_0^m r_{0m} \beta^2 + \beta^3 B^3 s_0^m r_m + 16 B^2 r_{00} r_0 \beta + 88 B r_{00} r_0 \beta - 16 B^2 r r_{00} \beta^2 + 8 B r r_{00} \beta^2 \\
& - 128 B^4 s_0^m s_{0m} \beta^2 + 128 B^3 s_0^m s_{0m} \beta^2 - 48 B^2 s_0^m s_{0m} \beta^2 - 208 B s_0^m s_{0m} \beta^2 + 16 B^3 r_m^2 r_{00} \beta^2 + 16 B^3 b^m r_{00|m} \beta^2 - 256 B^3 s_i^m s_m \beta^4 \\
& + 96 B^2 s_i^m s_m \beta^4 + 272 B s_i^m s_m \beta^4 + 96 B^3 B^2 s_0^m r_m - 12 B^2 r_m^2 r_{00} \beta^2 - 32 B^4 s_i^m s_m \beta^4 - 12 B^2 b^m r_{00|m} \beta^2 - 78 B r_m^2 r_{00} \beta^2 - 3 n r_0^2 \\
& - n r_{00|0} \beta - 78 B b^m r_{00|m} \beta^2 + 36 B s_0^m r_{0m} \beta^2 - 32 B^3 s_0^m r_{0m} \beta^2 - 168 \beta^3 B s_0^m r_m - 2 r_{00|0} \beta + 12 B^5 r_{00}^2 + 12 B^3 r_{00}^2 + 96 B^2 r_{00}^2 \\
& + 57 B r_{00}^2) - 8 \widehat{\text{Ric}} \beta^4 (2 B + 1)^2 (B + 2) (B - 4) \\
d''_{16} & := 2 (2 B + 1) (40 B^2 s_0^m \beta^2 r_{0m} - 14 r_0^2 \beta^2 + 10 r_{0|0} \beta^2 - 18 \beta^3 s_{|0} - 24 s_0^m |m \beta^3 - 36 \beta^3 B s_{|0} - 6 B^2 r_{00|0} \beta - 3 B r_{00|0} \beta - 4 B r_0^2 \beta^2 \\
& + 14 r r_{00} \beta^2 + 22 s_i^m s_m \beta^4 + 4 B^2 r_{0|0} \beta^2 + 22 B r_{0|0} \beta^2 - 96 B^2 s_0^m |m \beta^3 - 96 B s_0^m |m \beta^3 - 10 r_m^2 r_{00} \beta^2 - 10 b^m r_{00|m} \beta^2 - 20 s_0^m r_{0m} \beta^2 \\
& - 36 \beta^3 s_0^m r_m + 24 B r_{00} r_0 \beta + 4 B r r_{00} \beta^2 + 32 B^3 s_0^m s_{0m} \beta^2 - 48 B^2 s_0^m s_{0m} \beta^2 - 72 B s_0^m s_{0m} \beta^2 - 16 B^3 s_i^m s_m \beta^4 + 72 B^2 s_i^m s_m \beta^4 \\
& + 84 B s_i^m s_m \beta^4 - 4 B^2 r_m^2 r_{00} \beta^2 - 4 B^2 b^m r_{00|m} \beta^2 - 22 B r_m^2 r_{00} \beta^2 - 22 B b^m r_{00|m} \beta^2 - 20 B s_0^m r_{0m} \beta^2 - 72 \beta^3 B s_0^m r_m + 6 B^4 r_{00}^2 \\
& - 6 B^2 r_{00}^2 - 3 B r_{00}^2) - \widehat{\text{Ric}} \beta^2 (2 B + 1)^3 (2 B + 13) \\
d''_{18} & := 2 (2 B + 1)^2 ((2 B + 1) [4 \beta r_m s_0^m - 2 \beta^2 s_i^m s_m + 2 \beta s_0^m |m + 2 s_0^m s_{0m} + 2 s_0^m r_{0m} + 2 \beta s_{|0} + r_{00} r_m^2 + r_{00|m} b^m] - 2 r r_{00} \\
& + 2 r_0^2) + \widehat{\text{Ric}} (2 B + 1)^4
\end{aligned}$$

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