

HYPERSTRUCTURES AND SOME OF THE MOST RECENT APPLICATIONS

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ABSTRACT. After a brief history of Hypergroups, since the beginning around the 40s till today, one gives an excursus of the most recent applications of this topic to Fuzzy Sets and Chinese groups as HX -hypergroups.

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1. INTRODUCTION

Hyperstructures are a very important subject of Mathematics both on the theoretical point of view and for the numerous applications to several topics of Science.

Many remarkable results have been obtained on the structure of Hypergroups, for instance on the heart which is the least subhypergroup K of a hypergroup H , such that the quotient H/K is a group. and also on Hyperrings, hypermodules, hypervectorial spaces etc., on HX -Hypergroups.

Important for the history of Mathematics are the applications to Geometry. We can remember that a great mathematician of New York, Prenowitz, has constructed again more geometrical topics (Projective, Descriptive (as Euclidean, Hyperbolic), Spherical Geometries) as a Join Space structure, a commutative hypergroup satisfying a particular condition:

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$\forall a, b, c, d \in H$, if $a/b \cap c/d$ is not empty, also $a * d \cap b * c$ is not empty.

This structure has been utilized also in several other themes: Finite Geometries, Graph Theory, Hypergraphs, Median Algebras, Binary Relations, Cryptography, Fuzzy Sets.

Fuzzy Sets Theory is actually the most important application of Hypergroup Theory. Hypergroup Theory was born in 1934 when Marty, a French scientist, gave the definition of hypergroup at the Congress of Scandinavian Mathematicians and showed its utility in the study of groups, algebraic functions and rational fractions.

In the following years around the 40s several others worked on this subjects, in France, Krasner, Kuntzman, Croisot, in USA Dresher and Ore, Prenowitz, Wall, In Japan Utumi, in Spain San Juan, in Russia Vikhrov, in Uzbekistan Dietzman, in Italy Zappa.

In the 50s and 60s they worked on Hyperstructures in Romania Benado, in Czech Republic Drbohlav, in France Koskas, Sureau, in Greece Mittas, Stratigopoulos, in Italy Orsatti, Boccioni, in USA Prenowitz, Graetzer, in Japan Nakano, in Yugoslavia Dacic.

But it is above all since 70 that a more luxuriant flourishing of hyperstructures has been and is seen in Europe, Asia, America, Australia.

I shall remember only some names.

Europe

in *Greece*

at Thessaloniki: Mittas, Konstantinidou, Serafimidis, Kehagias

at Alexandroupolis: T. Vougiouklis

at Patras : Stratigopoulos

at Orestiada: Spartalis

at Athens: Ch. Massouros, G. Massouros

in *Romania*

at Iasi: V. Leoreanu (Fotea), Cristea, Tofan

at Cluj Napoca: Purdea, Pelea, Calugareanu

at Constanta: Stefanescu

in *Czech Republic*

at Praha: Kepka, Jezek, Drbohlav, Nemeč

at Brno: J. Chvalina, Hoskova

at Olomouc: Hort

at Vyskov: Moucka

in *Montenegro*

at Podgorica: Rasovic

in *Slovakia*

in Bratislava: Kolibiar

at Kosice: Jakubik

in *Italy*

at Udine: Corsini, Cristea

at Messina: De Salvo, Migliorato , Bonansinga, Chillemi, Freni

at Rome: Tallini, Scafati-Tallini, Rota, Procesi Ciampi

at Pescara: Maturo, Doria, Peroni

at Teramo: Eugeni

at LAquila: Innamorati, Berardi

at Brescia: Marchi

at Milano: Mercanti, Cerritelli, Gelsomini

at Lecce: Letizia

at Palermo: Falcone

in *France*

at Clermont-Ferran: Sureau, M. Gutan, C. Gutan

at Lyon: Bayon, Lygeros

in *Spain*

at Malaga: Martinez, Gutierrez, De Guzman, Cordero

in *Finland*

at Oulu: Nieminen, Niemenmaa.

America

in *USA*

at Charleston: Comer

at New York: Jantosciak

at Boulder: Roth

at Cleveland: Olson, Ward

in *Canada* at Montreal: Rosenberg

Asia

in *Thailand*

at Bangkok: Kemprasit, Punkla , Chaopraknoi, Triphop, Tumsoun

in *Iran* at Babolsar: Ameri, Mahjoob, Moghani

at Yazd: Davvaz

at Kerman: Zahedi, Molaiei, Torkzadeh, Borumand Saeid

at Kashan: Ashrafi

at Tehran: Darafsheh, Yavary, Iranmanesh, Madanshekaf, Ghorbani

at Zahedan: Borzooei, Hasankhani, Rezaei

in *Korea*

at Chinju: Young Bae Jun

at Taejon: Sang Cho Chung, Byung-Mun Choi

at Chungju: K.H. Kim

in *India*

at Kolkata: M.K. Sen, Dasgupta

at Tiruchendur: Asokkumar, Velrajan

in *China*

at Chongqing: Yuming Feng

at Xian: Xiao Long Xin

at Enshi: Janming Zhan, Xueling Ma

in *Japon*

at Tokyo: Machida

at Tagajo: Miyagi, Shoji Kyuno

in *Jordan*

at Karak: M.I. Al Ali

in *Turkey*

at Istanbul: Bayram Ersoy

2. APPLICATIONS TO FUZZY SETS AND CHINESE GROUPS AS HX-HYPERGROUPS

Definition of an HX-group:

Let (G, \cdot) be a group and $\mathcal{P}^*(G)$ be the set of all non empty subsets of G . An *HX-group* is a nonempty subset H of $\mathcal{P}^*(G)$, which is a group, with respect to the hyperoperation) :

$$\forall (A, B) \in \mathcal{P}^*(G) \times \mathcal{P}^*(G), \quad A \cdot B = \{xy \mid x \in A, y \in B\}.$$

Definition of a Chinese hypergroupoid:

Let \mathcal{G} be an HX-group, let $G^* = \bigcup_{A \in \mathcal{G}} A$.

We call Chinese hypergroupoid the hyperstructure $(G^*, \hat{\circ})$ defined by:

$$\forall (x, y) \in G^* \times G^*, \quad x \hat{\circ} y = \bigcup_{x \in A, y \in B; \{A, B\} \subseteq \mathcal{G}} A \cdot B.$$

If the following condition holds:

$$\forall (A, B) \in \mathcal{G} \times \mathcal{G}, \quad A \cap B \neq \emptyset \Rightarrow A = B,$$

then $(G^*, \hat{\circ})$ is a hypergroup.

Moreover, I have proved that $(G^*, \hat{\circ})$ is a join space.

I have proved [9] that to every fuzzy μ_H one can associate a join space $K_{\mu_H}^1$, by setting

$$(I) \quad \forall (x, y) \in H^2, \quad x \circ_1 y = \{z \mid \min\{\mu(x), \mu(y)\} \leq \mu(z) \leq \max\{\mu(x), \mu(y)\}\}.$$

I have proved also [16] that to every hypergroupoid K one associates a fuzzy set μ_K^1 . One can construct a sequence of fuzzy sets and of hypergroupoids which are join spaces.

If the basis set of the fuzzy set from which one starts or the first hypergroupoid is finite, one obtains a finite sequence.

The integer $n = \min\{h \mid K_{\mu_{H^{h-1}}} \simeq K_{\mu_H}^{h+1}\}$ is called the fuzzy grade of K^1 .

Set now $\forall z \in H$,

$$(II) \quad A(z) = \sum_{z \in x \circ y} 1/|x \circ y|, \quad \mu_1(z) = A(z)/q(z),$$

where

$$q(z) = |Q(z)|, \quad Q(z) = \{(a, b) \mid z \in a \circ b\}.$$

One has calculated the fuzzy grade of many hypergroups, for instance i.p.s. hypergroups, which are a particular type of strongly canonical hypergroups, which have several applications [2] complete hypergroups and so on.

Let us see now which is the fuzzy set (H, μ_1) associated with a hypergroupoid (h, \circ) .

(H, μ_1) is the fuzzy set associated with (H, \circ) .

To (H, μ_1) one associates μ_2 as in (I), then to μ_2 one associates (H, μ_2) as in (II) and so on. The fuzzy grade ∂H is the least integer k such that $\mu_k = \mu_{k+1}$.

One has considered the HX -group associated with several finite groups, for instance with $\mathbf{Z}/n\mathbf{Z}$.

Setting $n = aq$ one has considered the subset of $\mathcal{P}^*(\mathbf{Z}/n\mathbf{Z})$

$\mathcal{G}_a^n = \{A_i \mid 0 \leq i \leq q-1\}$, where $\forall i, A_i = \{i, a+i, 2a+i, \dots, (q-1)a+i\}$.

\mathcal{G}_a^n is clearly a group which satisfies the condition (1), whence ${}_G H_a^n$ is a hypergroup.

For instance, if $n = 8$, $\mathcal{G}_2^8 = \{(0, 4), (1, 5), (2, 6), (3, 7)\}$.

So K_2^8 is the following group:

K_2^8	(0, 4)	(1, 5)	(2, 6)	(3, 7)
(0, 4)	(0, 4)	(1, 5)	(2, 6)	(3, 7)
(1, 5)		(2, 6)	(3, 7)	(0, 4)
(2, 6)			(0, 4)	(1, 5)
(3, 7)				(2, 6)

whence we obtain the Chinese hypergroup H_2^8 :

${}_G H_2^8$	0	1	2	3	4	5	6	7
0	0, 4	1, 5	2, 6	3, 7	0, 4	1, 5	2, 6	3, 7
1		2, 6	3, 7	0, 4	1, 5	2, 6	3, 7	0, 4
2			0, 4	1, 5	2, 6	3, 7	0, 4	1, 5
3				2, 6	3, 7	0, 4	1, 5	2, 6
4					0, 4	1, 5	2, 6	3, 7
5						2, 6	3, 7	0, 4
6							0, 4	1, 5
7								2, 6

We find $\forall(j, k) \mu_1(j) = \mu_1(k)$, so ${}_1 H_2^8$ is total, whence $\partial_G H_2^8 = 1$.

If we set $a = 4$, we find $\mathcal{G}_4^8 = \{(0, 2, 4, 6), (1, 3, 5, 7)\}$ and we have also in this case $\partial_G H_4^8 = 1$.

Not in every case, the fuzzy grade of HX -structures is equal to one. We have found a hypergroupoid H such that the associated HX -hypergroupoid has the fuzzy grade equal to two, see [22].

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