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THE USE OF MATLAB PLATFORM TO COMPUTE SOME GEOMETRIC QUANTITIES OF SCHWARZSCHILD ROBERTSON-WALKER SPACE TIME

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Abstract. In this paper we compute some geometric quantities of Schwarzschild Robertson-Walker space time by using MATLAB platform to construct functions that compute These quantities.

Key Words: Schwarzschild Robertson-Walker space time, General relativity , geometric quantities.
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1. Introduction

Space times have many applications in general relatively and black holes. So the study of geometric quantities of space times is very important. In this paper we compute Christoffel symbols, Riemannian curvature tensor, Einstein tensor and weyl tensor for Schwarzschild Robertson-Walker space time.
The metric of Schwarzschild Robertson-Walker space time has 4 dimensional so it is difficult to calculate geometrical quantities, hence we use MATLAB platform and construct functions to compute geometric quantities.
In section two to calculate each quantities we introduce functions and

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get the nonzero components.
In section 4 MATLAB files listed in appendix at the end of the paper.

2. Calculate some geometric quantities of Schwarzschild Robertson-Walker space time

In the standard Schwarzschild (t,r,?, φ), and geometric system of units, the Schwarzschild Robertson-Walker space time are determined by the line element

$$ds^2 = -dt^2 + \frac{s^2}{1-kR^2}dr^2 + s^2R^2(d\theta^2 + sin^2\theta\ d\varphi^2)$$

and the coordinates are defined such that

$$-\infty \leq t \leq +\infty\ ,\ r \geq 0\ ,\ 0 \leq \theta \leq \pi\ \ and\ \ 0 \leq \varphi \leq 2\pi\ \ .$$

So the metric coefficients and the invers of matrix coefficients are as follows:

3. Chirstoffel symbols

The Christoffel symbols of first kind are defined by:

$$\Gamma_{ij}^{m}\ = \frac{1}{2}g^{km}\ (\ -\frac{\partial g_{ij}}{\partial x_k} + \frac{\partial g_{ik}}{\partial x_j} + \frac{\partial g_{jk}}{\partial x_i})$$

to calculate Γ_{ij}^{m} in Schwarzschild Robertson-Walker space time we construct function file SRW1 with 3 arguments that the first two arguments are lower indexes in Γ_{ij}^{m} And the third argument is upper indexes inΓ_{ij}^{m}
.
For example to compute Γ_{44}^{3} it is enough to call SRW1 with (4, 4,3) in MATLAB and we have :
$\gg$ SRW1(4,4,3)

```
the christoffel symbol  gama\_44\^{}3   is :
ans =
-sin(2*theta)/2
```

and we will have this equality :

$$\Gamma_{ij}^{m} = SRW1(i,j,m)$$

By this process we have obtained Christoffel symbols. The nonzero components are the following : $\Gamma_{22}^{2} = -\frac{kr}{kr^2-1}$

$$\Gamma_{23}^{3} = \Gamma_{24}^{4} = \Gamma_{32}^{3} = \Gamma_{42}^{4} = \frac{1}{r}\Gamma_{33}^{2} = r(kr^2-1)\Gamma_{34}^{4} =\ \cot\theta$$

$$\Gamma_{43}^{4} = \cot\theta \qquad \Gamma_{44}^{2} = rsin^{2}(\theta)\ kr^{2} - 1$$

$$\Gamma_{44}^{3} = -\frac{\sin(2\theta)}{2}.$$

Also with this method we can see that these Γ_{ij}^{m} satisfying the following equalities :

$$\Gamma_{ij}^{m} = \Gamma_{ji}^{m}$$

3.1. **Riemannian curvature tensor.** The Riemannian curvature tensor is given by :

$$R_{ijk}^{s} = \frac{\partial \Gamma_{ij}^{s}}{\partial x^{k}} - \frac{\partial \Gamma_{ik}^{s}}{\partial x^{j}} + \Gamma_{ij}^{r}\Gamma_{rk}^{s} - \Gamma_{ik}^{r}\Gamma_{rj}^{s}$$

$$R_{ijkl} = g_{si}R_{jkl}^{s}$$

It is clear that to calculate rimannian curvature tensor we need Christoffel symbols and metric coefficients. We have metric coefficients and calculated Christoffel symbols in 2.1 . to computeR_{ijk}^{s}

In Schwarzschild Robertson-Walker space time we construct function file SRW2 .

SRW 2 is a function with 4 arguments. The first three arguments are three lower indexes in R_{ijk}^{s}

And the forth argument is upper indexes in R_{ijk}^{s} .

for example to compute R_{224}^{4}it is enough to call SRW 2 with argument (2,2,4,4) or type the following command in MATLAB :

```
>>SRW2(2,2,4,4)
the Rieman tensor R\_224\^{}4 is :
ans =
K/(K*r\^{}2 - 1)
```

So we will have :

$$R_{ijk}^{s} = SRW2(i,j,k,s)$$

With this method we have obtained R_{ijk}^{s} 's that nonzero components are as follows :

$$R_{223}^{3} = R_{224}^{4} = \frac{k}{kr^{2}-1} R_{232}^{3} = R_{242}^{4} = -\frac{k}{kr^{2}-1}$$

$$R_{323}^{2} = kr^{2}\ R_{332}^{2} = R_{334}^{4} = -kr^{2}R_{343}^{4} = kr^{2}$$

$$R_{424}^{2} = R_{434}^{3} = kr^{2}sin^{2}(\theta)\, R_{442}^{2} = R_{443}^{3} = -kr^{2}sin^{2}(\theta)\,.$$

Also to calculate R_{ijkl} in Schwarzschild Robertson-Walker space time we construct SRW3 with 4 arguments. These arguments are R_{ijkl} indexes.

For example to calculate R_{4334} it is enough to call
SRW3 with (4,3,3,4) :

```
$>$$>$ SRW3(4,3,3,4)
the Rieman tensor R\_4334 is :
ans =
-16*K*r\^{}4*sin(theta)\^{}2
```

So we have:

$$R_{ijkl} = SRW3(i,j,k,l)$$

By this process we have obtained R_{ijkl} in Schwarzschild Robertson-Walker space time that nonzero components are the following :

$$R_{2323} = -\frac{16}{kr^2-1} - 16 R_{2332} = \frac{16}{kr^2-1} + 16$$

$$R_{2424} = -\frac{16kr^2 sin^2(\theta)}{kr^2-1} R_{2442} = \frac{16kr^2 sin^2(\theta)}{kr^2-1}$$

$$R_{3223} = \frac{16}{kr^2-1} + 16 R_{3232} = -\frac{16}{kr^2-1} - 16$$

$$R_{3434} = 16kr^4 sin^2(\theta)\, R_{3443} = -16kr^4 sin^2(\theta)$$

$$R_{4224} = \frac{16kr^2 sin^2(\theta)}{kr^2-1} R_{4242} = -\frac{16kr^2 sin^2(\theta)}{kr^2-1}$$

$$R_{4334} = -16kr^4 sin^2(\theta)\, R_{4343} = 16kr^4 sin^2(\theta)$$

Also with this method we can see that these $R'_{ijkl}s$ satisfying the following equalities:

$$R_{ijkl} = -R_{jikl} = -R_{ijlk} = R_{klij}$$

4. Ricci tensor

we know that the components of Ricci curvature tensor are defined by :

$$R_{jk} = R^i_{jik}$$

To compute Ricci tensor in Schwarzschild Robertson-Walker space time we construct function file SRW4 with two arguments and these arguments are R_{jk} indexes.
for example to compute R_{44}it is enough to call SRW4 with tow arguments(4,4) or type the following command in MATLAB.

```
\gg SRW4(4,4)
the RICCI tensor ( R\_44) is :
ans =
2*K*r\^{}2*sin(theta)\^{}2
```

So we will have:

$$R_{jk} = SRW4(j,k)$$

By this process we have obtained $R'_{jk}s$ That nonzero components are the following :

$$R_{22} = 2kr^2 sin^2(\theta)$$

$$R_{33} = 2kr^2 R_{44} = \frac{-2k}{kr^2-1}.$$

Finally the Ricci tensor is the following:

$$R_{jk} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2kr^2 sin^2(\theta) & 0 & 0 \\ 0 & 0 & 2kr^2 & 0 \\ 0 & 0 & 0 & \frac{-2k}{kr^2-1} \end{bmatrix}$$

4.1. **Scalar curvature.** The scalar curvature is defined by

$$S = R_{jk} g^{kj}$$

To compute the scalar curvature in Schwarzschild Robertson-Walker space time we construct the MATLAB script SRW5 and it is enough to run SRW5 to abtain scalar curvature. So we have: *the scalar curvature is*:

```
(6*K)/s\^{}2
```

Finally the scalar curvature is the following:

$$S = \frac{6k}{s^2}.$$

4.2. **Einstein tensor.** The Einstein tensor is defined by [1]

$$G_{jk} = R_{jk} - \frac{s}{2}\, g_{jk}$$

In this equation the $R_{jk}$are Ricci tensor that we calculated in 2.3 and g_{jk} 's are metric coefficients.

To compute Einstein tensor in Schwarzschild Robertson-Walker space time we construct SRW6 with two arguments and these arguments are $G'_{jk}s$ Indexes.

for example to compute G_{22} it is enough to call SRW6 with (2,2) argument or type SRWt6 (2,2) in MATLAB.
$\gg$SRW6(3,3)

```
the einstein tensor ( G\_ 22) is :
ans =
K/(K*r\^{}2 -- 1)
```

So we have:

$$G_{jk} = \text{SRW6}(j,k)$$

By this process we have obtained $G'_{jk}s$ that nonzero components are the following:

$$G_{11} = \frac{3k}{s^2} G_{22} = \frac{k}{kr^2-1}$$

$$G_{33} = -kr^2 G_{44} = -kr^2 sin^2(\theta)$$

So the Einstein tensor in Schwarzschild Robertson-Walker space time is :

$$G_{jk} = \begin{bmatrix} \frac{3k}{s^2} & 0 & 0 & 0 \\ 0 & \frac{k}{kr^2-1} & 0 & 0 \\ 0 & 0 & -kr^2 & 0 \\ 0 & 0 & 0 & -kr^2 sin^2(\theta) \end{bmatrix}$$

4.3. **Weyl tensor.** from [1] about Weyl tensor we have:

$$U_{abcd} = \frac{S}{n(n-1)} (g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$Z_{abcd} = \frac{1}{n-2} (R_{ac}g_{bd} + R_{bd}g_{ac} - R_{ad}g_{bc} - R_{bc}g_{ad}) - \frac{2S}{n(n-2)} (g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$W_{abcd} = R_{abcd} - U_{abcd} - Z_{abcd}$$

We use these equalities and to calculate Weyl tensor in Schwarzschild Robertson-Walker
space-time We construct function file SRW7 with 4 arguments, these arguments are W_{ijkl} indexes.
For example to compute W_{1313}it is enough to call SRW7 with (1,3,1,3) arguments or type SRW7 (1,3,1,3) in Matlab:

```
the WEYL tensor    W\_1313   is:
ans =
-5*K*r\^{}2
```

So we have:

$$W_{abcd} = SRW7(a,b,c,d)$$

By this method we have obtained $W'_{abcd}s$ that nonzero components are the following :

$$w_{1212} = \frac{5k}{kr^2-1} w_{1221} = -\frac{5k}{kr^2-1}$$

$$w_{1313} = -5kr^2 w_{1331} = 5kr^2$$

$$w_{1414} = 5kr^2 sin^2(\theta) \qquad w_{1441} = -5kr^2 sin^2(\theta)$$

$$w_{2112} = -\frac{5k}{kr^2-1} w_{2121} = -\frac{5k}{kr^2-1} w_{2323} = -\frac{kr^2(17s^2+32)}{2(kr^2-1)}$$

$$w_{2332} = \frac{kr^2(17s^2+32)}{2(kr^2-1)} w_{2424} = -\frac{kr^2 sin^2(\theta)(17s^2+32)}{2(kr^2-1)}$$

$$w_{2442} = \frac{kr^2 sin^2(\theta)(17s^2+32)}{2(kr^2-1)} w_{3113} = 5kr^2 w_{3131} = -5kr^2$$

$$w_{3223} = \frac{kr^2(17s^2+32)}{2(kr^2-1)}$$

$$w_{3232} = -\frac{kr^2(17s^2+32)}{2(kr^2-1)} w_{3434} = \frac{kr^4 sin^2(\theta)(17s^2+32)}{2}$$

$$w_{3443} = -\frac{kr^4 sin^2(\theta)(17s^2+32)}{2}$$

$$w_{4114} = 5kr^2 sin^2(\theta)$$

$$w_{4141} = -5kr^2 sin^2(\theta) w_{4224} = \frac{kr^2 sin^2(\theta)(17s^2+32)}{2(kr^2-1)} w_{4242} = -\frac{kr^2 sin^2(\theta)(17s^2+32)}{2(kr^2-1)}$$

$$w_{4334} = -\frac{kr^4 sin^2(\theta)(17s^2+32)}{2} w_{4343} = \frac{kr^4 sin^2(\theta)(17s^2+32)}{2}$$

5. Conclusion

In this paper we study the geometric quantities of Schwarzschild Robertson-Walker space time With 4 dimensional. by using MATLAB platform and construct functions to calculate nonzero components of quantities without difficult and long calculations. So we illustrated the possibility of the study of geometric objects for Schwarzschild Robertson-Walker space time using the facilities of computer platform.

This method by MATLAB can be used to compute other quantities in geometry or plot the geodesics without difficult calculations.

6. Appendix (M-Files)

SRW1:

```
function gama\_Z1Z2\_Z3 =SRW1(Z1,Z2,Z3)
syms t r theta phi s K ;
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
Z5=inv(g);
X=[t r theta phi];
gama\_Z1Z2\_Z3=0;
for d=1:4
W=1/2*Z5(d,Z3)*(diff(g(Z2,d),X(Z1))+diff(g(d,Z1),X(Z2))-diff(g(Z1,Z2),X(d)));
gama\_Z1Z2\_Z3=W+gama\_Z1Z2\_Z3;
Z4=(sum(gama\_Z1Z2\_Z3));
end
z1=' the christoffel symbol';
z2='  gama\_';
z3='   is :';
disp([z1  z2  num2str(Z1)  num2str(Z2)  '\^{}'  num2str(Z3)   z3]);
gama\_Z1Z2\_Z3 = (Z4);
end
--------------------------------------------------------------
```

SRW2:

```
function R\_I1I2I3\_I4 =SRW2(I1,I2,I3,I4)
t=sym('t');r=sym('r');theta=sym('theta');
phi=sym('phi');s=sym('s');K=sym('K');
X=[t r theta phi];
part1=diff(chris(I1,I3,I4),X(I2))-diff(chris(I1,I2,I4),X(I3));
P3=0;R\_I1I2I3\_I4=0;
for S=1:4
     part2=((chris(I1,I3,S))*(chris(S,I2,I4)))-((chris(I1,I2,S))
     *(chris(S,I3,I4)));
P3=P3+part2;
end
R\_I1I2I3\_I4=part1+P3+R\_I1I2I3\_I4;
function gama\_nb\_k = chris(n,b,k)
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
G=inv(g);
```

```
X=[t r theta phi];
gama\_nb\_k=0;
for d=1:4
      W=1/2*G(d,k)*(diff(g(b,d),X(n))+diff(g(d,n),X(b))-diff(g(n,b),X(d)));
      gama\_nb\_k=W+gama\_nb\_k;
      sum(gama\_nb\_k);
  end
 sum(gama\_nb\_k);
end
word1=' the Rieman tensor';word2=' R\_';word3=' is : ';
disp([word1  word2 num2str(I1) num2str(I2) num2str(I3) '\^{}'
num2str(I4)  word3]);
sum(R\_I1I2I3\_I4);
end
--------------------------------------------------------------------------------------------
```

SRW3:

```
function R\_M1M2M3M4 =SRW3(M1,M2,M3,M4)
t=sym('t');r=sym('r');theta=sym('theta');
phi=sym('phi');s=sym('s');K=sym('K');
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
R\_M1M2M3M4=0;
for Z=1:4
     I1=g(Z,M1).*(srw(M2,M3,M4,Z));
  R\_M1M2M3M4= R\_M1M2M3M4+I1;
end
  function R\_M2M3M4\_D =srw(M2,M3,M4,D)
X=[t r theta phi];
NN\_1=diff(GAMA(M2,M4,D),X(M3))-diff(GAMA(M2,M3,D),X(M4));
NN\_3=0;R\_M2M3M4\_D=0;
for S=1:4
    NN\_2=((GAMA(M2,M4,S))*(GAMA(S,M3,D)))-((GAMA(M2,M3,S))
    *(GAMA(S,M4,D)));
NN\_3=NN\_3+NN\_2;
end
R\_M2M3M4\_D=NN\_1+NN\_3+R\_M2M3M4\_D;
function gama\_nm\_k =GAMA(n,m,k)
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
```

```
G=inv(g);
X=[t r theta phi];
gama\_nm\_k=0;
for s=1:4
     N=1/2*G(s,k)*(diff(g(m,s),X(n))+diff(g(s,n),X(m))-diff(g(n,m),X(s)));
     gama\_nm\_k=N+gama\_nm\_k;
end
sum(gama\_nm\_k);
end
sum(R\_M2M3M4\_D);
end
W\_1=' the Rieman tensor';W\_2=' R\_';W\_3=' is : ';
disp([W\_1  W\_2 num2str(M1) num2str(M2) num2str(M3) num2str(M4)  W\_3]);
sum( R\_M1M2M3M4);
end
------------------------------------------------------------
```

SRW4 :

```
function RICCItensor =SRW4(Z1,Z2)
t=sym('t');r=sym('r');theta=sym('theta');
phi=sym('phi');s=sym('s');K=sym('K');
R\_Z1Z2=0;
for e=1:4
MS=sc\_ro\_wa\_2(Z1,e,Z2,e);
R\_Z1Z2=MS+R\_Z1Z2;
end
function R\_ABC\_D =sc\_ro\_wa\_2(A,B,C,D)
X=[t r theta phi];
LL\_1=diff(sc\_ro\_wa\_1(A,C,D),X(B))-diff(sc\_ro\_wa\_1(A,B,D),X(C));
LL\_3=0;R\_ABC\_D=0;
for S=1:4
LL\_2=((sc\_ro\_wa\_1(A,C,S))*(sc\_ro\_wa\_1(S,B,D)))-((sc\_ro\_wa\_1(A,B,S))
*(sc\_ro\_wa\_1(S,C,D)));
LL\_3=LL\_3+LL\_2;
end
R\_ABC\_D=LL\_1+LL\_3+R\_ABC\_D;
function chris\_nm\_k =sc\_ro\_wa\_1(n,m,k)
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
Z\_1=inv(g);
```

```
X=[t r theta phi];
chris\_nm\_k=0;
for V1=1:4
N\_1=1/2*Z\_1(V1,k)*(diff(g(m,V1),X(n))+diff(g(V1,n),X(m))-diff(g(n,m),X(V1)));
chris\_nm\_k=N\_1+chris\_nm\_k;
end
sum(chris\_nm\_k);
end
sum(R\_ABC\_D);
end
H\_1=' the RICCI tensor';H\_2=' R\_';H\_3=' is :';
disp([H\_1 '(' H\_2 num2str(Z1) num2str(Z2) ')' H\_3]);
RICCItensor=sum(R\_Z1Z2);
end
--------------------------------------------------------------
```

SRW5:

```
clear all;clc;
syms t r theta phi s K ;
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)];
M=inv(g);X=[t r theta phi];
R(2,2)=-(2*K)/(K*r\^{}2 - 1);R(3,3)=2*K*r\^{}2;
R(4,4)=2*K*r\^{}2*sin(theta)\^{}2;
R(1,1)=0;R(1,2)=0;R(1,3)=0;R(1,4)=0;R(2,1)=0;
R(2,3)=0;R(2,4)=0;R(3,1)=0;R(3,2)=0;R(3,4)=0;R(4,1)=0;R(4,2)=0;R(4,3)=0;
MS=0;
for T\_1=1:4
for T\_2=1:4
Z=(R(T\_1,T\_2))*(M(T\_2,T\_1));
MS=MS+Z;
end
end
WORD1=' the scalar curvature is  : ';
disp(WORD1);disp(MS);
------------------------------------------------------------------------------------------
```

SRW6 :

```
function EINSTEINtensor =SRW6(M1,M2)
syms t r theta phi s K ;
```

```
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)];
R(2,2)=-(2*K)/(K*r\^{}2 - 1);R(3,3)=2*K*r\^{}2;
R(4,4)=2*K*r\^{}2*sin(theta)\^{}2;
R(1,1)=0;R(1,2)=0;R(1,3)=0;R(1,4)=0;R(2,1)=0;
R(2,3)=0;R(2,4)=0;R(3,1)=0;R(3,2)=0;R(3,4)=0;
R(4,1)=0;R(4,2)=0;R(4,3)=0;
S=(6*K)/s\^{}2;
EINSTEINtensor=R(M1,M2)-((S/2)*g(M1,M2));
W\_1=' the EINSTEIN tensor ';
W\_2=' G\_ ';W\_3=' is :';
disp([W\_1 '(' W\_2 num2str(M1) num2str(M2) ')' W\_3]);
end
--------------------------------------------------------------------------------
```

SRW7 :

```
function WEYLtensor = SRW7(ms1,ms2,ms3,ms4)
t=sym('t');r=sym('r');theta=sym('theta');
phi=sym('phi');s=sym('s');K=sym('K');
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
R(2,2)=-(2*K)/(K*r\^{}2 - 1);R(3,3)=2*K*r\^{}2;
R(4,4)=2*K*r\^{}2*sin(theta)\^{}2;
R(1,1)=0;R(1,2)=0;R(1,3)=0;R(1,4)=0;R(2,1)=0;
R(2,3)=0;R(2,4)=0;R(3,1)=0;R(3,2)=0;R(3,4)=0;R(4,1)=0;
R(4,2)=0;R(4,3)=0;
SC\_CU=(6*K)/s\^{}2;
U=(SC\_CU/4*3)*((g(ms1,ms3) * g(ms2,ms4))-(g(ms1,ms4) * g(ms2,ms3)));
Z=((1/4-2)*( R(ms1,ms3)*g(ms2,ms4) + R(ms2,ms4)*g(ms1,ms3)
- R(ms1,ms4)*g(ms2,ms3) - R(ms2,ms3)*g(ms1,ms4) ))
 - ((2*SC\_CU/4*2)*( g(ms1,ms3)*g(ms2,ms4)
 - g(ms1,ms4)*g(ms2,ms3) ) );
WEYLtensor = S\_R\_W\_3(ms1,ms2,ms3,ms4) - U - Z ;
function R\_OABC = S\_R\_W\_3(O,A,B,C)
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
R\_OABC=0;
for v=1:4
N\_1=g(v,O).*(S\_R\_W\_2(A,B,C,v));
R\_OABC= R\_OABC+N\_1 ;
```

```
end
function R\_ABC\_D = S\_R\_W\_2(A,B,C,D)
X=[t r theta phi];
N\_2=diff(S\_R\_W\_1(A,C,D),X(B))-diff(S\_R\_W\_1(A,B,D),X(C));
N\_4=0;R\_ABC\_D=0;
for S=1:4
N\_3=((S\_R\_W\_1(A,C,S))*(S\_R\_W\_1(S,B,D)))-((S\_R\_W\_1(A,B,S))
*(S\_R\_W\_1(S,C,D)));
N\_4=N\_4+N\_3;
end
R\_ABC\_D=N\_2+N\_4+R\_ABC\_D;
function gama\_nm\_k = S\_R\_W\_1(n,m,k)
g=[-1 0 0 0;0 (s\^{}2)/(1-K*r\^{}2) 0 0;0 0 s\^{}2*r\^{}2 0;
0 0 0 s\^{}2*r\^{}2*((sin(theta))\^{}2)] ;
G=inv(g);X=[t r theta phi];gama\_nm\_k=0;
for s=1:4
N\_5=1/2*G(s,k)*(diff(g(m,s),X(n))+diff(g(s,n),X(m))-diff(g(n,m),X(s)));
gama\_nm\_k=N\_5+gama\_nm\_k;
end
sum(gama\_nm\_k);
end
sum(R\_ABC\_D);
end
sum( R\_OABC);
end
P\_1=' the WEYL tensor   ';P\_2=' W\_';P\_3='   is : ';
disp([P\_1  P\_2 num2str(ms1) num2str(ms2) num2str(ms3)
num2str(ms4)  P\_3]);
end
------------------------------------------------------------------------------------------------
```

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