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ON SOFT TOPOLOGICAL HYPERGROUPS

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ABSTRACT. Hyperstructure theory, initiated by Marty, is a generalization theory of classical algebraic structures, while soft set theory is a powerful mathematical approach for modeling uncertainties and imprecision. In this study, it is aimed to introduce the concept of soft topological hypergroups by presenting a soft approachto the concept of topological hypergroups, one of the topological algebraic hyperstructures. Also, by defining the concept of a soft topological transposition hypergroup, several special types of soft topological subhypergroups are presented, and then the relationships between these concepts are investigated. Later on, by constructing the category $\mathcal{C}_{STH}$ of soft topological hypergroups with soft topological homomorphisms, some related characterizations are established.

Key Words: Soft set, Topological hypergroup, Soft hypergroup, Soft topological hypergroup.

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1. Introduction

Hyperstructure theory was introduced in 1934 by F. Marty as a natural generalization of classical algebraic theory [3]. Later on, he defined the concept of hypergroups. Many different types of hyperstructures have been developed and many studies have been carried out on them

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[16]. In particular, the concept of hypergroups has reached a high working potential by applying to many branches of both theoretical and applied mathematics. Topological studies on hypergroups were initiated by Heidari and et.al. [18]. They obtained some important results by presenting the definition of topological hypergroups as a generalization of topological groups [18].

Another theory combined with hyperstructure theory is soft sets. Soft set theory is an important mathematical tool presented by Molodtsov to overcome the uncertainties that exist in some complex problems in engineering, economics, social sciences and medical science etc [1]. This theory is applicable to all areas of mathematics and many studies on it have been carried out rapidly [4-14]. Recently, the relations between hyperstructures and soft sets are examined. The concept of soft grupoids was defined by Yamak and et.al. and some important results were established [19]. Morever, the definitions of soft polygroups, normal soft polygroups, soft subpolygroups and normal soft subpolygroups were proposed by Wang and et.al [17]. They constructed three isomorphism theorems for soft polygroups. Selvachandran and Salleh investigated the theory of soft hyperalgebra by defining the concepts of soft hypergroups and soft subhypergroups [15]. Also, Selvachandran given the definitions of soft hyperring and soft subhyperring and examined several related characterizations [20].

In this chapter, our aim is to introduce the notion of soft topological hypergrups by examining the relationship between the topological hypergrups and soft set theory. By using this novel notion, we present the definition of soft topological subhypergroups. Morever, we define the homomorphisms of soft topological hypergrups and prove some properties of them.

2. Preliminaries

In this section, we rewiev some definitions and results concerning soft sets and topological hypergroups for the sake of convenience and completeness of our study. For a comprehensive overview of this theme, the reader is refered to [1, 2, 18].

Let $\mathcal{X}$ be an initial universe set and E be a set of parameters. Let $P(\mathcal{X})$ denotes the power set of $\mathcal{X}$ and $A \subset E$.

Definition 2.1. [1] A soft set over $\mathcal{X}$ is a pair $(\mathcal{F}, A)$ together with a mapping $\mathcal{F} : A \longrightarrow P(\mathcal{X})$.

In actually, a soft set over $\mathcal{X}$ can be regarded as a parameterized family of subsets of the universe $\mathcal{X}$.

Definition 2.2. [2] Let $(\mathcal{F}, A)$ and $(\mathcal{G}, B)$ be two soft sets over the common universe $\mathcal{X}$. Then, we say that $(\mathcal{F}, A)$ is a soft subset of $(\mathcal{G}, B)$ if

i. $A \subseteq B$;

ii. $\mathcal{F}(\varepsilon)$ and $\mathcal{G}(\varepsilon)$ are identical approximations for all $\varepsilon \in A$.

We denote it as $(\mathcal{F}, A)\widetilde{\subset}(\mathcal{G}, B)$.

Definition 2.3. [2] A soft set $(\mathcal{F}, A)$ over $\mathcal{X}$ is said to be a *null* soft set denoted by Φ, if $\mathcal{F}(\varepsilon) = \emptyset$ for all $\varepsilon \in A$.

Definition 2.4. [2] A soft set $(\mathcal{F}, A)$ over $\mathcal{X}$ is said to be an *absolute* soft set denoted by $\tilde{A}$, if $\mathcal{F}(\varepsilon) = X$ for all $\varepsilon \in A$.

Definition 2.5. [8] The support of a soft set $(\mathcal{F}, A)$ is defined as the set

$$Supp(\mathcal{F}, A) = \{\varepsilon \in A \mid \mathcal{F}(\varepsilon) \neq \emptyset\}.$$

If $Supp(\mathcal{F}, A)$ is not equal to the empty set, then $(\mathcal{F}, A)$ is said to be non-null.

Now, the following generalizations are presented for the nonempty family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ of soft sets over the common universe $\mathcal{X}$.

Definition 2.6. [21] The *restricted intersection* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = \widetilde{\bigcap}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \bigcap_{\alpha \in I} A_\alpha \neq \emptyset$ and $\mathcal{F}(\varepsilon) = \bigcap_{\alpha \in I} \mathcal{F}_\alpha(a)$ for all $\varepsilon \in A_\alpha$.

Definition 2.7. [21] The *restricted union* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = (\bigcup_{\mathcal{R}})_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \bigcap_{\alpha \in I} A_\alpha \neq \emptyset$ and $\mathcal{F}(\varepsilon) = \bigcup_{\alpha \in I} \mathcal{F}_\alpha(\varepsilon)$ for all $\varepsilon \in A_\alpha$.

Definition 2.8. [21] The *extended union* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = \widetilde{\bigcup}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \bigcup_{\alpha \in I} A_\alpha$ and $\mathcal{F}(\varepsilon) = \bigcup_{\alpha \in I(\varepsilon)} \mathcal{F}_\alpha(\varepsilon)$, $I(\varepsilon) = \{\alpha \in I \mid \varepsilon \in A_\alpha\}$ for all $\varepsilon \in A_\alpha$.

Definition 2.9. [21] The *extended intersection* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = (\bigcap_{\mathcal{E}})_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \bigcup_{\alpha \in I} A_\alpha$ and $\mathcal{F}(\varepsilon) = \bigcap_{\alpha \in I(\varepsilon)} \mathcal{F}_\alpha(\varepsilon)$, $I(\varepsilon) = \{\alpha \in I \mid \varepsilon \in A_\alpha\}$ for all $\varepsilon \in A_\alpha$

Definition 2.10. [21] The $\wedge-$*intersection* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = \widetilde{\bigwedge}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \Pi_{\alpha \in I} A_\alpha$ and $\mathcal{F}((\varepsilon_\alpha)_{\alpha \in I}) = \bigcap_{\alpha \in I} \mathcal{F}_\alpha(\varepsilon_\alpha)$ for all $(\varepsilon_\alpha)_{\alpha \in I} \in A_\alpha$.

Definition 2.11. [21] The $\vee-$*intersection* of the family $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ is a soft set $(\mathcal{F}, A) = \widetilde{\bigvee}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha)$ such that $A = \Pi_{\alpha\in I}A_\alpha$ and $\mathcal{F}((\varepsilon_\alpha)_{\alpha\in I}) = \bigcup_{\alpha\in I}\mathcal{F}_\alpha(\varepsilon_\alpha)$ for all $(\varepsilon_\alpha)_{\alpha\in I} \in A_\alpha$.

In the following we give the definitions of hypergroups and topological hypergroups and present some of their properties.

Definition 2.12. [22] Let $\mathcal{H}$ be a non-empty set and $P^*(\mathcal{H})$ denote the family of non-empty subsets of $\mathcal{H}$. Then, the mapping $\cdot : \mathcal{H} \times \mathcal{H} \longrightarrow P^*(\mathcal{H})$ is called a hyperoperation and the pair $(\mathcal{H}, \cdot)$ is also called hypergroupoid.

Definition 2.13. [22] A hypergroup is a hypergroupoid $(\mathcal{H}, \cdot)$ which satises the following axioms:

i. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in \mathcal{H}$

ii. $x \cdot \mathcal{H} = \mathcal{H} \cdot x$ for all $x \in \mathcal{H}$.

Shortly, the symbol $\mathcal{H}$ assigns to the hypergroupoid $(\mathcal{H}, \cdot)$.

Definition 2.14. [15] Let $(\mathcal{H}, \cdot)$ be a hypergroup and $\mathcal{K}$ be a non-empty subset of $\mathcal{H}$. Then $\mathcal{K}$ is called a subhypergroup if $(\mathcal{K}, \cdot)$ is itself a hypergroupoid.

Definition 2.15. [22]Let $(\mathcal{H}, \cdot)$ and $(\mathcal{H}', \star)$ be two hypergroups. Then the mapping $f : \mathcal{H} \longrightarrow \mathcal{H}'$ is said to be

- an *inclusion homomorphism* if $f(x \cdot y) \subseteq f(x) \star f(y)$ for all $x, y \in \mathcal{H}$;
- a *good homomorphism* if $f(x \cdot y) = f(x) \star f(y)$ for all $x, y \in \mathcal{H}$.

Also, a soft hypergroup is defined as follows by examining hypergroups with a soft approach:

Definition 2.16. [15] For a non-null soft set $(\mathcal{F}, A)$ over the hypergroup $\mathcal{H}$, $(\mathcal{F}, A)$ is said to be a soft hypergroup over $\mathcal{H}$ if $\mathcal{F}(\varepsilon)$ is a subhypergroup of $\mathcal{H}$ for all $\varepsilon \in Supp(\mathcal{F}, A)$.

Definition 2.17. [18] Let $(\mathcal{H}, \tau)$ be a topological space. Then, the collection $\mathcal{B}$ consisting of all sets $\mathcal{S}_V = \{U \in P^*(\mathcal{H}) : U \subseteq V, U \in \tau\}$ is a base for a topology on $P^*(\mathcal{H})$ denoted by τ^*.

From now on, the product topology on $\mathcal{H} \times \mathcal{H}$ and the topology τ^* on $P^*(\mathcal{H})$ will be considered. Let us present the definition of topological hypergroups here.

Definition 2.18. [18] Let $(\mathcal{H}, \cdot)$ be a hypergroup and $(\mathcal{H}, \tau)$ be a topological space. Then, the triplet $(\mathcal{H}, \cdot, \tau)$ is said to be a topological hypergroup if the following two conditions are satisfied:

i. The hyperoperation $\cdot : \mathcal{H} \times \mathcal{H} \longrightarrow P^*(\mathcal{H})$) is continuous.

ii. The mapping $\mathcal{H} \times \mathcal{H} \longrightarrow P^*(\mathcal{H})$ defined by $(x, y) \longmapsto x/y$ is continuous, where $x/y = \{z \in \mathcal{H} : x \in z \cdot y\}$.

3. Soft Topological Hypergroups

In this section, we will introduce the concept of soft topological hypergroups by combining soft sets and hypergroups with a topological perspective. Assume $(\mathcal{H}, \cdot)$ be a hypergroup and $P^*(\mathcal{H})$ denotes the power set of $\mathcal{H}$.

Definition 3.1. Let τ be a topology on the hypergroup $\mathcal{H}$. Let $\mathcal{F} : A \longrightarrow P(\mathcal{H})$ be a mapping, where $P(\mathcal{H})$ is the set of all subhypergroups of $\mathcal{H}$, and A is the set of parameters. The triplet $(\mathcal{F}, A, \tau)$ is said to be a soft topological hypergroup over $\mathcal{H}$ if the following axioms hold:

i. $F(\varepsilon)$ is a subhypergroup of $\mathcal{H}$ for all $\varepsilon \in Supp(\mathcal{F}, A)$;

ii. The hyperoperation $\cdot : \mathcal{F}(\varepsilon) \times \mathcal{F}(\varepsilon) \longrightarrow P^*(\mathcal{F}(\varepsilon))$ and the mapping $\mathcal{F}(\varepsilon) \times \mathcal{F}(\varepsilon) \longrightarrow P^*(\mathcal{F}(\varepsilon))$ defined by $(x, y) \longmapsto x/y$ are continuous with respect to the topologies induced by $\tau \times \tau$ and τ^* for all $\varepsilon \in Supp(\mathcal{F}, A)$.

Notice that if $\mathcal{H}$ is a topological hypergroup, it is sufficient that only the first axiom of the above definition holds in order to called the pair $(\mathcal{F}, A)$ as a soft topological hypergroup. In general, the soft topological hypergroup $(\mathcal{F}, A)$ can be regarded as a parameterized family of subhypergroups of the topological hypergroup $\mathcal{H}$.

Theorem 3.2. *Every soft hypergroup on a topological hypergroup is a soft topological hypergroup.*

Proof. Assume H be a topological hypergroup with the topology τ and $(\mathcal{F}, A)$ be a soft hypergroup over $\mathcal{H}$. In this case, $\mathcal{F}(\varepsilon)$ is a subhypergroup of $\mathcal{H}$ for all $\varepsilon \in A$. Hence, $\mathcal{F}(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$ with recpect to the topologies induced by τ and τ^* for all $\varepsilon \in A$. Hence, $(\mathcal{F}, A, \tau)$ is also a soft topological hypergroupoid over $\mathcal{H}$. □

Also, it is straightforward to see that

Remark 3.3. Each soft hypergroup $\mathcal{H}$ can be transformed into a soft topological hypergroup by equipping both $\mathcal{H}$ and $P^*(\mathcal{H})$ with discrete or indiscrete topology. However, every soft hypergroup over a hypergroup is not a soft topological hypergroup.

Theorem 3.4. *Let* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *be a non-empty family of soft topological hypergroups over* $\mathcal{H}$*.*

i. *The restricted intersection of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *with* $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$ *is a soft topological hypergroup over* $\mathcal{H}$ *if it is non-null.*

ii. *The extended intersection of the family* $\{(\mathcal{F}_\alpha, A_\alpha) \mid \alpha \in I\}$ *is a soft topological hypergroup over* $\mathcal{H}$ *if it is non-null.*

Proof. (i) The restricted intersection of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$ defined as the soft set $\widetilde{\bigcap}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau) = (\mathcal{F}, A, \tau)$ such that $\bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon)$ for all $\varepsilon \in A$. Choose $\varepsilon \in Supp(\mathcal{F}, A)$. Assume $\bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon) \neq \emptyset$ such that $\mathcal{F}_\alpha(\varepsilon) \neq \emptyset$ for all $\alpha \in I$. Since $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ is a non-empty family of soft topological hypergroups over $\mathcal{H}$, $\mathcal{F}_\alpha(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$ for all $\alpha \in I$. In this case, $\bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$. Thus, $(\mathcal{F}, A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$.

ii. The proof is similar to **i**. □

Theorem 3.5. *Let* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ *be a non-empty family of soft topological hypergroups over* $\mathcal{H}$*.*

i. *The extended union of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ *is a soft topological hypergroup over* $\mathcal{H}$ *if* $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ *or* $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ *for all* $\alpha, \beta \in I$, $\varepsilon \in \bigcup_{\alpha\in I} A_\alpha$*;*

ii. *The restricted union of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ *is a soft topological hypergroup over* $\mathcal{H}$ *if* $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ *or* $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ *for all* $\alpha, \beta \in I$, $\varepsilon \in \cap_{\alpha\in I} A_\alpha$ *with* $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$*.*

Proof. **i**. Suppose $(\mathcal{F}, A, \tau) = \widetilde{\bigcup}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ as the extended union of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid\alpha \in I\}$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$. Let $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ or $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ for all $\alpha, \beta \in I$, $\varepsilon \in \bigcup_{\alpha\in I} A_\alpha$. Since each $(\mathcal{F}_\alpha, A_\alpha)$ is non-null soft sets over $\mathcal{H}$, then $\bigcup_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha)$ is also a non-null soft set over $\mathcal{H}$ for all $\alpha \in I$. By the hypothesis, $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ or $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ for all $\alpha, \beta \in I$, $\varepsilon \in \bigcap_{\alpha\in I} A_\alpha$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$ so that $\mathcal{F}_\alpha(\varepsilon)$ and $\mathcal{F}_\beta(\varepsilon)$ are the topological subhypergroups of $\mathcal{H}$ and hence their union must be non-null too. Therefore, $\mathcal{F}(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$. Thus, we can conclude that $(\mathcal{F}, A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$.

ii. It can be proved analogously. □

The following result together with previous proposition is straightforward.

Corollary 3.6. *Let* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *be a non-empty family of soft topological hypergroups over* $\mathcal{H}$*. Then the extended union of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *is a soft topological hypergroup over* $\mathcal{H}$ *if* $A_\alpha \cap A_\beta \neq \emptyset$ *for all* $\alpha, \beta \in I$*,* $i \neq j$*.*

Theorem 3.7. *Let* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *be a non-empty family of soft topological hypergroups over* $\mathcal{H}$*.*

i. *The* $\wedge-$*intersection* $\widetilde{\bigwedge}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ *is a soft topological hypergroup over* $\mathcal{H}$ *if it is non-null;*

ii. *The* $\vee-$*union* $\widetilde{\bigvee}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ *is a soft topological hypergroup over* $\mathcal{H}$ *if* $\mathcal{F}_\alpha(\varepsilon_\alpha) \subseteq \mathcal{F}_\beta(\varepsilon_\beta)$ *or* $\mathcal{F}_\beta(\varepsilon_\beta) \subseteq \mathcal{F}_\alpha(\varepsilon_\alpha)$ *for all* $\alpha, \beta \in I$*,* $\varepsilon_\alpha \in A_\alpha$*.*

Proof. **i**. Take $(\mathcal{F}, A, \tau) = \widetilde{\bigwedge}_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ for a non-empty family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ of soft topological hypergroups over $\mathcal{H}$. Let $\varepsilon \in Supp(\mathcal{F}, A)$. By the hypothesis, $\bigcap_{\alpha \in I} \mathcal{F}_\alpha(\varepsilon_\alpha) \neq \emptyset$ which means that $\mathcal{F}_\alpha(\varepsilon_\alpha) \neq \emptyset$ for all $\alpha \in I$ and $(\varepsilon_\alpha)_{\alpha \in I} \in A_\alpha$. Hence, $\mathcal{F}_\alpha(\varepsilon_\alpha)$ is a topological subhypergroup of $\mathcal{H}$ for all $\alpha \in I$ so that their intersection must be a topological subhypergroup of $\mathcal{H}$ too. Therefore, $(\mathcal{F}, A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$.

ii. It can be proved analogously. □

Definition 3.8. Let $\{(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha) \mid \alpha \in I\}$ be a non-empty family of soft topological hypergroups over $\mathcal{H}_\alpha$. Then the cartesian product of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha) \mid \alpha \in I\}$ over $\Pi_{\alpha \in I}\mathcal{H}_\alpha$ is written as $\Pi_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha)$, is defined as $\Pi_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha) = (\mathcal{F}, A, \tau)$ where $A = \Pi_{\alpha \in I}A_\alpha$, the product topology $\tau = \prod_{\alpha \in I} \tau_\alpha$ and $\mathcal{F}(\varepsilon_\alpha) = \Pi_{\alpha \in I}\mathcal{F}_\alpha(\varepsilon_\alpha)$ for all $(\varepsilon_\alpha)_{\alpha \in I} \in A$.

Theorem 3.9. *The cartesian product of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha) \mid \alpha \in I\}$ *is a soft topological hypergroup over* $\Pi_{\alpha \in I}\mathcal{H}_\alpha$*.*

Proof. Assume that $(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha)$ is a soft topological hypergroup over $\mathcal{H}_\alpha$ for all $\alpha \in I$. In this case, $\mathcal{F}_\alpha(\varepsilon) \neq \emptyset$ and $\mathcal{F}_\alpha(\varepsilon_\alpha)$ a topological subhypergroup of $\mathcal{H}_\alpha$ for all $(\varepsilon_\alpha)_{\alpha \in I} \in Supp(\mathcal{F}_\alpha, A_\alpha)$. Also, $\Pi_{\alpha \in I}\mathcal{F}_\alpha(\varepsilon_\alpha) \neq \emptyset$ and $\Pi_{\alpha \in I}\mathcal{F}_\alpha(\varepsilon_\alpha)$ a topological subhypergroup of $\Pi_{\alpha \in I}\mathcal{H}_\alpha$ with the product topology $\Pi_{\alpha \in I}\tau_\alpha$. Thus, $\Pi_{\alpha \in I}(\mathcal{F}_\alpha, A_\alpha, \tau_\alpha)$ is a soft topological hypergroup over $\Pi_{\alpha \in I}\mathcal{H}_\alpha$. This completes the proof. □

3.1. **Soft Topological Hypergroup Homomorphisms.**

Definition 3.10. Let $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau')$ be soft topological hypergroups over $\mathcal{H}$ and $\mathcal{H}'$, respectively. Let $\phi : A \longrightarrow B$ and $\psi : \mathcal{H} \longrightarrow \mathcal{H}'$ be two mappings. Then the pair (ψ, ϕ) is is said to be a soft topological homomorphism if the following axioms hold:

i. ψ is a good homomorphism;

ii. $\psi(\mathcal{F}(\varepsilon)) = \mathcal{K}(\phi(\varepsilon))$ for all $\varepsilon \in Supp(\mathcal{F}, A)$;

iii. $\psi_a : (\mathcal{F}(\varepsilon), \tau_{\mathcal{F}(\varepsilon)}) \longrightarrow (\mathcal{K}(\phi(\varepsilon)), \tau'_{\mathcal{K}(\phi(\varepsilon))})$ continuous and open for all $\varepsilon \in Supp(\mathcal{F}, A)$.

It is obvious that a soft topological homomorphism (ψ, ϕ) is a mapping of soft topological hypergroups. Besides, we form a new category whose objects are soft topological hypergroups and whose arrows are soft topological homomorphisms.

Note that $(\mathcal{F}, A, \tau)$ is soft topologically isomorphic to $(\mathcal{K}, B, \tau')$ if the mappings ψ and ϕ are one to one and onto.

Example 3.11. Let $(\mathcal{K}, B, \tau)$ be a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ over $\mathcal{H}$. Together with the inclusion map $i : B \longrightarrow A$ and the identity map $\mathcal{I} : \mathcal{H} \longrightarrow \mathcal{H}$, the pair $(\mathcal{I}, i)$ is a soft topological homomorphism from $(\mathcal{K}, B, \tau)$ to $(\mathcal{F}, A, \tau)$.

Example 3.12. Let $(\mathcal{F}, A)$ and $(\mathcal{K}, B)$ be the two soft good homomorphic hypergroups defined over $\mathcal{H}$ and $\mathcal{H}'$, respectively. Then $(\mathcal{F}, A, \tau)$ is soft topological homomorphic to $(\mathcal{K}, B, \tau)$ such that τ is discrete or anti-discrete topology. Therefore, any soft good homomorphic hypergroups can be regarded as soft topological homomorphic hypergroups with the discrete or anti-discrete topology.

It is clear that we have the followings:

Theorem 3.13. *Let the pair (ψ, ϕ) be a soft good topological homomorphism from the soft topological hypergroups $(\mathcal{F}, A, \tau)$ to $(\mathcal{K}, B, \tau')$ over $\mathcal{H}$ and $\mathcal{H}'$, respectively. Then $(\psi(\mathcal{F}), B)$ is a soft topological hypergroup over $\mathcal{H}'$ if $\phi : A \longrightarrow B$ be an injective mapping.*

Proof. Suppose that $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau')$ are two soft topological hypergroups over $\mathcal{H}$ and $\mathcal{H}'$, respectively. In this case, $\mathcal{F}(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$ for all $\varepsilon \in Supp(\mathcal{F}, A)$. Since $(\psi, \phi) : (\mathcal{F}, A, \tau) \longrightarrow (\mathcal{K}, B, \tau')$ is a soft topological homomorphism, we obtain

$\phi(Supp(\mathcal{F}, A)) = Supp(\psi(\mathcal{F}), B)$. Consider $b \in Supp(\psi(\mathcal{F}), B)$. Then there exist $\varepsilon \in Supp(\mathcal{F}, A)$ such that $\phi(\varepsilon) = b$ and therefore we have $\mathcal{F}(\varepsilon) \neq \emptyset$. Also, it is clear that $\mathcal{F}(\varepsilon)$ is a topological subhypergroup of $\mathcal{H}$ with respect to the topology induced by τ. Since ψ is a good topological homomorphism, it follows that $\psi(\mathcal{F}(\varepsilon))$ is a topological subhypergroup of $\mathcal{H}'$ with respect to the topology induced by τ'. Thereby,we conclude that $(\psi(\mathcal{F}), B, \tau')$ is a soft topological hypergroup over $\mathcal{H}'$. □

Theorem 3.14. *Let the pair (ψ, ϕ) be a soft topological homomorphism from the soft topological hypergroups $(\mathcal{F}, A, \tau)$ to $(\mathcal{K}, B, \tau')$ over $\mathcal{H}$ and $\mathcal{H}'$, respectively. Then $(\psi^{-1}(\mathcal{K}), A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$ if it is non-null.*

Proof. Consider two soft topological hypergroups $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau')$ over $\mathcal{H}$ and $\mathcal{H}'$, respectively. Then it is simple to verify that

$$\phi(Supp(\psi^{-1}(\mathcal{K}), A)) = \phi^{-1}(Supp(\mathcal{K}, B))$$

for all $b \in Supp(\mathcal{K}, B)$. Taking $a \in Supp(\psi^{-1}(\mathcal{K}), A)$, we get $\phi(\varepsilon) \in Supp(\mathcal{K}, B)$. Hence, the nonempty set $\mathcal{K}(\phi(\varepsilon))$ is a topological subhypergroup of $\mathcal{H}'$ with respect to the topology induced by τ'. Besides, since ψ is a good topological homomorphism, we have that $\psi^{-1}(\mathcal{K}(\phi(\varepsilon))) = \psi^{-1}(\mathcal{K}(\varepsilon))$ is a topological subhypergroup of $\mathcal{H}$ with respect to the topology induced by τ. Therefore, we conclude that $(f^{-1}(\mathcal{K}), A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$. □

Theorem 3.15. *Let $(\mathcal{F}, A, \tau)$, $(\mathcal{K}, B, \tau')$ and $(\mathcal{N}, C, \tau'')$ be soft topological hypergroups over $\mathcal{H}$, $\mathcal{H}'$ and $\mathcal{H}''$, respectively. If $(\psi, \phi) : (\mathcal{F}, A, \tau) \longrightarrow (\mathcal{K}, B, \tau')$ and $(\psi', \phi') : (\mathcal{K}, B, \tau') \longrightarrow (\mathcal{N}, C, \tau'')$ are two soft topological homomorphisms, then $(f' \circ f, g' \circ g) : (\mathcal{F}, A) \longrightarrow (\mathcal{N}, C)$ is a soft topological homomorphism.*

Proof. Let $(\psi, \phi) : (\mathcal{F}, A, \tau) \longrightarrow (\mathcal{K}, B, \tau')$ and $(\psi', \phi') : (\mathcal{K}, B, \tau') \longrightarrow (\mathcal{N}, C, \tau'')$ be two soft topological homomorphisms. By Definition 3.10, it follows that $\psi : \mathcal{H} \longrightarrow \mathcal{H}'$ and $\psi' : \mathcal{H}' \longrightarrow \mathcal{H}''$ are two good homomorphisms, and $\phi : A \longrightarrow B$ and $\phi' : B \longrightarrow C$ are two mappings such that the axioms $\psi(\mathcal{F}(\varepsilon)) = \mathcal{K}(\phi(\varepsilon))$ and $\psi'(\mathcal{K}(\epsilon)) = \mathcal{N}(\phi'(\epsilon))$ hold for all $\varepsilon \in Supp(\mathcal{F}, A)$, $\epsilon \in Supp(\mathcal{K}, B)$. Hence, we easily verify that $\psi' \circ \psi : \mathcal{H} \longrightarrow \mathcal{H}''$ is also good topological homomorphism and $\phi' \circ \phi : A \longrightarrow C$ is a mapping so that the equation

$$(\psi' \circ \psi)(\mathcal{F}(\varepsilon)) = \psi'(\psi(\mathcal{F}(\varepsilon))) = \psi'(\mathcal{K}(\phi(\varepsilon))) = \mathcal{N}(\phi'(\phi(\varepsilon))) = \mathcal{N}((\phi' \circ \phi)(\varepsilon))$$

holds for all $\varepsilon \in Supp(\mathcal{F}, A)$. Consequently, the pair $(\psi' \circ \psi, \phi' \circ \phi)$ is a soft topological homomorphism from $(\mathcal{F}, A, \tau)$ to $(\mathcal{N}, C, \tau'')$. □

3.2. **Soft Topological Subhypergroups.**

Definition 3.16. Let $(\mathcal{F}, A, \tau)$ be a soft topological hypergroup over $\mathcal{H}$. Then, $(\mathcal{K}, B, \tau)$ is said to be a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if the following axioms hold:

i. $B \subseteq A$;

ii. $\mathcal{K}(b)$ is a subhypergroup of $\mathcal{F}(b)$ for all $b \in Supp(\mathcal{K}, B)$;

iii. The hyperoperation $\cdot : \mathcal{K}(b) \times \mathcal{K}(b) \longrightarrow P^*(\mathcal{K}(b))$ and the mapping $\mathcal{K}(b) \times \mathcal{K}(b) \longrightarrow P^*(\mathcal{K}(b))$ defined by $(x, y) \longmapsto x/y$ are continuous with respect to the topologies induced by $\tau \times \tau$ and τ^* for all $b \in Supp(\mathcal{K}, B)$.

Example 3.17. Suppose $(\mathcal{F}, A, \tau)$ is a soft topological hypergroup over $\mathcal{H}$. Then, $(\mathcal{F}|_B, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if $B \subseteq A$.

From the above definition, one easily deduces that

Theorem 3.18. *If $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ and $(\mathcal{N}, C, \tau)$ is a soft topological subhypergroup of $(\mathcal{K}, B, \tau)$, then $(\mathcal{N}, C, \tau)$ is the soft topological subhypergroup of $(\mathcal{F}, A, \tau)$.*

Proof. The proof is obvious. □

It is also straightforward to see that

Theorem 3.19. *Let $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau)$ be two soft topological hypergroups over $\mathcal{H}$. Then $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if $(\mathcal{K}, B)$ is a soft subset of $(\mathcal{F}, A)$.*

Proof. Suppose that $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau)$ are two soft topological hypergroups over $\mathcal{H}$. Then, if $(\mathcal{K}, B)$ is a soft subset of $(\mathcal{F}, A)$, we have $B \subseteq A$ and $\mathcal{K}(b) \subseteq \mathcal{F}(b)$ for all $b \in Supp(\mathcal{K}, B)$. Thus, $\mathcal{K}(b)$ is a topological subhypergroup of $\mathcal{F}(b)$ with respect to the topology induced by τ. Clearly, $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$. □

Now, we state and prove the generalized characterizations for soft topological subhypergroups.

Theorem 3.20. *Let $(\mathcal{F}, A, \tau)$ be a soft topological hypergroup over $\mathcal{H}$ and $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ be a non-empty family of soft topological subhypergroups of $(\mathcal{F}, A, \tau)$.*

i. *The restricted intersection of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ with $\bigcap_{\alpha \in I} A_\alpha \neq \emptyset$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if*

$\tilde{\bigcap}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau) \neq \emptyset$.

ii. *The extended intersection of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *is a soft topological subhypergroup of* $(\mathcal{F}, A, \tau)$ *if* $(\bigcap_{\mathcal{E}})_{\alpha\in I} \neq \emptyset$.

Proof. **i**. The restricted intersection of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$ given by the soft set $\tilde{\bigcap}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau) = (\mathcal{F}, A, \tau)$ such that $\mathcal{F}(\varepsilon) = \bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon)$ for all $\varepsilon \in A$. Take $\varepsilon \in Supp(\mathcal{F}, A)$. Suppose $\bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon) \neq \emptyset$ so that $\mathcal{F}_\alpha(\varepsilon) \neq \emptyset$ for all $\alpha \in I$. Since $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ is a non-empty family of soft topological subhypergroups of $(\mathcal{F}, A, \tau)$, this implies that $A_\alpha \subseteq A$ and $\mathcal{F}_\alpha(\varepsilon)$ is a topological subhypergroup of $\mathcal{F}(\varepsilon)$ with respect to the topology induced by τ for all $\alpha \in I$. Hence, $\bigcap_{\alpha\in I} A_\alpha \subseteq A$ and $\bigcap_{\alpha\in I} \mathcal{F}_\alpha(\varepsilon)$ is a topological subhypergroup of $\mathcal{F}(\varepsilon)$. That is, the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$

The second claim can be proved similarly. □

Theorem 3.21. *Let* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *be a non-empty family of soft topological subhypergroups of a soft topological hypergroup* $(\mathcal{F}, A, \tau)$ *over* $\mathcal{H}$.

i. *The extended union of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *is a soft topological subhypergroup of* $(\mathcal{F}, A, \tau)$ *if* $f_\alpha(\varepsilon) \subseteq f_\beta(\varepsilon)$ *or* $f_\beta(\varepsilon) \subseteq f_\alpha(\varepsilon)$ *for all* $\alpha, \beta \in I$, $\varepsilon \in \bigcup_{\alpha\in I} A_\alpha$.

ii. *The restricted union of the family* $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ *is a soft topological subhypergroup of* $(\mathcal{F}, A, \tau)$ *if* $f_\alpha(\varepsilon) \subseteq f_\beta(\varepsilon)$ *or* $f_\beta(\varepsilon) \subseteq f_\alpha(\varepsilon)$ *for all* $\alpha, \beta \in I$, $\varepsilon \in \bigcap_{\alpha\in I} A_\alpha$ *with* $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$.

Proof. **i**. Suppose that $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ is a non-empty family of soft topological subhypergroups of a soft topological hypergroup $(\mathcal{F}, A, \tau)$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$. Let $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ or $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ for all $\alpha, \beta \in I$, $\varepsilon \in \bigcup_{\alpha\in I} A_\alpha$. Take $\varepsilon \in Supp(\mathcal{F}, A)$. Since each $(\mathcal{F}_\alpha, A_\alpha)$ is non-null soft sets over $\mathcal{H}$, $\widetilde{\bigcup}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha)$ is also a non-null soft set over $\mathcal{H}$ for all $\alpha \in I$. . On the other hand, $\mathcal{F}_\alpha(\varepsilon) \subseteq \mathcal{F}_\beta(\varepsilon)$ or $\mathcal{F}_\beta(\varepsilon) \subseteq \mathcal{F}_\alpha(\varepsilon)$ for all $\alpha, \beta \in I$, $\varepsilon \in \bigcap_{i\in I} A_\alpha$ with $\bigcap_{\alpha\in I} A_\alpha \neq \emptyset$ implies that $\mathcal{F}_\alpha(\varepsilon)$ and $\mathcal{F}_\beta(\varepsilon)$ are the topological subhypergroups of $\mathcal{F}(\varepsilon)$ with respect to the topology induced by τ and so their union must be non-null too. Therefore, the extended union of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \mid \alpha \in I\}$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$.

ii. Similar to the proof of **i**. □

Here, it is not hard to see that

Corollary 3.22. *Let $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ be a non-empty family of soft topological subhypergroups of a soft topological hypergroup $(\mathcal{F}, A, \tau)$ over $\mathcal{H}$ with the topology τ. Then, the extended union of the family $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ with the topology τ if $A_\alpha \cap A_\alpha \neq \emptyset$ for all $\alpha, \beta \in I$, $\alpha \neq \beta$.*

Theorem 3.23. *Let $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ be a non-empty family of soft topological hypergroups over $\mathcal{H}$ and let $(\mathcal{K}_\alpha, B_\alpha, \tau)$ be a soft topological subhypergroup of $(\mathcal{F}_\alpha, A_\alpha, \tau)$ for all $\alpha \in I$.*

i. *The $\wedge-$intersection $\widetilde{\bigwedge}_{\alpha\in I}(\mathcal{K}_\alpha, B_\alpha, \tau)$ is a soft topological subhypergroup of $\widetilde{\bigwedge}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ if it is non-null.*

ii. *The $\vee-$union $\widetilde{\bigvee}_{\alpha\in I}(\mathcal{K}_\alpha, B_\alpha, \tau)$ is a soft topological subhypergroup of $\widetilde{\bigvee}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ if $\mathcal{K}_\alpha(\epsilon_\alpha) \subseteq \mathcal{K}_\beta(\epsilon_\beta)$ or $\mathcal{K}_\beta(\epsilon_\beta) \subseteq \mathcal{K}_\alpha(\epsilon_\alpha)$ for all $\alpha, \beta \in I$, $\epsilon_\alpha \in B_\alpha$.*

Proof. **i**. Consider $\{(\mathcal{F}_\alpha, A_\alpha, \tau) \; |\alpha \in I\}$ as a non-empty family of soft topological hypergroups over $\mathcal{H}$. By Theorem 3.5 (**ii**.), $\widetilde{\bigvee}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$ is a soft topological hypergroup over $\mathcal{H}$. Let $\epsilon_\alpha \in Supp(\mathcal{K}_\alpha, B_\alpha)$. Also, $\bigcap_{\alpha\in I} \mathcal{K}_\alpha(\epsilon_\alpha) \neq \emptyset$ which implies that $\mathcal{K}_\alpha(\epsilon_\alpha) \neq \emptyset$ for all $\alpha \in I$ and $(\epsilon_\alpha)_{\alpha\in I} \in B_i$. Further, $B_\alpha \subseteq A_\alpha$ and $\mathcal{K}_\alpha(\epsilon_\alpha)$ is a topological subhypergroup of $\mathcal{F}_\alpha(\epsilon_\alpha)$ with respect to the topology induced by τ for all $\alpha \in I$ such that $\bigcap_{\alpha\in I} B_\alpha \subseteq \bigcap_{\alpha\in I} A_\alpha$ and $\bigvee_{\alpha\in I}(\mathcal{K}_\alpha(\epsilon_\alpha))$ must be a topological subhypergroup of $\bigvee_{\alpha\in I}(\mathcal{F}_\alpha(\epsilon_\alpha))$ too. Hence, $\widetilde{\bigwedge}_{\alpha\in I}(\mathcal{K}_\alpha, B_\alpha, \tau)$ is a soft topological subhypergroup of $\widetilde{\bigwedge}_{\alpha\in I}(\mathcal{F}_\alpha, A_\alpha, \tau)$.

ii. It can be proved as in the previous case. □

Theorem 3.24. *Let $(\mathcal{F}, A, \tau)$ be a soft topological hypergroup over $\mathcal{H}$ and $(\mathcal{K}, B, \tau)$ be a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$.*

i. *The restricted intersection of $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if it is non-null.*

ii. *The restricted union of $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ if it is non-null.*

Proof. **i**. Assume that $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ over H. If it is non-null, then $B \subseteq A$ and $\mathcal{K}(\epsilon)$ is a topological subhypergroup of $\mathcal{F}(\epsilon)$ with respect to the topology induced by τ for all $\epsilon \in Supp(\mathcal{K}, B)$. So it is easy to see that $A \cap B \subseteq A$ and $\mathcal{K}(\epsilon) \cap \mathcal{F}(\epsilon)$ is also a topological subhypergroup of $\mathcal{F}(\epsilon)$ with respect to the topology induced by τ for all $\epsilon \in Supp(\mathcal{K}, B)$. Therefore, the restricted intersection $(\mathcal{F}, A, \tau)\tilde{\cap}(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$.

ii. It is similar to the proof of previous case. □

Theorem 3.25. *Let $f : \mathcal{H} \longrightarrow \mathcal{H}'$ be a good homomorphism of topological hypergroups with the topologies τ and τ', respectively, and let $(\mathcal{F}, A, \tau')$ and $(\mathcal{K}, B, \tau')$ be two soft topological hypergroups over $\mathcal{H}'$. Then $(f^{-1}(\mathcal{K}), B, \tau)$ is a soft topological subhypergroup of $(f^{-1}(\mathcal{F}), A, \tau)$ if $(\mathcal{K}, B, \tau')$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau')$.*

Proof. Assume $(\mathcal{K}, B, \tau')$ be a soft topological subhypergroup of $(\mathcal{F}, A, \tau')$ over $\mathcal{H}$. Take $\epsilon \in Supp(f^{-1}(\mathcal{K}), B)$. Since $(\mathcal{K}, B, \tau')$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau')$, then $B \subseteq A$ and $(\mathcal{K}(b))$ is a topological subhypergroup of $(\mathcal{F}(\epsilon)$ with respect to the topology induced by τ' for all $\epsilon \in Supp(f^{-1}(\mathcal{K}), B)$. On the other hand, since $f : \mathcal{H} \longrightarrow \mathcal{H}'$ be a good topological homomorphism, it follows that$f^{-1}(\mathcal{F})(\epsilon) = f^{-1}(\mathcal{F}(\epsilon))$ is a topological subhypergroup of $f^{-1}(\mathcal{K})(\epsilon) = f^{-1}(\mathcal{K}(\epsilon))$ with respect to the topology induced by τ for all $\epsilon \in Supp(f(\mathcal{K}), B)$. Thus, it has been proven that $(f^{-1}(\mathcal{K}), B, \tau)$ is a soft topological subhypergroup of $(f^{-1}(\mathcal{F}), A, , \tau)$. □

Theorem 3.26. *Let $f : \mathcal{H} \longrightarrow \mathcal{H}'$ be a good homomorphism of topological hypergroups with the topologies τ and τ', respectively, and let $(\mathcal{F}, A, \tau)$ and $(\mathcal{K}, B, \tau)$ be two soft topological hypergroups over $\mathcal{H}$. Then $(f(\mathcal{K}), B, \tau')$ is a soft topological subhypergroup of $(f(\mathcal{F}), A, \tau')$ over $\mathcal{H}'$ if $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$.*

Proof. Suppose that $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$ over $\mathcal{H}$. If $(\mathcal{K}, B, \tau)$ is a soft topological subhypergroup of $(\mathcal{F}, A, \tau)$, in this case $B \subseteq A$ and $(\mathcal{K}(\epsilon))$ is a topological subhypergroup of $(\mathcal{F}(\epsilon)$ with respect to the topology induced by τ for all $\epsilon \in Supp(\mathcal{K}, B)$. Also, since $f : \mathcal{H} \longrightarrow \mathcal{H}'$ be a good topological homomorphism, it follows that $f(\mathcal{F})(\epsilon) = f(\mathcal{F}(\epsilon))$ is a topological subhypergroup of $f(\mathcal{K})(\epsilon) = f(\mathcal{K}(\epsilon))$ with respect to the topology induced by τ' for all $\epsilon \in Supp(f(\mathcal{K}), B)$. Consequently, $(f(\mathcal{K}), B, \tau')$ is a soft topological subhypergroup of $(f(\mathcal{F}), A, \tau')$, and the proof is complete. □

References

[1] D. A. Molodtsov, *Soft set theory-First results*, Comput. Math. Appl., **37**(4-5) (1999), 19–31.

[2] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Comput. Math. Appl., **45**(4-5) (2003), 555–562.

[3] F. Marty, *Sur une Generalisation de la Notion de Groupe*, 8th Congress Mathematiciens Scandinaves, Stockholm, 1934, pp. 45–49.

[4] A. O. Atagun and A. Sezgin, *Soft substructures of rings, fields and modules*, Comput. Math. Appl., **61** (2011), 592–601.

[5] M. Shabir, M. Naz, *On soft topological spaces*, Comput. Math. Appl., **61**(7) (2011), 1786–1799.

[6] T. Shah, S. Shaheen, *Soft topological groups and rings*, Ann. Fuzzy Math. Inform., **7**(5)(2014), 725–743.

[7] P.K. Maji, A.R. Roy and R. Biswas, *An application of soft sets in a decision making problem*, Comput. Math. Appl., **44** (2002), 1077–1083.

[8] F. Feng, Y.B. Jun and X. Zhao, *Soft semirings*, Comput. Math. Appl., **56** (2008), 2621–2628.

[9] U. Acar, F. Koyuncu and B. Tanay, *Soft sets and soft rings*, Comput. Math. Appl., **59**(2010), 3458–3463.

[10] G, Oguz, M.H. Gursoy and I. Icen, *On soft topological categories*, Hacet. J. Math. Stat., **48**(6)(2019), 1675–1681.

[11] S. Nazmul ,S.K. Samanta, *Soft topological groups*, Kochi.j.Math., **5**(2010), 151–161.

[12] Y.B. Jun, *Soft BCK/BCI-algebras*, Comput. Math. Appl., **56** (2008), 1408–1413.

[13] G, Oguz, I. Icen and M.H. Gursoy,*Actions of soft groups*,Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat., **68**(1)(2019), 1163–1174.

[14] A. Aygunoglu, A. Aygun, *Some notes on soft topological spaces*, Neural Comput. Appl., **22**(1) (2012), 113–119.

[15] G. Selvachandran, A. R. Salleh,*Soft hypergroups and soft hypergroup homomorphism*, AIP Proceedings of the 20th National Symposium on Mathematical Sciences, 2013, pp. 821–827.

[16] P.Corsini, V. Leoreanu, *Applications of Hyperstructure Theory*, in: Advances in Math-ematics, vol. 5, Kluwer Academic Publishers Boston, Mass, 2003.

[17] J. Wang, M. Yin and W. Gu, *Soft polygroups*, Comput. Math. Appl., **62**(9) (2011), 3529–3537.

[18] D. Heidari, B. Davvaz and S.M.S. Modarres, *Topological hypergroups in the sense of Marty*, Communications in Algebra, **42**(11) (2014): 4712–4721.

[19] S.Yamak, O. Kazanci and B. Davvaz, *Soft hyperstructure*, Comput. Math. with Appl. **62**(2) (2011), 797-803.

[20] G. Selvachandran, *Introduction to the theory of soft hyperrings and soft hyperring homomorphism*, JP Journal of Algebra, Number Theory and Applications, **36**(3) (2015), 279–294.

[21] O. Kazanci, S. Yilmaz, S. Yamak,*Soft sets and soft BCH-algebras*, Hacettepe J. Math. Statistics **39** (2010), 205–217.

[22] D. Heidari, B.Davvaz and S. M. S. Modarres, *Topological polygroups*, Bull. Malaysian Math. Sci. Soc., **39**(2)(2016), 707–721.

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