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THE MODIFIED SIMPLE EQUATION METHOD FOR THE TWO SPACE-TIME NONLINEAR FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. Many important phenomena in various fields are described and generalized by a fractional partial differential equation. In this paper, the modified simple equation method which is widely applicable to handle nonlinear wave equations, is successfully implemented for constructing exact solutions of two nonlinear fractional equations, namely the space-time nonlinear fractional potential Kadomstev- Petviashvili (PKP) and Sharma-Tasso- Olver (STO) equations in the sense of the modified Riemann-Liouville derivative. As a result, some new exact solutions are successfully obtained for them

Key Words: Modified simple equation method, Space-time nonlinear fractional differential equation, Exact solution.

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1. Introduction

Fractional calculus is a field of mathematics that grows out of the traditional definitions of the calculus integral and derivative operators. Most of the mathematical theories are applicable to the study of fractional calculus. This calculus was developed prior to the turn of the 20th century. Recent monographs and symposia proceedings have highlighted the application of fractional calculus in physics, continuum mechanics, signal processing, and electromagnetics [7]. The advantage of

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the method of fractional derivatives in theory of viscoelasticity is that it affords possibilities for obtaining constitutive equations for elastic complex modulus of viscoelastic materials with only few experimentally determined parameters. Also the fractional derivative method has been used in studies of the complex moduli and impedances for various models of viscoelastic substances. Other applications of the fractional calculus could be pointed to modeling of speech signals fractional [28] and modeling the Cardiac Tissue Electrode interface [25], use of Fractional Order Controllers (FOC) to the path tracking problem in an autonomous electric vehicle, using time derivatives of fractional order to describe the behavior of sound waves in this kind of materials, including relaxation and frequency dependence [10], Wave propagation in viscoelastic horns using a fractional calculus rheology model [29], application of fractional calculus to fluid mechanics to the solution of time-dependent viscous diffusion fluid mechanics problems [21]. Many important phenomena in various fields are described and generalized by a fractional partial differential equation such as fluid flow, finance, Physics, fractal phenomena], biological processes, and Control Systems [4,8,17]. Recently, considerable attention has been given to find the exact solutions of these equations.

Here two nonlinear fractional partial differential equations are investigated as follows. The space-time nonlinear fractional potential Kadomstev- Petviashvili (PKP) equation

$$\frac{1}{4}D_x^{4\alpha}u+\frac{3}{2}D_x^{\alpha}uD_x^{2\alpha}u+\frac{3}{4}D_y^{2\alpha}u+D_t^{\alpha}u(D_x^{\alpha}u)=0,\quad 0<\alpha\leq 1. \tag{1.1}$$

The space-time nonlinear fractional Sharma-Tasso-Olver (STO) equation

$$D_t^{\alpha}u+3\beta\left(D_x^{\alpha}u\right)^2+3\beta u^2D_x^{\alpha}u+3\beta uD_x^{2\alpha}u+\beta D_x^{2\alpha}u=0,\quad 0<\alpha\leq 1.$$

(1.2)

There are several definitions of a fractional derivative of order $\alpha>0$. Two most commonly used definitions are the Riemann–Liouville and Caputo. Each definition uses Riemann–Liouville fractional integration and derivatives of whole order. Here, the Jumarie's fractional derivative [11, 14], which is a modified Riemann-Liouville derivative [26], will be used. Li and He [22] proposed a fractional complex transformation to convert fractional differential equations into ordinary differential equations (ODE), so all analytical methods which are devoted to the advanced calculus can easily be applied to the fractional calculus. Some of the current powerful methods for deriving exact solutions of the partial fractional differential equations are such as the fractional sub-equation method [1], the first integral method [2,3,5,12,24], the (G'/G)-expansion method [9,16], and so on [6,18,27].

In this paper, the modified simple equation method (MSE) has been applied to solve Eq. (1.1) and (1.2). MSE method is a powerful mathematical technique to find exact solutions of nonlinear differential equations. It has been developed by Kudryashov [19,20] and many authors used this method successfully for finding exact solutions of PDEs in mathematical physics [15,31]. In the MSE method, exact solutions is assumed to be expressed by a polynomial in φ'/φ, such that $\varphi = \varphi(\xi)$ is an unknown function to be determined later.

The rest of the paper is organized as follows.
In Section 2, some definitions and properties of the fractional calculus are reviewed. Description of the MSE method combined with fractional calculus is presented in section 3. In Section 4, this method is applied to the nonlinear fractional equations pointed out above and the conclusion of the article appears in Section 5.

2. Basic Definitions of fractional calculus

Let's give some definitions and properties of the fractional calculus. There are several definitions of a derivative of fractional order. The two most commonly used definitions are the Riemann–Liouville and Caputo. Each definition uses Riemann–Liouville fractional integration and derivatives of whole order. We will be applied in this paper the Jumarie's fractional derivative that is a modified Riemann-Liouville derivative [26].

Definition. Jumarie's fractional derivative is a modified Riemann-Liouville derivative [14], defined as:

(2.1)
$$D_x^\alpha f(x) = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_0^x (x-\xi)^{-\alpha-1} \left(f(\xi) - f(0)\right) d\xi, & \alpha < 0, \\ \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x (x-\xi)^{-\alpha} \left(f(\xi) - f(0)\right) d\xi, & 0 < \alpha < 1, \\ \left[f^{(\alpha-n)}(x)\right]^{(n)}, & n \leq \alpha < n+1,\ n \geq 1. \end{cases}$$

Where $f : \mathbb{R} \to \mathbb{R}$, $x \to f(x)$ denotes a continuous but not necessarily differentiable function. Some important properties of Jumaries modified Riemann-Liouville derivative are as follows.

(1) $D_x^\alpha c = 0, \quad \alpha > 0, \qquad c = \text{constant},$
(2) $D_x^\alpha [cf(x)] = cD_x^\alpha f(x), \quad \alpha > 0, \qquad c = \text{constant},$
(3) $D_x^\alpha x^\beta = \frac{\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)} x^{\beta-\alpha}, \quad \beta > \alpha > 0,$
(4) $D_x^\alpha [f(x)g(x)] = [D_x^\alpha f(x)]g(x) + f(x)[D_x^\alpha g(x)],$
(5) $D_x^\alpha f(x(t)) = f'(x) \cdot x^{(\alpha)}(t).$

3. Description of the MSE method combined with fractional calculus

Let's, consider the following general nonlinear fractional differential equations

$$F(u, D_t^\alpha u, D_x^\alpha u, ...) = 0, \quad 0 < \alpha \leq 1, \tag{3.1}$$

where $D_t^\alpha u$,$D_x^\alpha u$,. . . are the modified Riemann-Liouville derivatives, and F is a polynomial in $u(x,t)$and its fractional partial derivatives, in which the highest order derivatives and the nonlinear terms are involved. The main steps

of the MSE method for solving nonlinear fractional equations are given as follows.

Step1. By using the nonlinear fractional complex transformation [22,23]

$$\begin{aligned} &u(x,y,t) = u(\xi), \\ &\xi = \tfrac{\tau x^\beta}{\Gamma(1+\beta)} + \tfrac{\delta y^\psi}{\Gamma(1+\psi)} + \tfrac{\lambda t^\alpha}{\Gamma(1+\alpha)}, \end{aligned} \tag{3.2}$$

where τ, δ, and λ are nonzero arbitrary constants, Eq. (3.1) is reduced to an ordinary differential equation

$$Q(u, u', u'', u''', ...) = 0, \tag{3.3}$$

where the prime denotes the derivation with respect to ξ.

Step2. Suppose that the solution of Eq.(3.4) can be expressed by a polynomial in $\frac{\varphi'}{\varphi}$ as the following form:

$$u(\xi) = \sum_{i=0}^{N} c_i \left(\frac{\varphi'(\xi)}{\varphi(\xi)}\right)^i, \tag{3.4}$$

Where c_i's are constants to be determined, with $c_N \neq 0$, and $\varphi(\xi)$is an unknown function to be determined later, such that $\varphi'(\xi) \neq 0$.

Step3. The positive integer N in (3.4) is determined by balancing the highest order derivatives with the highest order nonlinear term in Eq. (3.3).

Step4. Eq.(3.4) is substituted in Eq.(3.3) and all the necessary derivatives u', u", u"',... of the unknown function $u(\xi)$ are calculated. As a result of this substitution, a polynomial of $\frac{\varphi'(\xi)}{\varphi(\xi)}$ and its derivatives are obtained. In this polynomial, the terms of the same power and its derivatives are collected, and all the coefficients of this polynomial will be equated to zero. This operation yields to develop a system of equations which can be solved without using the computer programs to findc_i's and$\varphi(\xi)$. Consequently, the exact solutions of Eq. (3.1) will be obtained.

4. Applications

In this section, the method proposed in Sec. 3, is applied to find the exact solutions of the space-time nonlinear fractional PKP and STO equations.

4.1. **The space-time nonlinear fractional PKP equation.** By using the nonlinear fractional complex transformation

$$\begin{aligned} &u\left(x,y,t\right)=u\left(\xi\right),\\ &\xi=\tfrac{k_1x^{\alpha}}{\Gamma(1+\alpha)}+\tfrac{k_2y^{\alpha}}{\Gamma(1+\alpha)}+\tfrac{ct^{\alpha}}{\Gamma(1+\alpha)}+\xi_0,\end{aligned}$$

Where k_1, k_2, c, ξ_0 are constants, Eq. (1.1) is reduced to the following ODE:

$$k_1^4u'''+3k_1u'^2+(3k_2^2+4ck_1)u'=0, \tag{4.1}$$

Balancing u''' with u'^2 gives $N=1$ and then Eq.(3.4) as a solution for Eq. (4.1) has the following form

$$u(\xi)=c_0+c_1(\frac{\varphi'}{\varphi}). \tag{4.2}$$

Where c_0 and c_1 are constants to be determined later, with $c_1\neq 0$ and $\varphi(\xi)$ is also to be determined.
So we drive

$$u'=c_1\left(\frac{\varphi''}{\varphi}-\frac{\varphi'^2}{\varphi^2}\right), \tag{4.3}$$

$$u'^2=c_1^2\left(\frac{\varphi''^2}{\varphi^2}-2\frac{\varphi''\varphi'^2}{\varphi^3}+\frac{\varphi'^4}{\varphi^4}\right), \tag{4.4}$$

$$u'''=c_1\left(\frac{\varphi^{(4)}}{\varphi}-4\frac{\varphi'''\varphi'}{\varphi^2}+12\frac{\varphi''\varphi'^2}{\varphi^3}-3\frac{\varphi''^2}{\varphi^2}-6\frac{\varphi'^4}{\varphi^4}\right), \tag{4.5}$$

Equating all the coefficients of $\varphi^0, \varphi^{-1}, \varphi^{-2}, \varphi^{-3}$and φ^{-4} to zero, following equations are obtained:

$$c_1\left(k_1^4\varphi^{(4)} + (3k_2^2 + 4ck_1)\varphi''\right) = 0, \tag{4.6}$$

$$-c_1\left(4k_1^4\varphi'''\varphi' + 3k_1^4\varphi''^2 - 3k_1c_1\varphi''^2 + (3k_2^2 + 4ck_1)\varphi'^2\right) = 0, \tag{4.7}$$

$$c_1\varphi''\varphi'^2\left(12k_1^4 - 6k_1c_1\right) = 0, \tag{4.8}$$

$$c_1\varphi'^4\left(-6k_1^4 + 3k_1c_1\right) = 0. \tag{4.9}$$

Since $c_1 \neq 0$ and $\varphi'(\xi) \neq 0$, from Eq. (4.8) or (4.9) it can be deduced that $c_1 = 2k_1^3$. Therefore, Eq. (4.7) will be reduced to

$$4k_1^4\varphi'''\varphi' - 3k_1^4\varphi''^2 + \omega\varphi'^2 = 0, \tag{4.10}$$

where $\omega = 3k_2^2 + 4ck_1$. By integrating Eq.(4.6) and using Eq.(4.10), the following equation will be found:

$$\frac{\varphi'''}{\varphi''} = \frac{\pm\sqrt{-\omega}}{k_1^2}, \tag{4.11}$$

with zero constant of integration, and $\omega \neq 0$. Therefore, it can be deduced that

$$\varphi' = \pm A\frac{k_1^2}{\sqrt{-\omega}}\exp\left(\pm\frac{\sqrt{-\omega}}{k_1^2}\xi\right), \tag{4.12}$$

$$\varphi = B + A\frac{k_1^4}{\omega}\exp\left(\pm\frac{\sqrt{-\omega}}{k_1^2}\xi\right), \tag{4.13}$$

where A and B are constants of integration.

Now, from Eq. (4.2) an exact solution to nonlinear fractional PKP equation can be written as follows

$$u(x,y,t) = c_0 \pm 2Ak_1\left(3k_2^2 + 4ck_1\right) \times \left(\frac{\sqrt{-\left(3k_2^2 + 4ck_1\right)}\exp\left(\pm\frac{\sqrt{-\left(3k_2^2+4ck_1\right)}}{k_1^2}\xi\right)}{B\left(3k_2^2 + 4ck_1\right) + Ak_1^4\exp\left(\pm\frac{\sqrt{-\left(3k_2^2+4ck_1\right)}}{k_1^2}\xi\right)}\right),$$

where c_0 is an arbitrary constant and $\xi = \frac{k_1x^\alpha}{\Gamma(1+\alpha)} + \frac{k_2y^\alpha}{\Gamma(1+\alpha)} + \frac{2k_1^3t^\alpha}{\Gamma(1+\alpha)} + \xi_0$.

4.2. **The space-time nonlinear fractional STO equation.** Applying transformation

$$u\left(x,t\right)=u\left(\xi\right),\ \xi=\tfrac{kx^{\alpha}}{\Gamma(1+\alpha)}+\tfrac{ct^{\alpha}}{\Gamma(1+\alpha)}+\xi_{0},$$

will change the Eq. (1.2) into the following form.

$$cu+3\beta k^{2}uu'+\beta ku^{3}+\beta k^{3}u''=0. \tag{4.14}$$

To apply MSE method, we consider the homogeneous balance between u'' with u^3 . So we drive $N=1$, and Eq. (3.4) turns to the following simple form

$$u(\xi)=c_{0}+c_{1}(\frac{\varphi'}{\varphi}). \tag{4.15}$$

Therefore, it can be derive

$$uu'=c_{0}c_{1}\frac{\varphi''}{\varphi}-c_{0}c_{1}\frac{\varphi'^{2}}{\varphi^{2}}+c_{1}^{2}\frac{\varphi'\varphi''}{\varphi^{2}}-c_{1}^{2}\frac{\varphi'^{3}}{\varphi^{3}}, \tag{4.16}$$

$$u''=c_{1}\left(\frac{\varphi'''}{\varphi}-3\frac{\varphi''}{\varphi}+2\frac{\varphi'^{3}}{\varphi^{3}}\right), \tag{4.17}$$

$$u^{3}=c_{1}^{3}\left(\frac{\varphi'}{\varphi}\right)^{3}+3c_{1}^{2}c_{0}\left(\frac{\varphi'}{\varphi}\right)^{2}+3c_{1}c_{0}^{2}\frac{\varphi'}{\varphi}+c_{0}^{3}, \tag{4.18}$$

By substituting (4.15)-(4.18) into Eq. (4.14), and equating all the coefficients of $\varphi^{0},\varphi^{-1},\varphi^{-2}$,and φ^{-3} to zero, the following equations are obtained:

$$c_{0}\left(c+\beta kc_{0}^{2}\right)=0, \tag{4.19}$$

$$c_{1}\left(c\varphi'+3\beta k^{2}c_{0}\varphi''+3\beta kc_{0}^{2}\varphi'+\beta k^{3}\varphi'''-3\beta k^{3}\varphi''\right)=0, \tag{4.20}$$

$$3\beta kc_{1}\varphi'\left((c_{1}c_{0}-kc_{0})\varphi'+kc_{1}\varphi''\right)=0, \tag{4.21}$$

$$\beta kc_{1}\varphi'^{3}\left(c_{1}^{2}-3kc_{1}+2k^{2}\right)=0, \tag{4.22}$$

Eq. (4.19) gives values 0 and $\pm\sqrt{\frac{-c}{\beta k}}$ for c_0. Since $\varphi'(\xi)\neq 0$and $c_1\neq 0$, from Eq. (4.22) two values $2k$ and k will be found for c_1. If $c_1=k$, then, from Eqs. (4.20) and (4.21) we will have$\varphi''=\varphi'''=0$. Inevitably, we set $c_1=2k$ and rewrite Eqs. (4.20) and (4.21) in a short form.

$$c\varphi'+3\beta k^{2}c_{0}\varphi''+3\beta kc_{0}^{2}\varphi'+\beta k^{3}\varphi'''-3\beta k^{3}\varphi''=0, \tag{4.23}$$

$$\varphi' = -\frac{2k}{c_0}\varphi'', \tag{4.24}$$

Eq. (4.24) implies $c_0 \neq 0$. By putting (4.24) into (4.23) and simplifying the resultant equation by assumption $c_0 = \pm\sqrt{\frac{-c}{\beta k}}$, following expression will be obtained.

$$\frac{\varphi'''}{\varphi''} = \frac{3\beta k^2\sqrt{k} + 5c\sqrt{k} \pm 6\sqrt{-\beta c}}{\pm k^2\sqrt{-\beta c}}, \tag{4.25}$$

Solving this ordinary differential equation gives:

$$\varphi = B + A\frac{1}{\lambda^2}\exp\left(\lambda\xi\right), \tag{4.26}$$

where $\lambda = \frac{3\beta k^2\sqrt{k}+5c\sqrt{k}\pm 6\sqrt{-\beta c}}{\pm k^2\sqrt{-\beta c}}$.$A$ and B are constant of integration. As a result, an exact solution to nonlinear fractional STO equation can be written as

$$u(x,y,t) = \pm\sqrt{\frac{-c}{\beta k}} + \frac{2kA\lambda\exp\left(\lambda\xi\right)}{B\lambda^2 + A\exp\left(\lambda\xi\right)}. \tag{4.27}$$

Where $\xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} + \xi_0$.

Some figures are given to illustrate the solutions of the equations (4.1) and (4.14) for different values of free parameters in Figures 1 and 2.

5. Conclusions

Fractional partial differential equations can be turned into ordinary differential equations (ODE) by modifying Riemann-Liouville derivatives and a complex transformation. So, all analytical methods which are devoted to the advanced calculus can easily be applied to the fractional calculus. In this paper, the modified simple equation method has successfully been implemented to find exact solutions for two fractional differential equations, the space-time nonlinear fractional potential Kadomstev-Petviashvili (PKP) and Sharma-Tasso-Olver (STO) equations. Via the proposed method, some new exact solutions for these equations have successfully been obtained without using any symbolic computation software. The method offers solutions with free parameters that might be important to explain some physical phenomena.

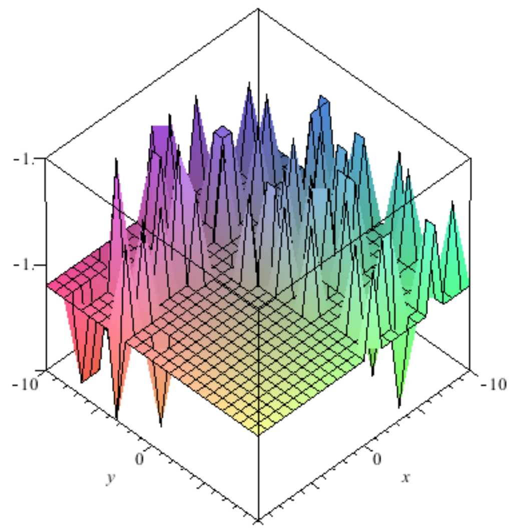

FIGURE 1. Solution of the equation (4.1) for $k_1 = 1, k_2 = 1, c_0 = 1, A = 1, B = 1, \alpha = \frac{1}{3}, c = -1, t = 1$

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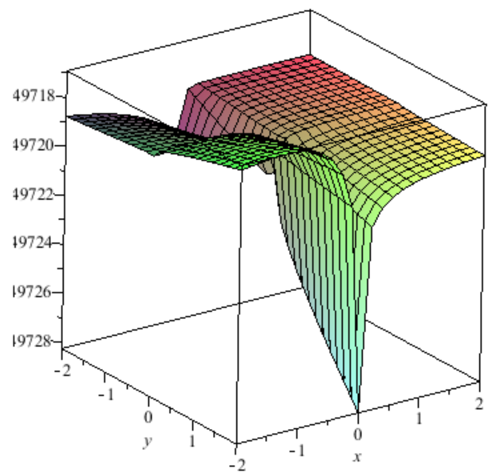

FIGURE 2. Solution of the equation (4.1) for $k_1 = -2, k_2 = 1, c_0 = 1, A = 1, B = 1, \alpha = \frac{1}{3}, c = 1, t = 1$

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