*Journal of Hyperstructures* 8 *(1) (2019), 69-80.*
*ISSN: 2251-8436 print/2322-1666 online*

# GENERALIZATION OF BI-POLAR FUZZY SOFT INTERIOR IDEALS OVER SEMIRINGS

M. MURALI KRISHNA RAO

Abstract. In this paper, we introduce the notion of a generalized bi-polar fuzzy set whose membership degree range is [-0.5,0.5], as a generalization of a fuzzy set and a bi-polar fuzzy set, the notion of generalized bi-polar fuzzy ideal, generalized bi-polar fuzzy interior ideal of semiring, generalized bi-polar fuzzy soft ideal and generalized bi-polar fuzzy soft interior ideal over semiring and study some of their algebraic properties and the relations between them.

**Key Words:** bi-polar fuzzy set, soft set, fuzzy soft set, generalized bi-polar fuzzy soft ideal, generalized bi-polar fuzzy soft interior ideal.
**2010 Mathematics Subject Classification:** 08A72, 16Y60.

## 1. Introduction

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was first introduced by Zadeh [29]. Molodtsov [14] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Acar et al. [1] gave the basic concept of soft ring. Aktas and Cagman [3] introduced the concept of fuzzy soft subgroup. Majumdar and Samantha extended soft sets to fuzzy soft set. Ghosh et al. [6] initiated the study of fuzzy soft rings and fuzzy soft ideals. Murali Krishna Rao [16-26] introduced and studied fuzzy soft ideals and fuzzy soft k -ideals over a $\Gamma$-semiring. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, interval

Received: 2019-03-09, Accepted: 2019-06-08. Communicated by: Ali Taghavi
∗Address correspondence: Department of Mathematics, GIT, GITAM University, India; E-mail: mmarapureddy@gmail.com.

valued fuzzy sets, vague sets, bipolar fuzzy sets, cubic sets etc. In 2001, Maji et al. [15] combined the concept of fuzzy set theory which was introduced by Zadeh in 1965 and the notion of soft set theory which was introduced by Molodstov in 1999. In 2000, Lee [10,11] used the term bipolar valued fuzzy sets and applied it to algebraic structures. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In 1994, Zhang [30] initiated the concept bipolar fuzzy sets as a generalization of fuzzy sets. The concept of bipolar fuzzy soft sets introduced by Akram [2]. Jun et al. [7,8] introduced the notion of bipolar fuzzy ideals and bipolar fuzzy filters in BCI-algebras. The concept of intuitionistic fuzzy set was first introduced by Atanassov [4] as a generalization of notion of fuzzy set. Murali Krishna Rao [27] introduced the notion of tri-polar fuzzy interior ideal of $\Gamma$-semigroup. In this paper, we introduce the notion of generalized bi-polar fuzzy soft ideals, generalized bi-polar fuzzy soft interior ideals over semiring and study some of their algebraic properties and the relations between them

## 2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

**Definition 2.1.** [23] A universal algebra $(M, +, \cdot)$ is called a semiring if and only if $(M, +)$ and $(M, \cdot)$ are semigroups which are connected by distributive laws, *i.e.*, $a(b+c) = ab+ac,\ (a+b)c = ac+bc,$ for all $a, b, c \in M.$

**Definition 2.2.** A semiring $M$ is said to be commutative semiring if $xy = yx$, for all $x, y \in M$.

**Definition 2.3.** [23] A semiring $M$ is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0x = x\ 0 = 0,$ for all $x \in M$.

**Definition 2.4.** [23] An element $1 \in M$ is said to be unity if for each $x \in M$ such that $x\ 1 = 1\ x = x$.

**Definition 2.5.** In a semiring $M$ with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exists $b \in M$ such that $ba = 1 (ab = 1)$.

**Definition 2.6.** [23] In a semiring $M$ with unity 1, an element $a \in M$ is said to be invertible if there exists $b \in M$ such that $ab = ba = 1$.

**Definition 2.7.** [23] A semiring $M$ with unity 1 is said to be division semiring if every non zero element of $M$ is invertible.

**Definition 2.8.** [23] An element $a \in M$ is said to be regular element of $M$ if there exists $x \in M$ such that $a = axa$.

**Definition 2.9.** [23] If every element of a semiring $M$ is a regular then $M$ is said to be regular semiring.

**Definition 2.10.** [23] An element $a \in M$ is said to be idempotent of $M$ if $a = aa$.

**Definition 2.11.** [23] Every element of $M$ is an idempotent of $M$ then $M$ is said to be idempotent semiring $M$.

**Definition 2.12.** [23] A non-empty subset $A$ of a semiring $M$ is called

(i) a subsemiring of $M$ if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.
(ii) a quasi ideal of $M$ if $A$ is a subsemiring of $M$ and $AM \cap MA \subseteq A$.
(iii) a bi-ideal of $M$ if $A$ is a subsemiring of $M$ and $AMA \subseteq A$.
(iv) an interior ideal of $M$ if $A$ is a subsemiring of $M$ and $MAM \subseteq A$.
(v) a left (right) ideal of $M$ if $A$ is a subsemiring of $M$ and $MA \subseteq A(AM \subseteq A)$.
(vi) an ideal if $A$ is a subsemiring of $M$, $AM \subseteq A$ and $MA \subseteq A$.
(vii) a $k-$ideal if $A$ is a subsemiring of $M$, $AM \subseteq A$, $MA \subseteq A$ and $x \in M$, $x + y \in A, y \in A$ then $x \in A$.
(viii) a left( right) bi- quasi ideal of $M$ if $A$ is a subsemiring of $M$ and $MA \cap AMA$ $(AM \cap AMA) \subseteq A$.
(ix) a bi- quasi ideal of $M$ if $A$ is a left bi- quasi ideal and a right bi-quasi ideal of $M$.

**Definition 2.13.** [23] A semiring $M$ is a left (right) simple semiring if $M$ has no proper left (right) ideal of $M$.

**Definition 2.14.** A semiring $M$ is said to be simple semring if $M$ has no proper ideals.

**Definition 2.15.** [4] An intuitionistic fuzzy set $f$ of a non-empty set $X$ is an object having the form $f = (\mu_f, \delta_f) = \{(x, \mu_f(x), \delta_f(x)) \mid x \in X\}$, where $\mu_f : X \to [0,1], \delta_f : X \to [0,1]$ are membership functions, $\mu_f(x)$ is a degree of membership, $\lambda_f(x)$ is a degree of non membership and $0 \le \mu_f(x) + \delta_f(x) \le 1$, for all $x \in X$.

**Definition 2.16.** [10] A bipolar fuzzy set $A$ of a non-empty set $X$ is an object having the form $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$, where $\mu_A : X \to [0, 1]; \delta_A : X \to [-1, 0]$. $\mu_A(x)$ represents degree of satisfaction of an element $x$ to the property corresponding to fuzzy set A and $\delta_A(x)$ represents degree of satisfaction of an element $x$ to the implicit counter property of fuzzy set A.

**Definition 2.17.** [15] Let $(f, A), (g, B)$ be fuzzy soft sets over a semiring $M$. The union of fuzzy soft sets $(f, A)$ and $(g, B)$ is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \text{ for all } c \in A \cup B. \\ f_c \cup g_c, & \text{if } c \in A \cap B, \end{cases}$$

**Definition 2.18.** [23] Let $(f, A), (g, B)$ be fuzzy soft sets over a semiring $M$. Then $(f, A)$ and $(g, B)$ is denoted by "$(f, A) \wedge (g, B)$" is defined by $(f, A) \wedge (g, B) = (f \cap g, C) = (h, C)$, where $C = A \times B$, $h_c(x) = \min\{f_a(x), g_b(x)\}$, for all $c = (a, b) \in A \times B$ and $x \in M$.

**Definition 2.19.** [23] Let $(f, A), (g, B)$ be fuzzy soft sets over a semiring $M$. Then $(f, A)$ or $(g, B)$ is denoted by $(f, A) \vee (g, B)$ is defined by $(f, A) \vee (g, B) = (h, C)$, where $C = A \times B$ and $h_c(x) = \max\{f_a(x), g_b(x)\}$, for all $c = (a, b) \in A \times B, x \in M$.

**Definition 2.20.** Let $(f, A), (g, B)$ be fuzzy soft sets over a semiring $M$. The intersection of fuzzy soft sets $(f, A)$ and $(g, B)$ is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \text{ for all } c \in A \cup B. \\ f_c \cap g_c, & \text{if } c \in A \cap B\ , \end{cases}$$

## 3. GENERALIZED BI-POLAR FUZZY SOFT IDEALS OVER SEMIRING

In this section, we introduce the notion of generalized bi-polar fuzzy sets to be able to deal with bi-polar information as a generalization of fuzzy sets and bipolar fuzzy set. We introduce the notion of generalized bi-polar fuzzy soft ideals and generalized bi-polar fuzzy soft interior ideals over semirings.

**Definition 3.1.** A generalized bi-polar fuzzy set $A$ of a universe set $X$ is an object having the form $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X$where

$\mu_A : X \to [0.5, 1]; \delta_A : X \to [-0.5, 0]$ The membership degree $\mu_A(x)$ characterizes the extent that the element $x$ satisfies to the property corresponding to bi-polar fuzzy set $A$, and $\delta_A(x)$ characterizes the extent that the element $x$ satisfies to the implicit counter property of bi-polar fuzzy set $A$. For simplicity $A = (\mu_A, \delta_A)$ has been used for $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$.

*Remark* 3.2. A generalized bi-polar fuzzy set $A$ is a generalization of a bipolar fuzzy set and a fuzzy set. A generalized bi-polar fuzzy set $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$ represents the taste of food stuffs. Assuming the sweet taste of food stuff as a positive membership value $\mu_A(x)$ i.e. the element $x$ is satisfying the sweet property. Then bitter taste of food stuff as a negative membership value $\delta_A(x)$ i.e. the element $x$ is satisfying the bitter property.

**Definition 3.3.** A generalized bi-polar fuzzy set $A = (\mu_A, \delta_A)$ of a semiring $M$ is called a generalized bi-polar fuzzy subsemiring of $M$ if $A$ satisfies the following conditions.

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(x)\}$
(ii) $\delta_A(x + y) \leq \max\{\delta_A(x), \delta_A(y)\}$
(iii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$
(iv) $\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\}$, for all $x, y \in M$.

**Definition 3.4.** A generalized bi-polar fuzzy subsemiring $A = (\mu_A, \delta_A)$ of a semiring $M$ is called a generalized bi-polar fuzzy ideal of $M$ if $A$ satisfies the following conditions

(i) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_{A)}(y)\}$
(ii) $\delta_A(xy) \leq \min\{\delta_A(x), \delta_A(y)\}$, for all $x, y \in M$

**Definition 3.5.** A generalized bi-polar fuzzy subsemiring $A = (\mu_A, \delta_A)$ of a semiring $M$ is called a generalized bi-polar fuzzy interior ideal of $M$ if $A$ satisfies the following conditions

(i) $\mu_A(xzy) \geq \mu_A(z)$
(ii) $\delta_A(xzy) \leq \delta_A(z)$, for all $x, y, z \in M$.

**Definition 3.6.** A generalized bi-polar fuzzy soft set $(f, A)$ over semiring $M$ is called a generalized bi-polar fuzzy soft semiring over $M$ if $f(a) = \{(\mu_{f(a)}(x), \delta_{f(a)}(x)) \mid x \in M, a \in A\}$, where $\mu_{f(a)}(x) : M \to [0, 0.5]; \delta_{f(a)}(x) : M \to [-0.5, 0]$ for all $x \in M$ satisfying the following conditions

(i) $\mu_{f(a)}(x + y) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(x)\}$

(ii) $\delta_{f(a)}(x+y) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$
(iii) $\mu_{f(a)}(xy) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$
(iv) $\delta_{f(a)}(xy) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$, for all $x, y \in M$ and $a \in A$.

**Definition 3.7.** A generalized bi-polar fuzzy soft set $(f, A)$ over a semiring $M$ is called a generalized bi-polar fuzzy soft ideal over $M$ if

(i) $\mu_{f(a)}(x+y) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$
(ii) $\delta_{f(a)}(x+y) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$
(iii) $\mu_{f(a)}(xy) \geq \max\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$
(iv) $\delta_{f(a)}(xy) \leq \min\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$, for all $x, y \in M$ and $a \in A$.

*Remark* 3.8. Every generalized bi-polar fuzzy soft ideal $(f, A)$ over a semiring $M$ is a generalized bi-polar fuzzy soft semiring $(f, A)$ over $M$ but the converse is not true.

*Example* 3.9. Let $M = \{x_1, x_2, x_3\}$. Then we define operations with the following tables:

| $+$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $x_1$ | $x_1$ | $x_2$ | $x_3$ |
| $x_2$ | $x_2$ | $x_2$ | $x_3$ |
| $x_3$ | $x_3$ | $x_3$ | $x_2$ |

;

| | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $x_1$ | $x_1$ | $x_3$ | $x_3$ |
| $x_2$ | $x_3$ | $x_2$ | $x_3$ |
| $x_3$ | $x_3$ | $x_3$ | $x_3$ |

Let $E = \{a, b, c\}$ and $B = \{a, b\}$. Then $(\phi, B)$ is generalized bi-polar fuzzy soft set defined as $(\phi, B) = \{\phi(a), \phi(b)\}$, where
$\phi(a) = \{(x_1, 0.7, -0.2), (x_2, 0.8, -0.3), (x_3, 0.5, -0.3)\}$
$\phi(b) = \{(x_1, 0.9, -0.3), (x_2, 0.8, -0.5), (x_3, 0.5, -0.2)\}$.
Then $(\phi, B)$is a generalized bi-polar fuzzy soft semiring over $M$ and $(\phi, B)$ is not a generalized bi-polar fuzzy soft ideal over $M$.
$(\phi, B)$ is generalized bi-polar fuzzy soft interior ideal over $M$.

**Definition 3.10.** A generalized bi-polar fuzzy soft $(f, A)$ over semiring $M$ is called a generalized bi-polar fuzzy soft interior ideal of $M$ if

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(z)$
(ii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(z)$, for all $x, y, z \in M$ and $a \in A$.

**Theorem 3.11.** *Every generalized bi-polar fuzzy soft ideal over a semiring $M$ is a generalized bi-polar fuzzy soft interior ideal over a semiring $M$.*

*Proof.* Let $(f, A)$ be generalized bi-polar fuzzy soft ideal over the semiring $M$. Then $f(a) = \{\mu_{f(a)}, \delta_{f(a)}\}$ is a generalized bi-polar fuzzy ideal of $M, a \in A$. Then

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(xz) \geq \mu_{f(a)}(z)$

(ii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(xz) \leq \delta_{f(a)}(z)$, for all $x, y, z \in M$ and $a \in A$.

Hence $(f, A)$ is a generalized bi-polar fuzzy soft interior ideal over $M$. □

**Theorem 3.12.** *Every generalized bi-polar fuzzy soft interior ideal over a regular semiring $M$ is a generalized bi-polar fuzzy soft ideal over $M$.*

*Proof.* Let $(f, A)$ be generalized bi-polar fuzzy soft interior ideal over the regular semiring $M$. Therefore $f(a) = \{\mu_{f(a)}, \delta_{f(a)}\}$ is a generalized bi-polar fuzzy ideal of $M, a \in A$.

Suppose $x, y \in M$. Then $xy \in M$ and there exists $z \in M$ such that $xy = xyzxy$.

$$\begin{aligned}
\mu_{f(a)}(xy) &= \mu_{f(a)}\big(xy(zxy)\big) \\
&\geq \mu_{f(a)}(y) \\
\mu_{f(a)}(xy) &= \mu_{f(a)}\big((xyz)xy\big) \\
&\geq \mu_{f(a)}(x).
\end{aligned}$$

Hence $\mu_{f(a)}$ is a fuzzy ideal of $M$.

$$\begin{aligned}
&\delta_{f(a)}(xy) = \delta_{f(a)}(xyzxy) \leq \delta_{f(a)}(y) \\
&\delta_{f(a)}(xy) \leq \delta_{f(a)}(x).
\end{aligned}$$

Hence $\delta_{f(a)}$ is a fuzzy ideal of $M$.
Therefore $f(a)$ is a generalized bi-polar fuzzy ideal of the semiring $M$.
Thus $(f, A)$ is a generalized bi-polar fuzzy soft ideal over the semiring $M$. □

**Theorem 3.13.** *If $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft ideals over a semiring $M$ then $(f, A) \wedge (g, B)$ is a generalized bi-polar fuzzy soft ideal over a semiring $M$.*

*Proof.* Suppose $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft ideals over the semiring $M$.
Then by Definition 2.18, $(f, A) \wedge (g, B) = (f \cap g, C)$, where $C = A \times B$

and $(f \wedge g)(a,b) = f(a) \cap g(b)$, for all $(a,b) \in C$. Then

$$\begin{aligned}
\mu_{f(a)\cap g(b)}(x+y) &= \min\{\mu_{f(a)}(x+y), \mu_{g(b)}(x+y)\} \\
&\geq \min\big\{\min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}, \min\{\mu_{g(b)}(x), \mu_{g(b)}(y)\}\big\} \\
&= \min\big\{\min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}, \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\} \\
&= \min\{\mu_{f(a)\cap g(b)}(x), \mu_{f(a)\cap g(b)}(y)\}. \\
\delta_{f(a)\cap g(b)}(x+y) &= \min\{\delta_{f(a)}(x+y), \delta_{g(b)}(x+y)\} \\
&\geq \min\big\{\max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}, \max\{\delta_{g(b)}(x), \delta_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\delta_{f(a)}(x), \delta_{g(b)}(x)\}, \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} \\
&= \max\{\delta_{f(a)\cap g(b)}(x), \delta_{f(a)\cap g(b)}(y)\}.
\end{aligned}$$

$$\begin{aligned}
\mu_{f(a)\cap g(b)}(xy) &= \min\{\mu_{f(a)}(xy), \mu_{g(b)}(xy)\} \\
&\geq \min\big\{\min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}, \min\{\mu_{g(b)}(x), \mu_{g(b)}(y)\}\big\} \\
&= \min\big\{\min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}, \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\} \\
&= \min\{\mu_{f(a)\cap g(b)}(x), \mu_{f(a)\cap g(b)}(y)\}. \\
\delta_{f(a)\cap g(b)}(xy) &= \min\{\delta_{f(a)}(xy), \delta_{g(b)}(xy)\} \\
&\leq \min\big\{\max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}, \max\{\delta_{g(b)}(x), \delta_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\delta_{f(a)}(x), \delta_{g(b)}(x)\}, \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} \\
&= \max\{\delta_{f(a)\cap g(b)}(x), \delta_{f(a)\cap g(b)}(y)\}.
\end{aligned}$$

Hence $(f,A) \wedge (g,B)$ is a generalized bi-polar fuzzy soft ideal over the semiring $M$. □

**Theorem 3.14.** *If $(f,A)$ and $(g,B)$ are two generalized bi-polar fuzzy soft interior ideals over a semiring $M$ then $(f,A)\wedge(g,B)$ is a generalized bi-polar fuzzy interior ideals over semiring $M$.*

*Proof.* Suppose $(f,A)$ and $(g,B)$ are two generalized bi-polar fuzzy soft interior ideals over the semiring $M$.
Obviously $(f,A) \wedge (g,B)$ is soft generalized bi-polar fuzzy subsemiring of $M$.
By Definition 2.18, $(f,A) \wedge (g,B) = (f \cap g, C)$, where $C = A \times B$.

Suppose $(a, b) \in C, x, y \in M$. Then

$$\begin{aligned}
\mu_{f\wedge g(a,b)}(xyz) &= \mu_{f(a)\cap g(b)}(xyz) \\
&= \min\{\mu_{f(a)}(xyz), \mu_{g(b)}(xyz)\} \\
&\geq \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\} \\
&= \mu_{f(a)\cap g(b)}(y) \\
&= \mu_{f\wedge g(a,b)}(y). \\
\delta_{f\wedge g(a,b)}(xyz) &= \delta_{f(a)\cap g(b)}(xyz) \\
&= \min\{\delta_{f(a)}(xyz), \delta_{g(b)}(xyz)\} \\
&\leq \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\} \\
&= \delta_{f(a)\cap g(b)}(y) \\
&= \delta_{f\wedge g(a,b)}(y).
\end{aligned}$$

Hence $(f, A) \wedge (g, B)$ is a soft generalized bi-polar fuzzy interior ideal of the semiring $M$. □

The proofs of following theorems are similar to Theorem 3.14.

**Theorem 3.15.** *If $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft interior ideals over a semiring $M$ then $(f, A) \cup (g, B)$ is a generalized bi-polar fuzzy interior ideals of a semiring $M$.*

**Theorem 3.16.** *If $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft interior ideals over a semiring $M$ then $(f, A) \cap (g, B)$ is a generalized bi-polar fuzzy interior ideals of a semiring $M$.*

**Theorem 3.17.** *If $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft ideals over semiring $M$ then $(f, A) \cup (g, B)$ is a generalized bi-polar fuzzy soft ideal over $M$.*

*Proof.* Suppose $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft ideals over the semiring $M$. Then by Definition 2.17, we have $(f, A) \cup (g, B) = (h, C)$, where $C = A \cup B$, and

$$h(c) = f \cup g(c) = \begin{cases} f(c) & \text{if } c \in A \setminus B; \\ g(c) & \text{if } c \in B \setminus A; \\ f(c) \cup g(c) & \text{if } c \in A \cap B, \end{cases} \quad \text{for all } \ c \in A \cup B.$$

**case(i):** If $c \in A \setminus B$ then $f \cup g(c) = f(c)$. Thus we have

$$\begin{aligned}
\mu_{f\cup g(c)}(x+y) &= \mu_{f(c)}(x+y)\\
&\geq \min\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}\\
&= \min\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}.\\
\delta_{f\cup g(c)}(x+y) &= \delta_{f(c)}(x+y)\\
&\leq \max\{\delta_{f(c)}(x), \delta_{f(c)}(y)\}\\
&= \max\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.\\
\mu_{f\cup g(c)}(xy) &= \mu_{f(c)}(xy)\\
&\geq \max\{\mu_{f(c)}(xy), \mu_{f(c)}(xy)\}\\
&= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}.\\
\delta_{f\cup g(c)}(xy) &= \delta_{f(c)}(xy)\\
&\leq \min\{\delta_{f(c)}(x), \delta_{f(c)}(y)\}\\
&= \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.
\end{aligned}$$

**case(ii):** If $c \in B \setminus A$ then $f \cup g(c) = g(c)$.
Since $g(c)$ is a generalized bi-polar fuzzy ideal of the semiring $M, f \cup g(c)$ is a bi-polar fuzzy ideal of semiring $M$.
case(iii): If $c \in A \cap B$ then $f \cup g(c) = f(c) \cup g(c)$.

$$\begin{aligned}
\mu_{f\cup g(c)}(x+y) &= \max\{\mu_{f(c)}(x+y), \mu_{g(c)}(x+y)\}\\
&\geq \max\{\min\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}, \min\{\mu_{g(c)}(x), \mu_{g(c)}(y)\}\}\\
&= \min\{\max\{\mu_{f(c)}(x), \mu_{g(c)}(x)\}, \max\{\mu_{f(c)}(y), \mu_{g(c)}(y)\}\}\\
&= \min\{\mu_{f(c)\cup g(c)}(x), \mu_{f(c)\cup g(c)}(y)\}\\
&= \min\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}\\
\delta_{f\cup g(c)}(x+y) &\leq \max\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.\\
\mu_{f\cup g(c)}(xy) &= \mu_{f(c)\cup g(c)}(xy)\\
&= \max\{\mu_{f(c)}(xy), \mu_{g(c)}(xy)\}\\
&\geq \max\{\max\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}, \max\{\mu_{g(c)}(x), \mu_{g(c)}(y)\}\}\\
&= \max\{\max\{\mu_{f(c)}(x), \mu_{g(c)}(x)\}, \max\{\mu_{f(c)}(y), \mu_{g(c)}(y)\}\}\\
&= \max\{\mu_{f(c)\cup g(c)}(x), \mu_{f(c)\cup g(c)}(y)\}\\
&= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}
\end{aligned}$$

Similarly we can prove

$$\delta_{f\cup g(c)}(xy) \leq \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.$$

Therefore $f \cup g(c)$ is a generalized bi-polar fuzzy ideal of $M$.
Hence $(f, A) \cup (g, B)$ is a generalized bi-polar fuzzy soft ideal over $M$. $\square$

**Corollary 3.18.** *If $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft ideals over a semiring $M$ then $(f, A) \cap (g, B)$ is a generalized bi-polar fuzzy soft ideal over $M$.*

## 4. Conclusion

In this paper,we introduced the notion of generalized bi-polar fuzzy soft subsemiring, generalized bi-polar fuzzy soft ideal, generalized bi-polar fuzzy soft interior ideals over semirings and proved that if $(f, A)$ and $(g, B)$ are two generalized bi-polar fuzzy soft interior ideals over a semiring $M$ then $(f, A) \wedge (g, B)$,$(f, A) \cup (g, B)$ and $(f, A) \cap (g, B)$ are generalized bi-polar fuzzy soft interior ideals over a semiring $M$.

## References

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and Soft rings, Comput. Math. Appli., 59(2010), 3458–3463.

[2] M. Akram, Bipolar fuzzy soft Lie algebras, Quasigroups and Related Systems 21 (2013), 1 - 10

[3] H. Aktas and N. Cagman, Soft sets and soft groups, Information Science, 177(2007), 2726–2735.

[4] K. T. Attanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20 (1) (1986), 87–96.

[5] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appli., 56(2008), 2621–2628.

[6] J. Ghosh, B. Dinda and T. K. Samanta, Fuzzy soft rings and Fuzzy soft ideals, Int.J. P. App.Sc.Tech., 2(2)(2011), 66–74.

[7] Y.B. Jun, H.S. Kim and K.J. Lee, Bipolar fuzzy translation in BCK/BCIalgebra, J. of the Chungcheong Math. Soc., (22) (3) (2009), 399 - 408.

[8] K. H. Kim and Y. B. Jun, Intuitionistic fuzzy interior ideals of semigroups, Int. J. of Math. and Math. Sci., 27 (5) (2001), 261 - 267.

[9] K.J. Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCIalgebras, Bull. Malays. Math. Soc., (32) (2009), 361 - 373.

[10] K. M. Lee, Bipolar valued fuzzy sets and their applications, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand (2000), 307 - 312.

[11] K. M. Lee, Comparison of interval valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets, J. Fuzzy Logic intelligent Systems, (14)(2)(2004), 125 - 129.

[12] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, The J. of Fuzzy Math., 9(3)(2001), 589 - 602.

[13] D. Mandal, Fuzzy ideals and fuzzy interior ideals in ordered semirings, Fuzzy Inf. Engg., 6(1) (2014), 101 114

[14] D. A. Molodtsov, Soft set theory first results, Computers and math. with appli., 37 (1999), 19 - 31.

[15] P.K. Maji, R.Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math., 9(3) (2001), 589 - 602

[16] M. Murali Krishna Rao, r- Fuzzy Ideals of Γ-incline, Annals of Fuzzy Mathematics and Informatics, 17(2) (2019).

[17] M. Murali Krishna Rao, Fuzzy soft $\Gamma-$semiring homomorphism, Anl. of Fuzzy Math. and Info., 12(4) (2016), 479-489.

[18] M. Murali Krishna Rao, Fuzzy soft $\Gamma-$semiring and fuzzy soft $k-$ideal over $\Gamma-$semiring, Anl. of Fuzzy Math. and Info., 9 (2)(2015), 12–25.

[19] M. Murali Krishna Rao and B. Venkateswarlu, An intuitionistic normal fuzzy soft $k-$ideal over a $\Gamma-$semiring, Anl. of Fuzzy Math. and Info., 11(3) ( 2016), 361-376.

[20] M. Murali Krishna Rao, Fuzzy ideal graphs of a semigroup, Annals of Fuzzy Mathematics and Informatics,Volume 16( 3) (2018)pp. 363371.

[21] Marapureddy Murali Krishna Rao, Fuzzy graph of semigroup, Bull. Int. Math. Virtual Inst. 8 (3) (2018) 439–448.

[22] Marapureddy Murali Krishna Rao,Fuzzy left and right bi-quasl ideals of semiring, Bull. Int. Math. Virtual Inst., Vol 8(3), (2018), 449-460.

[23] M. Murali Krishna Rao and B. Venkateswarlu, Regular $\Gamma-$incline and field $\Gamma-$semiring, Novi Sad J. Math.,45(2) (2015), 155-171.

[24] M. Murali Krishna Rao, Left bi-quasi ideals of semirings, Bull. Int. Math. Virtual Inst , DOI: Vol. 8(2018),45-53. 10.7251/BIMVI1801045R

[25] M. Murali Krishna Rao and B. Venkateswarlu, Fuzzy soft k-ideals over semiring and Fuzzy soft semiring homomorphism ,Journal of Hyperstructures, 4 (2) (2015), 93-116.

[26] M. Murali Krishna Rao, Fuzzy soft ideal, Fuzzy soft bi-ideal, Fuzzy soft quasi-ideal and fuzzy soft interior ideal over ordered $\Gamma-$Semiring, Asia Pacific J. Math, Vol 5(1), (2018), 60-84.

[27] M. Murali Krishna Rao,Tripolar fuzzy interior ideals of -semigroup , Annals of Fuzzy Mathematics and Informatics,Volume 15(2) 2018, pp. 199206.

[28] A. Rosenfeld, Fuzzy groups, J. Math.Anal.Appl. 35(1971), 512-517.

[29] L. A. Zadeh, Fuzzy sets, Information and control, 8(1965), 338-353.

[30] W. R. Zhang, Bipolar fuzzy sets, Proc. of FUZZ-IEEE, (1998), 835-840.

**M. Murali Krishna Rao**

Department of Mathematics, GIT, GITAM University, Visakhapatnam- 530 045, A.P., India.

`Email: mmarapureddy@gmail.com`