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VAGUE SOFT NEAR-RINGS

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ABSTRACT. Soft set theory, proposed by Molodstov has been regarded as an effective mathematical tool to deal with uncertainties. Vague set is a set of objects, each of which has a grade of membership whose value is a continuous subinterval of [0,1]. Such a set is characterized by a truth membership function and a false membership function. In this paper we introduce and study the concept of vague soft near-rings, vague soft ideals of near-rings. Also, we derive some properties of vague soft near-rings and vague soft ideals of near-rings.

Key Words: Fuzzy set, Soft set, Vague Set, Vague Soft Set, Vague Soft Near-rings, Vague Soft Ideals, Near-rings.
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1. Introduction

The real world is complex and complexity in the world arises from uncertainty. Knowledge, which describes the real world, is usually imprecise and vague. The mathematical tools available to represent the real world are not suitable to describe rigorous and precise information. Thus, there is always some difference between vagueness of reality and its rigorousness of mathematical model.

The fuzzy set theory was initiated by L. A. Zadeh [9] in 1960s. Since a membership in a fuzzy set theory is a matter of degree, we can represent the gradual membership of an element of a set describing the fuzzy attributes like cold, hot, tall, short etc in a better way. Fuzzy set theory

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is applicable in control theory, rohobotics and other complex engineering systems. Molodtsov [3] initiated the concept of soft set. In 2001 Maji et al [11] combined fuzzy sets and soft sets models and introduced the concept of fuzzy soft sets. Also Borah, Neog and Sut[10] have studied algebraic properties of fuzzy soft sets. Varol, Aygun and Aygunnoglu [2] have introduced fuzzy soft ring and discussed its properties.

The fuzzy set assigns a single value to each object and this number is inadequate to tell its accuracy. To overcome this situation Gau and Buehrer [14] in 1993 introduced and studied vague sets. Vague sets are more generalization of fuzzy sets. Now the research on vague set theory is rapidly progressing. Xu et al [13] introduced the notion of vague soft sets which is a combination of vague sets and soft sets as an improvement to the notion of fuzzy soft sets. On the other hand, vague soft rings and the related properties are discussed by Selvachandran and Salleh[4].

Near-ring is a generalization of a ring. If in a ring we ignore the commutativity of addition and one distributive law then we get a near-ring. G. Pilz [12], J. D. P. Meldrum [7] and many other researchers have contributed the near-ring theory.

The aim of this paper is to further extend the concept of vague soft rings by using near-rings. We have also derived its basic properties. We have defined vague soft ideals over near-rings and discussed some properties. Inspite of this, we have defined vague soft ideals of vague soft near-rings. Further we have studied properties related to these concepts.

2. Preliminaries

We recall some basic definitions for the sake of completeness.

Definition 2.1. ([12]) By a near-ring we mean a non-empty set N with two binary operations '+' and '.' satisfying the following axioms:

(i) $(N, +)$ is a group,
(ii) $(N, .)$ is a semigroup,
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z \ \forall \ x, y, z \in N$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-rings" instead of "left near-ring". We denote xy instead of $x \cdot y$. Note that $x0 = 0$ and $x(-y) = -xy$, but $0x \neq 0$ for $x, y \in N$.

Definition 2.2. ([3]) Let U be an initial universe set and E be the set of parameters. Let A be a subset of E. Let $P(U)$ denote the power set of

U. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. For each $x \in A$, $F(x)$ is the set of x-approximate elements of the soft set (F, A). A soft set (F, E) over U is also denoted by a triple (F, E, U).

Definition 2.3. ([15]) For a soft set (F, A) the set Supp $(F, A) = \{x \in A | F(x) \neq \phi\}$ is called the support of the soft set (F, A). If Supp $(F, A) \neq \phi$ then the soft set (F, A) is called non-null.

Definition 2.4. ([1]) Let (F, A) be a non-null soft set over a near-ring N. Then (F, A) is called soft near-ring over N if $F(x)$ is a subnear-ring of N for all $x \in Supp(F, A)$.

Definition 2.5. ([4]) Let X be a set of objects and $x \in X$. A vague set V in X is characterized by a truth membership function $t_V : X \to [0, 1]$ and a false-membership function $f_V : X \to [0, 1]$. The value $t_V(x)$ is a lower bound on the grade of membership of x derived from the evidence for x and is $f_V(x)$ a upper bound on the negation of x derived from the evidence against x. The values $t_V(x)$ and $f_V(x)$ both associate a real number in the interval $[0, 1]$ with each point in X, where $t_V(x)+f_V(x) \leq 1$. This approach bounds the grade of membership of x to a subinterval $[t_V(x), 1 - f_V(x)]$ of $[0, 1]$. Hence a vague set is a form of fuzzy set.

Definition 2.6. ([13]) Let U be a universe, E a set of parameters, $V(U)$ the power set of vague sets on U, and $A \subseteq E$. A pair $(\hat{F}, A)$ is called a vague soft set over U, where $\hat{F}$ is a mapping given by $\hat{F} : A \to V(U)$.

Definition 2.7. ([13]) A vague soft set $(\hat{F}, A)$ over U is said to be an absolute vague soft set denoted by $\widetilde{A}$, if $\forall a \in A, t_{\hat{F}(a)}(x) = 1, 1 - f_{\hat{F}(a)}(x) = 1, x \in U$.

Definition 2.8. ([5]) Let $(\hat{F}, A)$ be a vague soft set over U. Then for all $\alpha, \beta \in [0, 1]$, where $\alpha \leq \beta$ the (α, β)-cut or vague soft (α, β)-cut of $(\hat{F}, A)$ for each $a \in A$ is a subset of U which is defined as;
$(\hat{F}, A)_{(\alpha,\beta)} = \left\{x \in U : t_{\hat{F}_a} \geq \alpha, 1 - f_{\hat{F}_a} \geq \beta\right\}$.
It can be written as,
$(\hat{F}, A)_{(\alpha,\beta)} = \left\{x \in U : \hat{F}_a(x) \geq [\alpha, \beta]\right\}$.

Definition 2.9. ([4], [13]) The union (resp.intersection) of two vague soft sets $(\hat{F}, A)$ and $(\hat{G}, B)$ over a universe X is a vague soft set $(\hat{H}, C)$ which is denoted by $(\hat{F}, A)\widetilde{\bigcup}\left(resp.\widetilde{\bigcap}\right)(\hat{G}, B) = (\hat{H}, C)$ where $C =$

$A \bigcup B$ and for all $c \in C, x \in X$,

$$t_{\hat{H}_c}(x) = \begin{cases} t_{\hat{F}_c}(x) & \text{if } c \in A - B \\ t_{\hat{G}_c}(x) & \text{if } c \in B - A \\ \max\,(resp.\min)\left[t_{\hat{F}_c}(x), t_{\hat{G}_c}(x)\right] & \text{if } c \in A \bigcap B \end{cases}$$

$$1 - f_{\hat{H}_c}(x) = \begin{cases} 1 - f_{\hat{F}_c}(x) & \text{if } c \in A - B \\ 1 - f_{\hat{G}_c}(x) & \text{if } c \in B - A \\ \max\,(\text{resp.}\min)\left[1 - f_{\hat{F}_c}(x), 1 - f_{\hat{G}_c}(x)\right] & \text{if } c \in A \bigcap B \end{cases}$$

Definition 2.10. ([4],[13]) If $(\hat{F}, A)$ and $(\hat{G}, B)$ be two vague soft sets over the universe X then $(\hat{F}, A)\text{AND}(\hat{G}, B)$ is a vague soft set denoted by $(\hat{F}, A) \bigwedge (\hat{G}, B)$ and is defined by,
$(\hat{F}, A) \bigwedge (\hat{G}, B) = (\hat{H}, A \times B)$
where

$$t_{\hat{H}_{(a,b)}}(x) = \min\left(t_{\hat{F}_a}(x), t_{\hat{G}_b}(x)\right)$$

and

$$1 - f_{\hat{H}_{(a,b)}}(x) = \min\left(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{G}_b}(x)\right)$$

for all $(a, b) \in A \times B$ and $x \in X$.

Definition 2.11. ([4],[13]) If $(\hat{F}, A)$ and $(\hat{G}, B)$ be two vague soft sets over the universe X then $(\hat{F}, A)\text{OR}(\hat{G}, B)$ is a vague soft set denoted by $(\hat{F}, A) \bigvee (\hat{G}, B)$ and is defined by,
$(\hat{F}, A) \bigvee (\hat{G}, B) = (\hat{H}, A \times B)$
where

$$t_{\hat{H}_{(a,b)}}(x) = \max\left(t_{\hat{F}_a}(x), t_{\hat{G}_b}(x)\right)$$

and

$$1 - f_{\hat{H}_{(a,b)}}(x) = \max\left(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{G}_b}(x)\right)$$

for all $(a, b) \in A \times B$ and $x \in X$.

Definition 2.12. ([5]) The restricted or bi-intersection of two vague soft set $(\hat{F}, A)$ and $(\hat{G}, B)$ over universe X is a vague soft set denoted by $(\hat{F}, A)\widetilde{\Cap}(\hat{G}, B) = (\hat{H}, C)$ where $C = A \cap B$ and for all $c \in A \cap B, x \in X$ we have,

$$t_{\hat{H}_c}(x) = \min\left(t_{\hat{F}_c}(x), t_{\hat{G}_c}(x)\right)$$

and

$$1 - f_{\hat{H}_c}(x) = \min\Big(1 - f_{\hat{F}_c}(x), 1 - f_{\hat{G}_c}(x)\Big)$$

3. Vague Soft Near-rings

Definition 3.1. Let N be a near-ring and $(\hat{F}, A)$ be a non-null vague soft set over N. Then $(\hat{F}, A)$ is called vague soft near-ring if and only if for $a \in A$ and $x, y \in N$

(i) $\hat{F}_a(x+y) \geq \min\Big(\hat{F}_a(x), \hat{F}_a(y)\Big)$,
i.e $t_{\hat{F}_a}(x+y) \geq \min\Big(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\Big)$ and $1 - f_{\hat{F}_a}(x+y) \geq \min\Big(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\Big)$

(ii) $\hat{F}_a(-x) \geq (\hat{F}_a(x))$,
i.e $t_{\hat{F}_a}(-x) \geq t_{\hat{F}_a}(x)$ and $1 - f_{\hat{F}_a}(-x) \geq 1 - f_{\hat{F}_a}(x)$

(iii) $\hat{F}_a(xy) \geq \min(\hat{F}_a(x), \hat{F}_a(y))$
i.e $t_{\hat{F}_a}(xy) \geq \min\Big(t_{\hat{F}_a}(x), t_{\hat{F}_a}(y)\Big)$ and $1 - f_{\hat{F}_a}(xy) \geq \min\Big(1 - f_{\hat{F}_a}(x), 1 - f_{\hat{F}_a}(y)\Big)$

Example 3.2. Let $N = \{0, a, b, c\}$ be a non-empty set with two binary oparations '+' and '·' defined as fallows,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	0	b
c	a	a	a	c

Let $A = \{e_1, e_2\}$ be a subset of set of parameters. Consider a vague soft set $(\hat{F}, A)$ over a near-ring N given by,

$\hat{F}(e_1) = \{[0.2, 0.9]/0, [0.2, 0.9]/a, [0.1, 0.8]/b, [0.1, 0.8]/c\}$

$\hat{F}(e_2) = \{[0.4, 1]/0, [0.4, 1]/a, [0.3, 0.5]/b, [0.3, 0.5]/c\}$.

Then $(\hat{F}, A)$ is a vague soft near-ring.

Theorem 3.3. *Let $(\hat{F}, A)$ be a non-null vague soft set over a near-ring N. Then $(\hat{F}, A)$ is a vague soft near-ring over N if and only if for each $a \in A$ and $x, y \in N$ the following conditions hold:*

(i) $\hat{F}_a(x-y) \geq \min\Big[\hat{F}_a(x), \hat{F}_a(y)\Big]$,

(ii) $\hat{F}_a(xy) \geq \min\Big[\hat{F}_a(x), \hat{F}_a(y)\Big]$.

Proof. Let $(\hat{F}, A)$ be a vague soft near-ring over N. Let $a \in A$ and $x, y \in N$. Then

$$\begin{aligned}\hat{F}_a(x-y) &= \hat{F}_a(x+(-y)) \\ &\geq min(\hat{F}_a(x), \hat{F}_a(-y)) \\ &\geq min(\hat{F}_a(x), \hat{F}_a(y))\end{aligned}$$

Since $(\hat{F}, A)$ is a vague soft near-ring over N, the second condition holds. Conversely let $(\hat{F}, A)$ be a vague soft set over a near-ring N satisfying the given conditions. Now consider

$$\begin{aligned}\hat{F}_a(0) &= \hat{F}_a(x-x) \\ &\geq min(\hat{F}_a(x), \hat{F}_a(x)) \\ &\geq \hat{F}_a(x).\end{aligned}$$

Thus $\hat{F}_a(0) \geq \hat{F}_a(x)$ for all $x \in N$. Also,

$$\begin{aligned}\hat{F}_a(-x) &= \hat{F}_a(0-x) \\ &\geq \hat{F}_a(0+(-x) \\ &\geq min(\hat{F}_a(x), \hat{F}_a(x)) \\ &\geq \hat{F}_a(x).\end{aligned}$$

Thus $\hat{F}_a(x-y) = \hat{F}_a(x+(-y)) \geq min(\hat{F}_a(x), \hat{F}_a(-y))$
Hence $(\hat{F}, A)$ is a vague soft near-ring over N. □

Definition 3.4. Let $(\hat{F}, A)$ be a vague soft set over N. Let $\phi : N \to N$ be a map.
Then we define,

$$\hat{F}^{\phi}(x) = \hat{F}(\phi(x))$$

.
That is, ${t_a}^{\phi}(x) = t_a(\phi(x))$ and $1 - {f_a}^{\phi}(x) = 1 - f_a(\phi(x))$ for all $a \in A$. $(\hat{F}^{\phi}, A)$ is also vague soft set over N.

Theorem 3.5. *Let $(\hat{F}, A)$ be a vague soft near-ring over N and ϕ be a homomorphism of N. Then $(\hat{F}^{\phi}, A)$ is a vague soft near-ring over N.*

Proof. Let $(\hat{F}, A)$ be a vague soft near-ring over N.
Let $a \in A$ and $x, y \in N$.
To prove that $(\hat{F}^{\phi}, A)$ is a vague soft near-ring over N.

$$\begin{aligned}
\hat{F_a}^{\phi}(x-y) &= \hat{F_a}\left(\phi(x-y)\right) \\
&= \hat{F_a}\left[\phi(x)-\phi(y)\right] \\
&\geq \min\left[\hat{F_a}\left(\phi(x)\right), \hat{F_a}\left(\phi(y)\right)\right] \\
&= \min\left[\hat{F_a}^{\phi}(x), \hat{F_a}^{\phi}(y)\right]
\end{aligned}$$

Also we have,

$$\begin{aligned}
\hat{F_a}^{\phi}(xy) &= \hat{F_a}\left(\phi(xy)\right) \\
&= \hat{F_a}\left[\phi(x)\phi(y)\right] \\
&\geq \min\left[\hat{F_a}\left(\phi(x)\right), \hat{F_a}\left(\phi(y)\right)\right] \\
&= \min\left[\hat{F_a}^{\phi}(x), \hat{F_a}^{\phi}(y)\right]
\end{aligned}$$

Hence $(\hat{F}^{\phi}, A)$ is a vague soft near-ring over N. □

Theorem 3.6. *Let $(\hat{F}, A)$ be a vague soft set over N. Then $(\hat{F}, A)$ be a vague soft near-ring over N if and only if for all $a \in A$ and for any $\alpha, \beta \in [0,1]$, the vague soft (α, β)-cut of $(\hat{F}, A)$ is a soft near-ring.*

Proof. Suppose $(\hat{F}, A)$ be a vague soft near-ring over N.
Let any $a \in A$.
Let any $\alpha, \beta \in [0,1]$ with $\alpha \leq \beta$.
To show that $(\hat{F}, A)_{(\alpha,\beta)}$ is a soft near-ring over N.
let $x, y \in \left(\hat{F}_a\right)_{(\alpha,\beta)}$. Then $\hat{F}_a(x) \geq [\alpha, \beta]$ and $\hat{F}_a(y) \geq [\alpha, \beta]$.
Therefore $\hat{F}_a(x-y) \geq \min\left(\hat{F}_a(x), \hat{F}_a(y)\right) \geq [\alpha, \beta]$.
Also $\hat{F}_a(xy) \geq \min\left(\hat{F}_a(x), \hat{F}_a(y)\right) \geq [\alpha, \beta]$
Hence $x-y, xy \in \left(\hat{F}_a\right)_{(\alpha,\beta)}$.
Thus, $(\hat{F}, A)_{(\alpha,\beta)}$ is a soft near-ring over N.
Conversely, Let $(\hat{F}, A)_{(\alpha,\beta)}$ is a soft near-ring over N.
To show that $(\hat{F}, A)$ be a vague soft near-ring over N.

Define, $[\alpha,\beta]=\min\left(\hat{F}_a(x),\hat{F}_a(y)\right)$ for all $a\in A$ and $x,y\in N$.
Then $x,y\in\left(\hat{F}_a\right)_{(\alpha,\beta)}$.
By assumption, $x-y,xy\in\left(\hat{F}_a\right)_{(\alpha,\beta)}$.
Therefore $\hat{F}_a(x-y)\geq[\alpha,\beta]=\min\left(\hat{F}_a(x),\hat{F}_a(y)\right)$ and $\hat{F}_a(xy)\geq[\alpha,\beta]=\min\left(\hat{F}_a(x),\hat{F}_a(y)\right)$ Hence $(\hat{F},A)$ be a vague soft near-ring over N. □

4. IDEALS OF VAGUE SOFT NEAR-RING

Let N be a near ring. Let $(\hat{I},A)$ be a non-null vague soft set over N. Then $(\hat{I},A)$ is called vague soft ideal over N if and only if for each $a\in A$ and $x,y,i\in N$ the following conditions hold:

(i) $(\hat{I},A)$ is vague soft near-ring over N ,
(ii) $\hat{I}_a[(x+i)y-xy]\geq\hat{I}_a(i)$,
(iii) $\hat{I}_a(xy)\geq\hat{I}_a(y)$.

If $\hat{I}_a$ satisfies (i), (ii) then it is called vague soft right ideal over N and if it satisfies (i), (iii) then it is called vague soft left ideal over N.

Example 4.1. Let $N=\{a,b,c,d\}$ be a non-empty set with two binary operations '+' and '·' defined as fallows,

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	a	b

let $A=\{e_1,e_2\}$ be a subset of set of parameters.
Consider a vague soft set $(\hat{F},A)$ over a near-ring N,

$$\hat{F}(e_1)=\{[0.2,0.8]/a,[0.1,0.13]/b,[0.1,0.3]/c,[0.1,0.3]/d\},$$
$$\hat{F}(e_2)=\{[0.4,0.9]/a,[0.3,0.5]/b,[0.3,0.5]/c,[0.3,0.5]/d\}.$$

Then $(\hat{F},A)$ is a vague soft ideal over N.

Theorem 4.2. *Let $(\hat{F},A)$ and $(\hat{G},B)$ be two vague soft ideals over a near- ring N. Then $(\hat{F},A)\bigwedge(\hat{G},B)$ is vague soft ideal over N.*

Proof. Let $(\hat{F}, A) \bigwedge (\hat{G}, B) = (\hat{H}, A \times B)$
Then for all $(a, b) \in A \times B$ we have,

$$\begin{aligned} \hat{H}_{(a,b)}(x-y) &= \hat{F}_a(x-y) \bigcap \hat{G}_b(x-y) \\ &= \min \left[\hat{F}_a(x-y), \hat{G}_b(x-y) \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\}, \min \left\{ \hat{G}_b(x), \hat{G}_b(y) \right\} \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{G}_b(x) \right\}, \min \left\{ \hat{F}_a(y), \hat{G}_b(y) \right\} \right] \end{aligned}$$

Hence $\hat{H}_{(a,b)}(x-y) \geq \min \left(\hat{H}_{(a,b)}(x), \hat{H}_{(a,b)}(y) \right)$.

$$\begin{aligned} \hat{H}_{(a,b)}(xy) &= \hat{F}_a(xy) \bigcap \hat{G}_b(xy) \\ &= \min \left[\hat{F}_a(xy), \hat{G}_b(xy) \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{F}_a(y) \right\}, \min \left\{ \hat{G}_b(x), \hat{G}_b(y) \right\} \right] \\ &\geq \min \left[\min \left\{ \hat{F}_a(x), \hat{G}_b(x) \right\}, \min \left\{ \hat{F}_a(y), \hat{G}_b(y) \right\} \right] \end{aligned}$$

Hence $\hat{H}_{(a,b)}(xy) \geq \min \left(\hat{H}_{(a,b)}(x), \hat{H}_{(a,b)}(y) \right)$.

$$\begin{aligned} \hat{H}_{(a,b)}(xy) &= \hat{F}_a(xy) \bigwedge \hat{G}_b(xy) \\ &= \min \left[\hat{F}_a(xy), \hat{G}_b(xy) \right] \\ &\geq \min \left[\hat{F}_a(y), \hat{G}_b(y) \right] \end{aligned}$$

Hence $\hat{H}_{(a,b)}(xy) \geq \hat{H}_{(a,b)}(y)$.

$$\begin{aligned} \hat{H}_{(a,b)} \left\{ (x+i)y - xy \right\} &= \hat{F}_a \left\{ (x+i)y - xy \right\} \bigwedge \hat{G}_b \left\{ (x+i)y - xy \right\} \\ &\geq \hat{F}_a(i) \bigwedge \hat{G}_b(i) \\ &= \hat{H}_{(a,b)}(i). \end{aligned}$$

Hence $(\hat{F}, A) \bigwedge (\hat{G}, B)$ is vague soft ideal over N. □

Theorem 4.3. *Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two vague soft ideals over a near-ring N. Then $(\hat{F}, A) \bigvee (\hat{G}, B)$ is vague soft ideal over N.*

Proof. Proof is similar to above theorem. □

Theorem 4.4. *Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two vague soft ideals over a near-ring N. Then $(\hat{F}, A)\widetilde{\bigcap}(\hat{G}, B)$ is vague soft ideal over N.*

Proof. Let $(\hat{F}, A)\widetilde{\bigcap}(\hat{G}, B) = (\hat{H}, C)$ where $C = A\bigcup B$ and for all $c \in C, x \in N$,

$$\begin{aligned} \hat{H}_c(x) &= \hat{F}_c(x) && \text{if } c \in A - B \\ &= \hat{G}_c(x) && \text{if } c \in B - A \\ &= \hat{F}_c(x)\bigcap \hat{G}_c(x) && \text{if } c \in A\bigcap B \end{aligned}$$

Case1: If $c \in A - B$ then $\hat{H}_c(x) = \hat{F}_c(x)$ and since $(\hat{F}, A)$ is a vague soft ideal over N, $(\hat{H}, C)$ is a vague soft ideal over N.
Case2: If $c \in B - A$ then $\hat{H}_c(x) = \hat{G}_c(x)$ and since $(\hat{G}, B)$ is a vague soft ideal of N, $(\hat{H}, C)$ is a vague soft ideal over N.
Case3: If $c \in A\bigcap B$ then $\hat{H}_c(x) = \hat{F}_c(x)\bigcap \hat{G}_c(x)$ we have,

$$\begin{aligned} \hat{H}_c(x-y) &= \hat{F}_c(x-y)\bigcap \hat{G}_c(x-y) \\ &= \min\left[\hat{F}_c(x-y), \hat{G}_c(x-y)\right] \\ &\geq \min\left[\min\left\{\hat{F}_c(x), \hat{F}_c(y)\right\}, \min\left\{\hat{G}_c(x), \hat{G}_b(y)\right\}\right] \\ &= \min\left[\min\left\{\hat{F}_c(x), \hat{G}_c(x)\right\}, \min\left\{\hat{F}_c(y), \hat{G}_c(y)\right\}\right] \end{aligned}$$

Hence $\hat{H}_c(x-y) \geq \min\left(\hat{H}_c(x), \hat{H}_c(y)\right)$.

$$\begin{aligned} \hat{H}_c(xy) &= \hat{F}_c(xy)\bigcap \hat{G}_c(xy) \\ &= \min\left[\hat{F}_c(xy), \hat{G}_c(xy)\right] \\ &\geq \min\left[\min\left\{\hat{F}_c(x), \hat{F}_c(y)\right\}, \min\left\{\hat{G}_c(x), \hat{G}_b(y)\right\}\right] \\ &= \min\left[\min\left\{\hat{F}_c(x), \hat{G}_c(x)\right\}, \min\left\{\hat{F}_c(y), \hat{G}_c(y)\right\}\right] \end{aligned}$$

Hence $\hat{H}_c(xy) \geq \min\left(\hat{H}_c(x), \hat{H}_c(y)\right)$.

$$\hat{H}_c(xy) = \hat{F}_c(xy)\bigcap \hat{G}_c(xy) = \min\left\{\hat{F}_c(xy), \hat{G}_c(xy)\right\} \geq \min\left\{\hat{F}_c(y), \hat{G}_c(y)\right\} = \hat{H}_c(y).$$

$$\begin{aligned}\hat{H}_c\{(x+i)y-xy\} &= \hat{F}_c\{(x+i)y-xy\}\bigcap \hat{G}_c\{(x+i)y-xy\}\\ &\geq \hat{F}_c(i)\bigcap \hat{G}_c(i)\\ &= \hat{H}_c(i).\end{aligned}$$

Hence $(\hat{F},A)\widetilde{\bigcap}(\hat{G},B)$ is a vague soft ideal over N. □

Theorem 4.5. *Let $(\hat{F},A)$ and $(\hat{G},B)$ be two vague soft ideals over a near-ring N.Then $(\hat{F},A)\widetilde{\Cap}(\hat{G},B)$ is vague soft ideal over N.*

Proof. Proof is straightforward. □

Theorem 4.6. *Let $(\hat{F},A)$ and $(\hat{G},B)$ be two vague soft ideals over a near-ring N. Then $(\hat{F},A)\widetilde{\bigcup}(\hat{G},B)$ is a vague soft ideal over N if A and B are disjoint.*

Proof. Proof is similar to above theorem. □

Theorem 4.7. *Let $(\hat{F},A)$ is a vague soft left ideal over N and ϕ be a homomorphism of N. Then $(\hat{F}^{\phi},A)$ is a vague soft left ideal over N.*

Proof. Let $(\hat{F},A)$ be a vague soft left ideal over N.
Let $a\in A$ and $x,y\in N$.
Also we have, $(\hat{F}^{\phi},A)$ is a vague soft near-ring over N.
To prove that $(\hat{F}^{\phi},A)$ is a vague soft left ideal over N, it remains to show that ${\hat{F}_a}^{\phi}(xy)\geq {\hat{F}_a}^{\phi}(y)$. Consider,

$$\begin{aligned}{\hat{F}_a}^{\phi}(xy) &= \hat{F}_a[\phi(xy)]\\ &= \hat{F}_a[\phi(x)\phi(y)]\\ &\geq \hat{F}_a[\phi(y)] \qquad \text{since } \hat{F}_a(xy)\geq \min\left(\hat{F}_a(x),\hat{F}_a(y)\right)\forall x,y\in N.\end{aligned}$$

Hence $(\hat{F}^{\phi},A)$ is a vague soft left ideal over N. □

Definition 4.8. Let $(\hat{F},A)$ and $(\hat{G},B)$ be two vague soft subsets over near-ring N. Then $(\hat{F},A)\circ(\hat{G},B)=(\hat{F}\circ\hat{G},C)$ is a vague soft subset over N defined by for all $c\in A\bigcap B$.

$$(\hat{F}\circ\hat{G})_c(z)=\begin{cases}\sup\left\{\min[\hat{F}_c(x),\hat{G}_c(x)]\right\} & ;\text{if z is expressed as } z=xy\\ 0 & ;\text{Otherwise}\end{cases}$$

Theorem 4.9. *Let N be a near-ring, E be set of parameters and A be a subset of E. Then $(\hat{F}, A)$ is vague a soft left ideal over N if and only if for each $a \in A$ the corresponding vague set over N satisfies the following conditions*

(i) $(\hat{F}, A)$ *is a vague soft near-ring over N,*

(ii) $\widetilde{A} \circ (\hat{F}, A) \subseteq (\hat{F}, A)$.

Proof. Suppose $(\hat{F}, A)$ is vague soft left ideal over N.
Then clearly $(\hat{F}, A)$ is a vague soft near-ring over N.
Let z be any element of N.
Now if $z = xy$

$$\begin{aligned}\widetilde{A} \circ (\hat{F}, A)(z) &= \sup\Big\{\min[\widetilde{A}(x), \hat{F}_c(y)]\Big\} \\ &= \hat{F}(y) \\ &\leq \hat{F}(xy) \\ &= \hat{F}(z)\end{aligned}$$

If z cannot be expressed as $z = xy$ then $\widetilde{A} \circ (\hat{F}, A)(z) = 0 \leq \hat{F}(z)$.
Therefore, in any case $\widetilde{A} \circ (\hat{F}, A)(x) \leq \hat{F}(x)$.
Conversely, let $(\hat{F}, A)$ be a vague soft set over N such that for each $a \in A$. The corresponding vague subset $\hat{F_a}$ of N satisfy given conditions. Let $x, y \in N$. Then we have

$$\begin{aligned}\hat{F}(xy) &\geq (\widetilde{A} \circ \hat{F})(xy) \\ &= \sup\Big\{\min(\widetilde{A}(x), \hat{F}(y))\Big\} \\ &\geq \min(\widetilde{A}(x), \hat{F}(y)) \\ &= \hat{F}(y).\end{aligned}$$

This shows that, $(\hat{F}, A)$ is vague soft left ideal over N. □

The vague soft ideal of a vague soft near-ring

Definition 4.10. Let $(\hat{F}, A)$ be a vague soft near-ring over N. A non-null vague soft set $(\hat{G}, B)$ over N is called a vague soft ideal of $(\hat{F}, A)$ if it satisfies the following conditions:

(i) $B \subset A$,
(ii) $(\hat{G}, B)$ is a vague soft sub near-ring of $(\hat{F}, A)$,

(iii) $\hat{G}_b(xy) \geq \hat{G}_b(y)$,
(iv) $\hat{G}_b\left[(x+i)y - xy\right] \geq \hat{G}_b(i)$. for all $x, y, i \in Supp(\hat{G}, B)$ and $\forall b \in B$.

Theorem 4.11. *Let $(\hat{G_1}, B_1)$ and $(\hat{G_2}, B_2)$ be two vague soft ideals of a vague soft near-ring $(\hat{F}, A)$ over N. Then $(\hat{G_1}, B_1)\widetilde{\bigcap}(\hat{G_2}, B_2)$ is a vague soft ideal of $(\hat{F}, A)$ if it is non-null.*

Proof. As $(\hat{G_1}, B_1)$ and $(\hat{G_2}, B_2)$ be two vague soft ideals of a vague soft near-ring $(\hat{F}, A)$ over N,
we have
$B_1 \subset A$ and $B_2 \subset A$. Also $(\hat{G_1}, B_1)$ and $(\hat{G_2}, B_2)$ be two vague soft subsets of $(\hat{F}, A)$.
Clearly $(\hat{G_1}, B_1)\widetilde{\bigcap}(\hat{G_2}, B_2)$ is non-null vague soft subset of $(\hat{F}, A)$.
let $(\hat{G_1}, B_1)\widetilde{\bigcap}(\hat{G_2}, B_2) = (\hat{H}, C)$ where $C = B_1 \bigcap B_2$ and for all $c \in C, x \in supp(\hat{H}, C)$
we have,

$$\begin{aligned}
\hat{H}_c(x) &= \hat{G_{1c}}(x) && \text{if } c \in B_1 - B_2 \\
&= \hat{G_{2c}}(x) && \text{if } c \in B_2 - B_1 \\
&= \hat{G_{1c}}(x) \bigcap \hat{G_{2c}}(x) && \text{if } c \in B_1 \bigcap B_2
\end{aligned}$$

Therefore, there are three cases to be considered and they are as given below:
Case1: If $c \in B_1 - B_2$, then $\hat{H}_c(x) = \hat{G_{1c}}(x)$ and since $(\hat{G_1}, B_1)$ is a vague soft ideal of $(\hat{F}, A)$, $(\hat{H}, C)$ is a vague soft ideal of $(\hat{F}, A)$.
Case2: If $c \in B_2 - B_1$, then $\hat{H}_c(x) = \hat{G_{2c}}(x)$ and since $(\hat{G_2}, B_2)$ is a vague soft ideal of $(\hat{F}, A)$, $(\hat{H}, C)$ is a vague soft ideal of $(\hat{F}, A)$.
Case3: If $c \in B_1 \bigcap B_2$ then $\hat{H}_c(x) = \hat{G_{1c}}(x) \bigcap \hat{G_{2c}}(x)$
So, we have

$$\begin{aligned}
\hat{H}_c(x-y) &= \hat{G_{1c}}(x-y) \bigcap \hat{G_{2c}}(x-y) \\
&= \min\left[\hat{G_{1c}}(x-y), \hat{G_{2c}}(x-y)\right] \\
&\geq \min\left[\min\left\{\hat{G_{1c}}(x), \hat{G_{1c}}(y)\right\}, \min\left\{\hat{G_{2c}}(x), \hat{G_{2c}}(y)\right\}\right] \\
&= \min\left[\min\left\{\hat{G_{1c}}(x), \hat{G_{2c}}(x)\right\}, \min\left\{\hat{G_{1c}}(y), \hat{G_{2c}}(y)\right\}\right] \\
&= \min\left(\hat{H}_c(x), \hat{H}_c(y)\right)
\end{aligned}$$

Hence $\hat{H}_c(x-y) \geq \min\Big(\hat{H}_c(x), \hat{H}_c(y)\Big)$

$$\begin{aligned}\hat{H}_c(xy) &= \hat{G_{1c}}(xy) \bigcap \hat{G_{2c}}(xy)\\ &= \min\Big[\hat{G_{1c}}(xy), \hat{G_{2c}}(xy)\Big]\\ &\geq \min\Big[\min\Big\{\hat{G_{1c}}(x), \hat{G_{1c}}(y)\Big\}, \min\Big\{\hat{G_{2c}}(x), \hat{G_{2c}}(y)\Big\}\Big]\\ &= \min\Big[\min\Big\{\hat{G_{1c}}(x), \hat{G_{2c}}(x)\Big\}, \min\Big\{\hat{G_{1c}}(y), \hat{G_{2c}}(y)\Big\}\Big]\\ &= \min\Big(\hat{H}_c(x), \hat{H}_c(y)\Big)\end{aligned}$$

Hence $\hat{H}_c(xy) \geq \min\Big(\hat{H}_c(x), \hat{H}_c(y)\Big)$.

Also, $\hat{H}_c(xy) = \hat{G_{1c}}(xy) \bigcap \hat{G_{2c}}(xy) = \min\Big(\hat{G_{1c}}(xy), \hat{G_{2c}}(xy)\Big) \geq \min\Big\{\hat{G_{1c}}(y), \hat{G_{2c}}(y)\Big\} = \hat{H}_c(y)$.

$$\begin{aligned}\hat{H}_c\{(x+i)y - xy\} &= \hat{G_{1c}}\{(x+i)y - xy\} \bigcap \hat{G_{2c}}\{(x+i)y - xy\}\\ &\geq \hat{G_{1c}}(i) \bigcap \hat{G_{2c}}(i)\\ &= \hat{H}_c(i).\end{aligned}$$

Thus in any case $(\hat{G}_1, B_1)\widetilde{\bigcap}(\hat{G}_2, B_2) = (\hat{H}, C)$ is a vague ideal of $(\hat{F}, A)$. □

Theorem 4.12. *Let $(\hat{G}_1, B_1)$ and $(\hat{G}_2, B_2)$ be two vague soft ideals of a vague soft near-ring $(\hat{F}, A)$ over N. Then $(\hat{G}_1, B_1)\widetilde{\bigcup}(\hat{G}_2, B_2)$ is a vague soft ideal of $(\hat{F}, A)$ if it is non-null.*

Proof. Proof is similar to above theorem. □

Conclusion

In this paper, we extend our work to a vague soft near-rings. We defined the vague soft ideals over a near-rings. We studied and discussed its properties. Also, we introduced the composition of vague soft sets and vague soft ideals of a vague soft near-rings.

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