

MATHEMATICAL MODELING OF SNOW DRIFTS TRANSPORT BY WIND OVER THE MOUNTAIN TERRAIN

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ABSTRACT. In this paper, estimation of snow particles motion velocity and snow-drift density by mathematical modeling is studied. The snow particles flow is modeling from fluid dynamic concepts and micro-continuum approach (couple stresses fluid [1]). The present approach is compared to conventional classical approach of logarithmic law of snow-drift density ([2], [3]). It is shown that, the present approach yield better results than that of conventional approach.

Key Words: Snow particles velocity, Snow drift density, Couple stress fluids.

2010 Mathematics Subject Classification: Primary: 13A15; Secondary: 13F30, 13G05.

1. INTRODUCTION

Snow drifting is a phenomenon that is triggered when wind speeds are strong enough to induce the movement of snow particles on the ground. Snow drifting is an important factor in the estimation of the surface mass balance of an ice sheet. Budd et al. [4] estimated the snow drift transport rate based on observations of the snow drift flux. There are three types of snow movement: creep, saltation, and turbulent diffusion [3, 5]. The correlation between different wind conditions and patterns of snow deposition was analyzed for a potential avalanche release zone. Because of its importance, a lot of progress has been made in modeling

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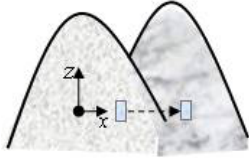


FIGURE 1. Flow over mountain

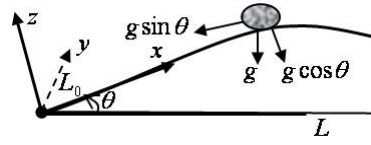


FIGURE 2. Body force representation

and understanding of snow drift. The development of such models is of extreme interest for the improvement of avalanche forecasting, and deserves major attention in studying of snow drift. Additionally, the interaction of wind and topography during snowfall promotes enhanced snow loading on leeward slopes due to the preferential deposition of the precipitation [6].

Wind-transported snow is a common phenomenon in cold windy areas, creating snowdrifts and contributing significantly to the loading of avalanche release areas. It is therefore necessary to take into account snowdrift formation both in terms of predicting and controlling drift patterns [2, 7, 8].

Previous investigations of snow drift can be grouped into three categories: field measurements, physical simulations and numerical simulations. Most field work has been carried out on planes, e.g., in Antarctica [9], and concern mostly the determination of the transport rate for steady-state conditions, e.g., [10]. Only few attempts were made to measure snow drift in mountain areas, e.g., [10], making drift flux measurements on a crest. Fohn and Meister [11] looked into the snow distribution across a mountain crest.

2. ANALYSIS

The transport due to blowing and drifting snow can be described as follows (see Fig. 1). If the wind blowing over the snow surface becomes sufficiently strong, and wind shear exceeds a certain critical value, some grains are set in motion by the wind. The basic governing momentum Equation for one-dimension flow with micro continuum approach is:

$$(2.1) \quad \rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu \nabla^2 u + \rho f_x - \eta \nabla^4 u$$

Where ρ is snow density, u is the axial and radial velocity, μ is viscosity of snow, p is the pressure, η is the couple stress parameter, f_x is the body force. Body force is taken in the form of (see Fig. 2)

$$(2.2) \quad f_x = g (\sin \theta + \psi \cos \theta)$$

Where, ψ is coefficient of friction ($\psi = -\mu$). Equation (2.1) is further simplified by following assumption:

- Flow is steady and laminar.
- Flow is one dimensional.
- $\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial z}$

So that equation (2.1) simplifies to

$$(2.3) \quad \eta \frac{\partial^4 u}{\partial z^4} - \mu \frac{\partial^2 u}{\partial z^2} + \frac{\partial p}{\partial x} - \rho f_x = 0$$

To solving equation (2.3), we need four conditions. Following initial and boundary conditions are assumed in the present model.

1. $u|_{z=z_0} = 0$
2. $\frac{du}{dz}|_{z=z_1} = 0$
3. $u|_{z=z_1} = U_0$
4. $\frac{d^2 u}{dz^2}|_{z=z_1} = \frac{\omega_0}{L_0(1+\bar{\eta})}$

Where $\bar{\eta}$ ($\frac{\dot{\eta}}{\eta}$) and $\dot{\eta}$ is gradient viscosity, z_0 is the surface roughness, U_0 is stands for the free-air flow speed, is the snow particle fall velocity, L_0 is Cross-Section of element surface and z_1 is Maximum height. In order to, estimate of pressure gradient by using the Bernoulli's Equation and empirical relation [3] is:

$$(2.4) \quad \frac{dp}{dx} = -\frac{0.17}{x} \rho U_0^2 \left(\frac{3x}{L \sin^2 \theta} \right)^{0.34}$$

Where $U_0 = 10m/sec$, $w_0 = 0.0191 1/sec$, $z_0 = 0.002 m$, $z_1 = 5 m$, $L_0 = 1 m$ and L is windward base distance from the ridge crest ($L = (x_1 - x_0) \cos \theta = 12 \cos \theta$).

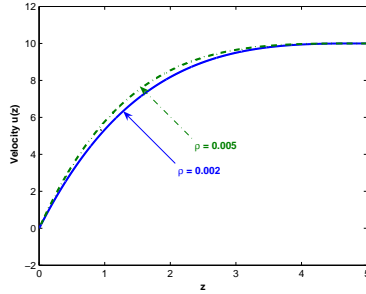
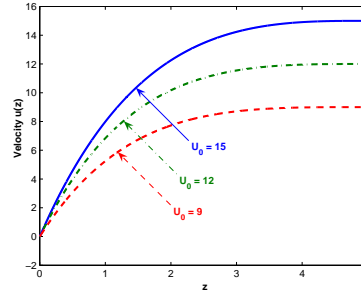
The other parameters required for the computation have been taken from [1, 3, 5, 12] are shown in Table 1.

3. RESULTS

Snow particle velocity has been computed for its variation with height (z), density (ρ), couple stress parameters ($\bar{\eta}$), and for different U_0 . The

TABLE 1. Data of various flow parameters

Symbol	Description	Range	Typical Value
θ	Angle	0 – 60	30°
η	Couple stress	-1 – 1	0.5 kg/m.s
$\bar{\eta}$	Couple stress	-1 – 1	1
ρ	Snow density	0.0002 – 0.0003	0.0002 kg/m ³
μ	Viscosity	0.16 – 0.3	0.2 kg/m.s

FIGURE 3. Variation of velocity with z for different drift densityFIGURE 4. Variation of velocity with z for different maximum velocity

results are shown in Figures 3 to 5. The results have also been compared to classical conventional results of logarithmic Wind profile [3]

$$(3.1) \quad u_z = \frac{u_*}{k} \ln \left(\frac{z}{z_0} \right)$$

Where u_* is the frictional velocity taken to be $u_* = 0.05U_0$, k is Von-Karman constant (0.41). The computed values have been shown in Fig. 6 [The graph includes the results computed by the present method also]. Snow drift density is computed by using the relations [3, 5]

$$(3.2) \quad \rho(z) = \rho_{z_0} \left(\frac{z_0}{z} \right)^{\frac{w}{ku_*}}$$

Where $w = 0.5 \text{ m.s}^{-1}$. The compared results are shown in Figs. 7&8.

4. CONCLUSION

The proposed model is estimated snow particles motion velocity in hilly terrain. The flow parameters are compared with the published results [3, 13, 14]. The results indicate that, the present model agree in

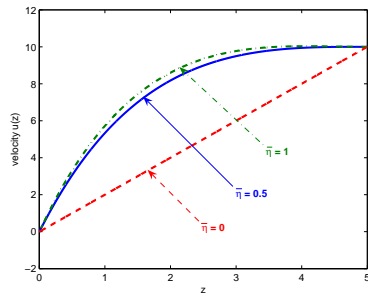


FIGURE 5. Variation of velocity with z for different couple stress parameter

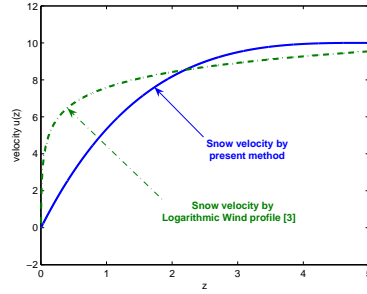


FIGURE 6. Variation of velocity with z [Comparison of Logarithmic Wind profile with the present approach]

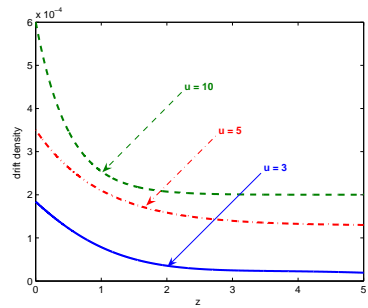


FIGURE 7. Variation of snow drift density with z for different velocity (with present method)

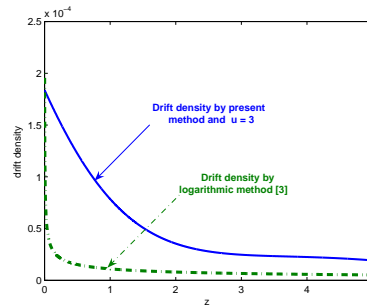


FIGURE 8. Variation of snow drift density with z [Comparison of Logarithmic drift density with the present approach]

quantitative approach to the one empirical modeled by Fohn [3] however the qualitative values are different. The present model is refined over the earlier observations [3]. The results indicate that, the Blowing and drifting snow of present values are higher in comparison to Ref. [3]. This observation also agrees with the physics of flows since snow particle flow in air reduces the density i. e. when height increases snow-drifts density decreases. Hence the present results can be used for the estimation of snow mass concentration on the leeward zones and estimation of snow avalanche.

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