Journal of Hyperstructures 11 *(2)* (*2022*)*, 196-213.*
ISSN: 2322-1666 print/2251-8436 online

ON QUASI-HYPERIDEALS AND BI-HYPERIDEALS IN MULTIPLICATIVE HYPERSEMIRINGS

MD. SALIM

Abstract. In this paper we introduce the notion of quasi-hyperideal in multiplicative hypersemirings which is a generalization of one-sided hyperideal and study some of its properties and obtain some characterizations of quasi-hyperideal in multiplicative hypersemirings. Also, we introduce the notion of bi-hyperideal in multiplicative hypersemirings. We prove that in a multiplicative hypersemiring every quasi-hyperideal is a bi-hyperideal, but the converse is not true. Lastly, we characterize regular multiplicative hypersemiring with the help of quasi-hyperideal and bi-hyperideal.

Key Words: Multiplicative hypersemiring, minimal left hyperideal, minimal right hyperideal, quasi-hyperideal, bi-hyperideal, regular multiplicative hypersemiring.
2010 Mathematics Subject Classification: Primary: 20N99; Secondary: 20N20, 16Y99.

1. Introduction

Algebraic hyperstructure which is based on the notion of hyperoperation was introduced by Marty [5] in 1934 and studied extensively by many researchers who have observed that the theory of hyperstructure has many applications in many disciplines such as theoretical physics, computer science, information science and coding theory etc. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements

Received: 13 September 2022, Accepted: 21 December 2022. Communicated by Yuming Feng;
∗Address correspondence to Md. Salim; E-mail: smpmath746@yahoo.in.

is a set. In [18, 19, 20, 21] Corsini and Fotea and in [2] Davvaz and Fotea points out their applications in rough set theory, cryptography, geometry, binary relations, graphs and hypergraphs. In 1934, Dresher and Ore [10] introduced the notion of multigroup which is an algebraic system that satisfied all the axioms of group except that multiplication is multivalued. In the work of Ibrahim and Ejegwa [1] it was revealed that "A hypergroup is also known as a multigroup, although some call a multigroup a hypergroup with a designed identity element, as well as a designated inverse for every element with respect to the identity". In 2006, Marshall [14] introduced the notions of multirings, multifield and studied their properties. Hamidi et. al [11] constructed relation between multigroups and multiring on every nonempty set. In [12], Hamidi et. al generalized the concept of multirings to general multirings and studied their properties.

The notion of multiplicative hyperring has been introduced by Rota [24] in which the addition is a binary operation and multiplication is a binary hyperoperation. In [13] Krasner also introduced the notion of hyperring, called Krasner hyperring. In Krasner hyperring $(R, +, \cdot)$, '+' is a binary hyperoperation and '$\cdot$' is a binary operation, in which the zero element is absorbing zero. The hyperstructure hypersemiring [23], introduced by Ciampi and Rota in the year 1987, is a straight hyperstructural generalization of the notion of semiring. In [25], Dasgupta studied multiplicative hypersemirings in his Ph.D thesis. Ameri, Kordi and Sarka-Mayerova [22] introduced the notion of coprime hyperideals in multiplicative hypersemiring. In the recent year the theory of hyperstructures is further developed by many researchers [7, 8, 4]. In 2015, Salim et. al [15] introduced the class of multiplicative ternary hyperring. After that, in 2018, Tamang and Mandal [16] defined ternary hypersemiring, which is a generalization of the concept of multiplicative ternary hyperring and studied prime and primary hyperideal in ternary hypersemirings.

In 1956, Steinfeld [17] introduced the notion of quasi-hyperideal. Quasi-ideals of semirings was studied by Dönges in [3], Sioson [6]. Quasi-ideals and bi-ideals of ternary semigroup was studied by Dixit et. al [26]. In [9], Hila et al. introduced the notion of quasi-hyperideal in semihypergroup and studied it. In this paper we introduce quasi-hyperideal and bi-hyperideal of multiplicative hypersemiring which are generalizations of one-sided hyperideal and quasi-hyperideal respectively. Furthermore we investigate their properties in multiplicative hypersemiring and obtain the relation between them. Also some intersection properties of quasi-hyperideal have been studied. We obtain some characterization of quasi-hyperideal and bi-hyperideal in a multiplicative hypersemiring. Finally we prove that in a regular multiplicative hypersemiring the notions of quasi-hyperideal and bi-hyperideal coincide.

2. Preliminaries

Definition 2.1. By a hyperoperation '$\circ$' on a nonempty set H, we shall mean a mapping $\circ$: $H \times H \to P^*(H)$ where $P^*(H)$ is the set of all nonempty subsets of H. For $x, y \in H$, the image of the element $(x, y) \in H \times H$ under the mapping '$\circ$' will be denoted by $x \circ y$ (which is called the hyperproduct x, y).

A nonempty set H equipped with a single hyperoperation $\circ$ is called hypergroupoid $(H, \circ)$. An element a of hypergroupoid $(H, \circ)$ is called a scalar in H if $|a \circ x| = |x \circ a| = 1$, for all $x \in H$.

A hypergroupoid $(H, \circ)$ is commutative, when $x \circ y = y \circ x$, $\forall x, y \in H$. If $(H, \circ)$ is a hypergroupoid, then for any $x \in H$ and $A, B \in P(H)$ (power set of H), we define that $A \circ B = \bigcup\limits_{(a,b) \in A \times B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$.

A hypergroupoid $(H, \circ)$ is said to be semihypergroup if the hyperoperation '$\circ$' satisfies the associative law: $x \circ (y \circ z) = (x \circ y) \circ z$, for all $x, y, z \in H$.

Let $A, B \in P(S)$ then $A + B = \{a + b : a \in A \text{ and } b \in B\}$, where $P(S)$ is the power set of S.

Definition 2.2. [25] A multiplicative hypersemiring $(S, +, \circ)$ is an additive commutative semigroup $(S, +)$ endowed with a hyperoperation '$\circ$' such that the following conditions hold :

- **(i):** $(S, \circ)$ is semihypergroup;
- **(ii):** $(x + y) \circ z \subseteq x \circ z + y \circ z$, and $x \circ (y + z) \subseteq x \circ y + x \circ z$, $\forall x, y, z \in S$;

where if the inclusions in (ii) are replaced by equalities, then the multiplicative hypersemiring is called a strongly distributive multiplicative hypersemiring.

We have the following remark.

Remark 2.3. It is immediate to see that the notion of multiplicative hypersemiring coincides with the notion of a semiring if and only if $|a \circ b| = 1$ for all $a, b \in S$.

On the other hand if $(S, +, \circ)$ is a semiring then $(S, +, .)$ can be regarded as a strongly distributive multiplicative hypersemiring if we take $a \circ b = \{a \cdot b\}$ for all $a, b \in S$.

Thus the above notion of a multiplicative hypersemiring is a generalization of the notion of semiring.

Example 2.4. Let $S = \{a, b\}$. Then the binary operation '+' and hyperoperation '$\circ$' defined respectively as follows:

+	a	b
a	a	a
b	a	b

$\circ$	a	b
a	$\{a\}$	S
b	$\{a\}$	$\{b\}$

Then $(S, +, \circ)$ is a multiplicative hypersemiring.

Definition 2.5. The additive identity '0' of a multiplicative hypersemiring $(S, +, \circ)$ is said to be a zero (strong zero) of $(S, +, \circ)$ if $0 \in a \circ 0 = 0 \circ a$ (resp.$\{0\} = a \circ 0 = 0 \circ a$) for all $a \in S$.

Example 2.6. Consider the semiring $(\mathbb{Z}_0^+, +, .)$ of the set of all non-negative integers with respect to the usual addition and multiplication of integers. Then $(Z_0^+, +, \circ)$ forms a multiplicative hypersemiring with zero element, if the '+' is the usual addition of integers and $x \circ y = \{x \cdot y + nk : n, k \in \mathbb{Z}_0^+\}$.

Example 2.7. Consider the ternary semiring $(\mathbb{Z}_0^-, +, .)$ of the set of all non-positive integers with respect to the usual addition and multiplication of integers. Corresponding to any subset A of the set of negative integers there exists a multiplicative hypersemiring $(Z_A^-, +, .)$, where $Z_A^- = \mathbb{Z}_0^-$ and for any $x, y \in \mathbb{Z}_A^-, +$ is the usual addition of integers and $x \circ y = \{x \cdot a \cdot y : a \in A\}$.

The above multiplicative hypersemiring is called a multiplicative hypersemiring induced by A.

Definition 2.8. An additive subsemigroup T of a multiplicative hypersemiring $(S, +, \circ)$ is called a subhypersemiring without zero if $x \circ y \subseteq T$ for all $x, y \in T$.

and is called subhypersemiring with zero if $0 \in T$ and $x \circ y \subseteq T$ for all $x, y \in T$.

Definition 2.9. [25] Let $(S, +, \circ)$ be multiplicative hypersemiring. An additive subsemigroup I of S is called

(i): a left hyperideal of S if $s \circ x \subseteq I$, for all $x \in I$ and for all $s \in S$;
(ii): a right hyperideal of S if $x \circ s \subseteq I$, for all $x \in I$ and for all $s \in S$;
(iii): a hyperideal of S if I is both a left and a right hyperideal of S;

Let S be a multiplicative hypersemiring. If A and B are two nonempty subsets of S, then $A \circ B = \bigcup \{ \sum_{finite} a_i \circ b_i : a_i \in A, b_i \in B\}$.

Definition 2.10. [25] An element $e \in S \backslash (S \circ 0 \cup 0 \circ S)$ is called a hyperidentity of the multiplicative hypersemiring $(S, +, \circ)$, if $x \in e \circ x = x \circ e$, for all $x \in S$.

Proposition 2.11. [25] *Let $(S, +, \circ)$ be a multiplicative hypersemiring and A be a nonempty subset of S. Then $S \circ A$, $A \circ S$ and $S \circ A \circ S$ are respectively a left hyperideal, a right hyperideal and a hyperideal of S. If the multiplicative hypersemiring S has a hyperidentity e, then left hyperideal generated by A is $\langle A \rangle_l = S \circ A$, right hyperideal generated by A is $\langle A \rangle_r = A \circ S$ and hyperideal generated by A is $\langle A \rangle = S \circ A \circ S + S \circ A + A \circ S$.*

Definition 2.12. An element a of a multiplicative hypersemiring $(S, +, \circ)$ is called idempotent if $a \circ a = a$.

A hyperideal I of a multiplicative hypersemiring $(S, +, \circ)$ is called idempotent if $I \circ I = I$.

Definition 2.13. A left hyperideal L of a multiplicative hypersemiring S is said to be a minimal left hyperideal of S if L does not properly contain any left hyperideal of S.

Definition 2.14. A right hyperideal L of a multiplicative hypersemiring S is said to be a minimal right hyperideal of S if L does not properly contain any right hyperideal of S.

3. Quasi-hyperideal in multiplicative hypersemirings

Definition 3.1. An additive subsemigroup Q of a multiplicative hypersemiring $(S, +, \circ)$ is called a quasi-hyperideal of S if $Q \circ S \cap S \circ Q \subseteq Q$.

Example 3.2. (i) The set S and the hyperideal $\langle 0 \rangle$ both are the quasi-hyperideals of S.
(ii) In Example 2.4, $Q_1 = \{a\}$ and $Q_2 = \{b\}$ are quasi-hyperideal of S.

Example 3.3. Let $S = \{a, b, c\}$. Then $(S, +, \circ)$ is a multiplicative hypersemiring with restpect to the binary operation '+' and hyperoperation '$\circ$' defined respectively as follows:

+	a	b	c
a	a	b	c
b	b	c	a
c	c	b	a

$\circ$	a	b	c
a	$\{a\}$	$\{c\}$	S
b	$\{a, b\}$	$\{b\}$	$\{b, c\}$
c	$\{b\}$	$\{a\}$	$\{c\}$

Here we can easily verify that $Q_1 = \{a\}$, $Q_2 = \{b, c\}$ and $Q_3 = \{c\}$ are the quasi-hyperideals of S.

Example 3.4. Let $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c \in \mathbb{Z}_0^+ \right\}$ where $\mathbb{Z}_0^+$ is set of all non-negative integers. Then S is a semigroup with respect to matrix addition.

Let $A = \left\{ \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$, there exists a multiplicative hypersemiring $(S_A, +, \circ)$ where $S_A = S$ and for any $a, b \in S_A$, $a \circ b \ = \{a \cdot i \cdot b : i \in A\}$ where $a, b \in (S, +)$. This multiplicative hypersemiring is called the multiplicative hypersemiring over the semiring S induced by A. Let $Q = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix} : a \in \mathbb{Z}_0^+ \right\}$. Then Q is a quasi-hyperideal of $(S_A, +, \circ)$.

Lemma 3.5. *Every left, right and a hyperideal of a multiplicative hypersemiring S is a quasi-hyperideal of S.*

Proof. Suppose Q is a left hyperideal of S, then $S \circ Q \subseteq Q$ and obviously $S \circ S \subseteq S$. Now $Q \circ S \cap S \circ Q \subseteq S \circ S \cap Q \subseteq S \cap Q \subseteq Q$. Thus Q is a quasi-hyperideal of S.

Similarly we can prove the other cases. □

Remark 3.6. The converse of Lemma 3.5 is not true. We give an example of a quasi-hyperideal Q of a multiplicative hypersemiring which is neither a left hyperideal nor a right hyperideal of S.

Example 3.7. Let $T = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c \in \mathbb{Z}_0^+ \right\}$ where $\mathbb{Z}_0^+$ is set of all non-negative integers. Then T is a semigroup with respect to matrix addition.

Let $A = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$, there exists a multiplicative hypersemiring $(T_A, +, \circ)$ where $T_A = T$ and for any $a, b \in T_A$, $a \circ b \ = \{a \cdot i \cdot b : i \in A\}$. Let $Q = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{Z}_0^+ \right\}$. Then we can easily verify that

Q is a quasi-hyperideal, but Q is neither a right hyperideal nor a left hyperideal of $(T, +, \circ)$.

From above Example 3.7 we observe that the class of quasi-hyperideals in multiplicative hypersemiring is the generalization of the class of one-sided hyperideal of multiplicative hypersemiring.

Lemma 3.8. *Each Quasi-hyperideal of multiplicative hypersemiring $(S, +, \circ)$ is subhypersemiring of S.*

Proof. Let Q be a quasi-hyperideal of a multiplicative hypersemiring S. Then Q is an additively subsemigroup. Let $x, y \in Q$. Then $x \circ y \subseteq Q \circ S$ for $x \in Q$ and $y \in S$. Again $x \circ y \subseteq S \circ Q$ for $x \in S$ and $y \in Q$. Thus $x \circ y \subseteq Q \circ S \cap S \circ Q \subseteq Q$ and Q is subhypersemiring of S. □

Lemma 3.9. *Let S be a multiplicative hypersemiring, then*

- **(a):** *Let Q be a quasi-hyperideal of a multiplicative hypersemiring S and T be a subhypersemiring of S, then $Q \cap T$ is either empty or a quasi-hyperideal of T.*
- **(b):** *The intersection of arbitrary collection of quasi-hyperideals of a multiplicative hypersemiring S is either empty or a quasi-hyperideal of S.*

Proof. (a) If $Q \cap T$ is a nonempty, then $Q \cap T$ is a subset of T. Now $((Q \cap T) \circ T) \cap (T \circ (Q \cap T)) \subseteq (T \cap T) \cap (T \circ T) \subseteq T \cap T = T$. Again $((Q \cap T) \circ T) \cap (T \circ (Q \cap T)) \subseteq (Q \circ T) \cap (T \circ Q) \subseteq (Q \circ S) \cap (S \circ Q) \subseteq Q$. Hence $((Q \cap T) \circ T) \cap (T \circ (Q \cap T)) \subseteq Q \cap T$. This shows that $Q \cap T$ is a quasi-hyperideal of T.

(b) Let Q_i be a quasi-hyperideal of S for $i \in I$. We show $\bigcap_{i \in I} Q_i$ is either empty or a quasi-ideal of S . Assusme that $\bigcap_{i \in I} Q_i \neq \phi$. We have $(\bigcap_{i \in I} Q_i) \circ S \cap S \circ (\bigcap_{i \in I} Q_i) \subseteq Q_i \circ S \cap S \circ Q_i \subseteq Q_i$ for all $I \in I$. Hence $\bigcap_{i \in I} Q_i$ is a quasi-hyperideal of S. □

Theorem 3.10. *An additive subsemigroup Q of a multiplicative hypersemiring S is a quasi-hyperideal of S if Q is the intersection of a right hyperideal and a left hyperideal of S.*

Proof. Let R be a right hyperideal and L be a left hyperideal of S such that $Q = R \cap L$. Then by Lemma 3.9, it is obvious that Q is a quasi-hyperideal of S. □

Claim 3.11. *A quasi-hyperideal Q said to have a intersection property if Q is the intersection of a right hyperideal and a left hyperideal of S.*

Lemma 3.12. *Let S be a multiplicative hypersemiring and $e \in S$. The following statements hold ture*

(i) *$S \circ e$ is a left hyperideal of S and hence quasi-hyperideal of S.*
(ii) *$e \circ S$ is a right hyperideal of S and hence quasi-hyperideal of S.*
(iii) *$e \circ S \cap S \circ e$ is a quasi-hyperideal of S.*

Proof. (i) It follows from $S \circ (S \circ e) \subseteq (S \circ S) \circ e \subseteq S \circ e$ and rest follows from Lemma 3.5.

(ii) Similiar to (i).

(iii) It follows from Lemma 3.9 [b]. □

Definition 3.13. A quasi-hyperideal L of a multiplicative hypersemiring S is said to be a minimal quasi-hyperideal of S if L does not properly contain any quasi-hyperideal of S.

Theorem 3.14. *An additive subsemigroup Q of multiplicative hypersemiring S is a minimal quasi-hyperideal of S if and only if Q is the intersection of a minimal right hyperideal and a minimal left hyperideal of S.*

Proof. Let R be a minimal right hyperideal and L be a minimal left hyperideal of S such that $Q = R \cap L$. Then, by Theorem 3.10, we find that Q is a quasi-hyperideal of S. Now we show that Q is minimal. Suppose $Q' \subseteq Q$ be any other quasi-hyperideal of S. Then $Q' \circ S$ is a right hyperideal of S and $Q' \circ S \subseteq Q \circ S \subseteq R \circ S \subseteq R$, since R is a minimal right hyperideal of S, we have $Q' \circ S = R$. Similarly, we can prove that $S \circ Q' = L$. Therefore $Q = R \cap L = Q' \circ S \cap S \circ Q' \subseteq Q'$. Consequently $Q = Q'$ and hence Q is a minimal quasi-hyperideal of S.

Conversely, let Q be a minimal quasi-hyperideal of S. Then $Q \circ S \cap S \circ Q \subseteq Q$. Let $q \in Q$, by Lemma 3.12, $q \circ S$ is a right hyperideal of S and $S \circ q$ is a left hyperideal of S. Then by Lemma 3.12, $q \circ S \cap S \circ q$ is a quasi-hyperideal of S. Now $q \circ S \cap S \circ q \subseteq Q \circ S \cap S \circ Q \subseteq Q$. Since Q is minimal quasi-hyperideal of S, we get $q \circ S \cap S \circ q = Q$. Now we show that $q \circ S$ and $S \circ q$ is a minimal right hyperideal and minimal left hyperideal of S respectively. Suppose that R is any other right hyperideal of S such that $R \subseteq q \circ S$. Then $R \circ S \subseteq R \subseteq q \circ S$. Now $R \circ S \cap S \circ Q \subseteq q \circ S \cap S \circ Q \subseteq Q \circ S \cap S \circ Q \subseteq Q$. Since Q is minimal quasi-hyperideal of S, then $R \circ S \cap S \circ Q = Q$. This implies that $Q \subseteq R \circ S$. Again $q \circ S \subseteq Q \circ S \subseteq (R \circ S) \circ S \subseteq R \circ S \subseteq R$.

Thus $R = q \circ S$. Consequently $q \circ S$ is a minimal right hyperideal of S. Similarly we can prove that $S \circ q$ is a minimal left hyperideal of S. $\square$

Corollary 3.15. *Let S be a multiplicative hypersemiring. Then S has at least one minimal quasi-hyperideal if and only if S has at least one minimal right hyperideal and at least one minimal left hyperideal.*

Proof. Suppose that Q is a minimal quasi-hyperideal of S, then for any $q \in Q$, $q \circ S$ is a right hyperideal of S and $S \circ q$ is a left hyperideal of S. By Theorem 3.14, $q \circ S$ is the minimal right hyperideal of S and $S \circ q$ is the minimal left hyperideal of S. Thus S has at least one minimal right hyperideal and at least one minimal left hyperideal.

Converse is straightforward. $\square$

Theorem 3.16. *Let $(S, +, \circ)$ be a multiplicative hypersemiring. Then*

(i) *A right hyperideal R is minimal if and only if $a \circ S = R$ for all $a \in R$;*

(ii) *A left hyperideal L is minimal if and only if $S \circ a = L$ for all $a \in L$;*

(iii) *A quasi-hyperideal Q is minimal if and only if $a \circ S \cap S \circ a = Q$ for all $a \in Q$.*

Proof. (i) Suppose that R is minimal right hyperideal. Let $a \in R$. Then $a \circ S \subseteq R \circ S \subseteq R$. By Lemma 3.12, $a \circ S$ is a right hyperideal of S. Since R is the minimal hyperideal of S we have $a \circ S = R$.

Conversely, let $a \circ S = R$ for all $a \in R$. Let $R^{'}$ is another right hyperideal of S such that $R^{'} \subseteq R$. Let $x \in R' \subseteq R$. Then $R = x \circ S \subseteq R' \circ S \subseteq R'$. Hence $R = R'$. Therefore R is minimal.

(ii) It is similiar to (i).

(iii) Let Q be a minimal quasi-hyperideal of S. Let $a \in Q$. Then $S \circ a$ is a left hyperideal and $a \circ S$ is a right hyperideal of S. By Theorem 3.10, we have $(a \circ S) \cap (S \circ a)$ is a quasi-hyperideal of S. Again $(a \circ S) \cap (S \circ a) \subseteq (Q \circ S) \cap (S \circ Q) \subseteq Q$ implies that $Q = a \circ S \cap S \circ a$.

Conversely, Let $Q = (a \circ S) \cap (S \circ a)$ for all $a \in Q$. Let Q' be another hyperideal of S such that $Q' \subseteq Q$. Let $x \in Q$. Then $Q = x \circ S \cap S \circ x \subseteq Q' \circ S \cap S \circ Q' \subseteq Q'$. Therefore $Q = Q'$, and Q is minimal quasi-hyperideal of S. $\square$

4. Quasi-hyperideal and regularity of multiplicative hypersemiring

Definition 4.1. [25] Let $(S, +, \circ)$ be a multiplicative hypersemiring. An element $x \in S$ is said to be regular if $x \in x \circ S \circ x$, that is, there exists $s_i \in S(i = 1, 2, \cdots, n)$ such that $a \in \sum_{i=1}^{n} a \circ s_i \circ a$.

A multiplicative hypersemiring $(S, +, \circ)$ is called regular if each of its elemets is regular. If S is strongly distributive multiplicative hypersemiring, an element $x \in S$ is regular if only if there exsists $a \in S$ such that $x \in x \circ a \circ x$.

Lemma 4.2. *Let S be a multiplicative hypersemiring. If Q is a quasi-hyperideal of S then $Q \circ S \circ Q \subseteq Q$*

Proof. Let Q be a quasi-hyperideal of S. Now $Q \circ S \circ Q \subseteq Q \circ S \circ S \subseteq Q \circ S$, again $Q \circ S \circ Q \subseteq S \circ S \circ Q \subseteq S \circ Q$. Therefore $Q \circ S \circ Q \subseteq Q \circ S \cap S \circ Q \subseteq Q$. □

Theorem 4.3. *Let $(S, +, \circ)$ be a multiplicative hypersemiring. Then the following statements are equivalent:*

- **(i):** *S is a regular;*
- **(ii):** *For any right hyperideal R and left hyperideal L of S, $R \circ L = R \cap L$;*
- **(iii):** *Each right hyperideal R and each left hyperideal L of S satisfies*
  - (a) *$R \circ R = R$;*
  - (b) *$L \circ L = L$;*
  - (c) *$R \circ L$ is a quasi-hyperideal of S*
- **(iv):** *The set $\mathcal{Q}$ of all quasi-hyperideals of S is a regular hypersemigroup with respect to hyperoperation '$\circ$';*
- **(v):** *For any quasi-hyperideal Q of S satisfies $Q = Q \circ S \circ Q$.*

Proof. $(i) \Rightarrow (ii)$ Suppose S is a regular multiplicative hypersemiring. Let R and L be a right hyperideal and a left hyperideal of S respectively. Obviously $R \circ L \subseteq R \cap L$ (1). Now let $x \in R \cap L \Rightarrow x \in R$ and $x \in L$. Then we have $x \in x \circ S \circ x$, since S is regular. Now $x \in R$ and $S \circ x \subseteq S \circ L \subseteq L$ implies that $x \in x \circ (S \circ x) \subseteq R \circ L$. Thus we have $R \cap L \subseteq R \circ L$ (2). From (1) and (2), it follows that $R \cap L = R \circ L$.

$(ii) \Rightarrow (iii)$ For (a), Let R be a right hyperideal of S and $\langle R \rangle_l = R + S \circ R$ be a left hyperideal of S. By (ii), $R = R \cap \langle R \rangle_l = R \circ \langle R \rangle_l =$

$R\circ(R+S\circ R)\subseteq R\circ R+R\circ S\circ R\subseteq R\circ R+R\circ R\subseteq R\circ R\subseteq R$. Hence $R\circ R=R$.

(b). It is similiar to (a).

(c). By Theorem 3.10, $R\circ L=R\cap L$ is a quasi-hyperideal of S.

$(iii)\Rightarrow(iv)$ Obviously '$\circ$' is associative on $\mathcal{Q}$. Let $Q_1,Q_2\in\mathcal{Q}$ be quasi-hyperideals of S. By (iii) implies that $S\circ S=S$. Let $R=Q_1\circ Q_2\circ S$ be a right hyperideal of S and $L=S\circ Q_1\circ Q_2$ be a left hyperideal of S. Now $Q_1\circ Q_2\circ S\cap S\circ Q_1\circ Q_2=Q_1\circ Q_2\circ S\circ S\circ Q_1\circ Q_2\subseteq Q_1\circ Q_2\circ(S\circ S\circ S)\circ Q_2\subseteq Q_1\circ(Q_2\circ S\circ Q_2)\subseteq Q_1\circ Q_2$, by Lemma 4.2. Hence $Q_1\circ Q_2$ is a quasi-hyperideal of S.

Again $\langle Q\rangle_r=Q\circ S+Q$ is a right hyperideal of S and $\langle Q\rangle_l=S\circ Q+Q$ is left hyperideal of S. Then by $(iii)(a)$, $Q\subseteq(Q\circ S+Q)=(Q\circ S+Q)\circ(Q\circ S+Q)\subseteq Q\circ S$. Similarly we can show $Q\subseteq S\circ Q$. Again $Q\circ S\cap S\circ Q\subseteq Q$. Consequently $Q=Q\circ S\cap S\circ Q$. By using (iii)(c), we get $R\circ L=(R\circ L\circ S)\cap(S\circ R\circ L)$. Now $Q=Q\circ S\cap S\circ Q=(Q\circ S\circ S\circ Q)\circ S\cap S\circ(Q\circ S\circ S\circ Q)=Q\circ S\circ S\circ Q=Q\circ S\circ Q$. Hence $\mathcal{Q}$ is regular multiplicative hypersemigroup.

$(iv)\Rightarrow(v)$ Follows from (iv) and Lemma 4.2.

$(v)\Rightarrow(i)$ For each element $s\in S$, then by Theorem 3.10, $\langle s\rangle_r\cap\langle s\rangle_l$ is a quasi-hyperideal of S. Now by (v) $s\in\langle s\rangle_r\cap\langle s\rangle_l=(\langle s\rangle_r\cap\langle s\rangle_l)\circ S\circ(\langle s\rangle_r\cap\langle s\rangle_l)\subseteq\langle s\rangle_r\circ S\circ\langle s\rangle_l\subseteq s\circ S\circ s$. i.e $s\in s\circ S\circ s$. Hence $s\in S$ is regular in S. Consequently S is regular multiplicative hypersemiring. □

Theorem 4.4. *If every quasi-hyperideal Q of a multiplicative hypersemiring $(S,+,\circ)$ is idempotent, then S is regular.*

Proof. Let R be a right hyperideal and L be a left hyperideal of S. Then by Lemma 3.10, $R\cap L$ is a quasi-hyperideal of S. Obviously $R\circ L\subseteq R\cap L$. Now by hypothesis $R\cap L=(R\cap L)\circ(R\cap L)\subseteq R\circ L$. Thus $R\cap L=R\circ L$. Then by Theorem 4.3, S is a regular multiplicative hypersemiring. □

Lemma 4.5. *Every hyperideal M of a regular multiplicative hypersemiring $(S,+,\circ)$ is a regular multiplicative subhypersemiring.*

Proof. Each element $m\in I\subseteq S$ is regular in S, so $m\in m\circ S\circ m\subseteq m\circ(S\circ m\circ S)\circ m\subseteq m\circ I\circ m$ (as M is a hyperideal of S and $S\circ m\circ S\subseteq I$). This follows that M is a regular multiplicative subhypersemiring. □

5. Bi-hyperideal in multiplicative hypersemirings

Definition 5.1. A multiplicative subhypersemiring B of a multiplicative hypersemiring $(S,+,\circ)$ is called a bi-hyperideal of S if $B\circ S\circ B\subseteq B$.

Example 5.2. Let S be a set of negative integers. Let $A = \{-3, -5\}$. Define a hyperopeartion '$\circ$' on S by $a \circ b = \{a \cdot x \cdot y : x \in A\}$ for all $a, b \in S$. Then $(S, +, \circ)$ is a multiplicative hypersemiring. Let $B = 2S$. Then we can check B is a bi-hyperideal of S.

Example 5.3. Let $S = \{a, b\}$. Then the binary operation $+$ and hyperoperation $\circ$ defined respectively as follows:

$+$	a	b
a	a	a
b	a	b

$\circ$	a	b
a	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$

Then $(S, +, \circ)$ is a multiplicative hypersemiring. Clearly $B_1 = \{a\}$ is a bi-hyperideal of S but $B_2 = \{b\}$ is not a bi-hyperideal.

Example 5.4. [25] Let S be the set of all non-negative real numbers. Then, $(S, +)$ is a commutative semigroup, On S, '$+$' and hyperoperation $\circ$ are defined by $a + b = min\{a, b\}$ and $a \circ b = [0, a]$ for all $a, b \in S \backslash \{0\}$ and $a \circ 0 = 0 \circ a = \{0\}$ for any $a \in S$. Then, $(S, +, \circ)$ is a non-commutative multiplicative hypersemiring having a strongly absorbing zero. Let $B = [0, 1]$. Then B is a bi-hyperideal of S.

Lemma 5.5. *Every quasi-hyperideal of a multiplicative hypersemiring $(S, +, \circ)$ is a bi-hyperideal of $(S, +, \circ)$.*

Proof. It follows from Lemma 4.2. □

Claim 5.6. *Converse of the Lemma 5.5 does not hold, in general, that is, a bi-hyperideal of a multiplicative hypersemiring S may not be a quasi-hyperideal of S.*

Example 5.7. Let S be a set of all natural number. Then $(S, +)$ is a commutative semigroup. On S, '$\circ$' defined on S by $a \circ b = a + b + 3n$ for $n \in \mathbb{N}$ for all $a, b \in S$. Then $(S, +, \circ)$ forms a commutative multiplicative hypersemiring. Let $B = \{4\} \cup \{n_0 \in \mathbb{N} : n_0 \geq 10\}$. Then B is a bi-hyperideal of S but B is not a quasi-hyperideal of S, since $4 \circ 1 \subseteq B \circ S \cap S \circ B \nsubseteq B$.

Remark 5.8. Since every right hyperideal and left hyperideal of S is a quasi-hyperideal of S, it follows that every right hyperideal and left hyperideal of S is a bi-hyperideal of S, but the converse is not true.

Proposition 5.9. *If B is a bi-hyperideal of multiplicative hypersemiring S and T is a multiplicative subhypersemiring of S, then $B \cap T$ is a bi-hyperideal of T.*

Proof. It follows from $(B \cap T) \circ T \circ (B \cap T) \subseteq (B \cap T) \circ S \circ (B \cap T) \subseteq B \circ S \circ B \subseteq B$ and $(B \cap T) \circ T \circ (B \cap T) \subseteq T \circ T \circ T \subseteq T$. Consequently $(B \cap T) \circ T \circ (B \cap T) \subseteq B \cap T$ and hence $B \cap T$ is a bi-hyperideal of T. □

Lemma 5.10. *If B is a bi-hyperideal of a multiplicative hypersemiring $(S, +, \circ)$ and T is a multiplicative subhypersemiring of S, then $B \circ T$ and $T \circ B$ are bi-hyperideal of S.*

Proof. It follows from $(B \circ T) \circ S \circ (B \circ T) \subseteq (B \circ (T \circ S) \circ B) \circ T \subseteq (B \circ S \circ B) \circ T \subseteq B \circ T$, since $T \circ S \subseteq S \circ S \subseteq S$ and B is a bi-hyperideal of S. Hence $B \circ T$ is a bi-hyperideal of S. □

Corollary 5.11. *If B_1, B_2 are two bi-hyperideals of a multiplicative hypersemiring S, then $B_1 \circ B_2$ is a bi-hyperideal of S.*

Proof. It follows from $(B_1 \circ B_2) \circ S \circ (B_1 \circ B_2) \subseteq (B_1 \circ (B_2 \circ S) \circ B_1) \circ B_2 \subseteq (B_1 \circ (S \circ S) \circ B_1) \circ B_2 \subseteq (B_1 \circ S \circ B_1) \circ B_2 \subseteq B_1 \circ B_2$, since B_1 is a bi-hyperideal of S. □

Corollary 5.12. *If Q_1, Q_2 are two quasi-hyperideals of a multiplicative hypersemiring S, then $Q_1 \circ Q_2$ is a bi-hyperideal of S.*

Proof. Since Q_1 and Q_2 are quasi-hyperideals of S. Then by Lemma 3.9[b], $Q_1 \circ Q_2$ is a quasi-hyperideal of S. Thus from Corollary 5.11, $Q_1 \circ Q_2$ is bi-hyperideal of S. □

Theorem 5.13. *Let B be a bi-hyperideal of a multiplicative hypersemiring S, and C be a bi-hyperideal of B such that $C \circ C = C$. Then C is a bi-hyperideal of S.*

Proof. Since B is bi-hyperideal in S, then $B \circ S \circ B \subseteq B$ and C is bi-hyperideal in B, then $C \circ B \circ C \subseteq C$.

Now $C \circ S \circ C = C \circ (C \circ S \circ C) \circ C \subseteq C \circ (B \circ S \circ B) \circ C \subseteq C \circ B \circ C \subseteq C$. Hence C is a bi-hyperideal of S. □

Proposition 5.14. *Let X and Y be two multiplicative subhypersemiring of a multiplicative hypersemiring S and $B = X \circ Y$. Then B is a bi-hyperideal of S if at least one of X, Y a right and a left hyperideal of S.*

Proof. Let $B = X \circ Y$. Suppose that X is a left hyperideal of S. Then we get $(X \circ Y) \circ S \circ (X \circ Y) \subseteq (S \circ S) \circ S \circ (X \circ Y) \subseteq (S \circ X) \circ Y \subseteq X \circ Y$. Hence $B = X \circ Y$ is a bi-hyperideal of S.

Suppose that Y is a left hyperideal of S. Then we get $(X \circ Y) \circ S \circ (X \circ Y) \subseteq X \circ (S \circ S \circ S) \circ Y \subseteq X \circ (S \circ Y) \subseteq X \circ Y$. This shows that $B = X \circ Y$ is a bi-hyperideal of S.

Similarly we can prove the other cases. □

An immediate corollary of the Proposition 5.14 is the following one.

Corollary 5.15. *A multiplicative subhypersemiring B of a multiplicative hypersemiring S is a bi-hyperideal of S if $B = R \circ L$, where R is a right hyperideal and L is a left hyperideal of S.*

Theorem 5.16. *Let S be a regular multiplicative hypersemiring. Then the following statements hold ture:*

- **(i):** *Each quasi-hyperideal Q satisfies $Q = R \cap L = R \circ L$, where R is a right hyperideal and L is a left hyperideal of S,*
- **(ii):** *Each quasi-hyperideal Q of S satisfies $Q \circ Q = Q \circ Q \circ Q$,*
- **(iii):** *Each bi-hyperideal B of S is a quasi-hyperideal of S.*

Proof. (i) Let Q be a quasi-hyperideal of S, it has a intersection property. Let $R = \langle Q \rangle_r$ be a right hyperideal of S and $L = \langle Q \rangle_l$ be a left hyperideal of S. Then $Q \subseteq \langle Q \rangle_r \subseteq Q \circ S$ and $Q \subseteq \langle Q \rangle_l \subseteq S \circ Q$ by Theorem 4.3 (iii). So $Q \subseteq \langle Q \rangle_r \cap \langle Q \rangle_l \subseteq Q \circ S \cap S \circ Q \subseteq Q$. Hence $Q = \langle Q \rangle_r \cap \langle Q \rangle_l$ i.e $Q = R \cap L$, since S is regular then $Q = R \cap L = R \circ L$.

(ii) Obviously $Q \circ Q \circ Q \subseteq Q \circ Q$. By Theorem 4.3 (iv), $Q \circ Q$ is a quasi-hyperideal of S. So $Q \circ Q \subseteq (Q \circ Q) \circ S \circ (Q \circ Q) = Q \circ (Q \circ S \circ Q) \circ Q \subseteq Q \circ Q \circ Q$. Therefore $Q \circ Q = Q \circ Q \circ Q$.

(iii) Let B be a bi-hyperideal of S. Let $R = \langle B \rangle_r$ be a right hyperideal of S and $L = \langle B \rangle_l$ be a left hyperideal of S, Theorem 4.3, implies that $\langle B \rangle_r \cap \langle B \rangle_l = \langle B \rangle_r \circ \langle B \rangle_l \subseteq B + B \circ S \circ B \subseteq B$. So B is a quasi-hyperideal of S.

□

Corollary 5.17. *Let $(S, +, \circ)$ be a regular multiplicative hypersemiring, then $R \cap L$ is bi-hyperideal of S, for any right hyperideal R and left hyperideal L of S.*

Proof. It follows from Theorem 4.3 and Proposition 5.14. □

Proposition 5.18. *Let B be a subhypersemiring of multiplicative hypersemiring S. If R is a right hyperideal and L is a left hyperideal of S such that $R \circ L \subseteq B \subseteq R \cap L$, then B is a bi-hyperideal of S.*

Proof. It follows from $B \circ S \circ B \subseteq (R \cap L) \circ S \circ (R \cap L) \subseteq (R \circ S) \circ L \subseteq R \circ L \subseteq B$. □

Theorem 5.19. *Let S be a multiplicative hypersemiring. Then the following are equivalent*

- **(i):** *Each right hyperideal R and each left hyperideal L of S satisfies $R \circ L = L \cap R \subseteq L \circ R$.*
- **(ii):** *The set $\mathcal{Q}$ of all quasi-hyperideal of S is an idempotent semigroup with respect to hyperoperation '$\circ$'.*
- **(iii):** *Each quasi-hyperideal Q of S satisfies $Q = Q \circ Q$.*

Proof. $(i) \Rightarrow (ii)$ The set $\mathcal{Q}$ is a regular semigroup with respect multiplicative hyperoperation '$\circ$' by Theorem 4.3. Now it remains to show that $\mathcal{Q}$ is idempotent. By Theorem 4.3, we have $Q = Q \circ S \circ Q$ and $S \circ S = S$ combining this two we get $Q = Q \circ S \circ Q = Q \circ S \circ Q \circ S \circ Q \circ S \circ Q = (Q \circ S \circ Q) \circ (S \circ S) \circ (Q \circ S \circ Q) = (Q \circ S) \circ ((Q \circ S) \circ (S \circ Q)) \circ (S \circ Q) \subseteq (Q \circ S) \circ ((S \circ Q) \circ (Q \circ S)) \circ (S \circ Q) = (Q \circ S \circ Q) \circ (Q \circ S \circ Q) = Q \circ Q$ by (i). Obviously $Q \circ Q \subseteq Q$. Hence $Q \circ Q = Q$.

$(ii) \Rightarrow (iii)$ It is straightforward.

$(iii) \Rightarrow (i)$ Let R be a right hyperideal and L be a left hyperideal of S then $R \circ L \subseteq R \cap L = L \cap R$ and the intersection $L \cap R$ is quasi-hyperideal of S. By (iii), $L \cap R = (L \cap R) \circ (L \cap R) \subseteq R \circ L$ and $L \cap R = (L \cap R) \circ (L \cap R) \subseteq L \circ R$. Hence $R \circ L = L \cap R \subseteq L \circ R$. $\square$

The following theorem gives a characterization of regular multiplicative hypersemiring S in terms of quasi-hyperideal and bi-hyperideal of S.

Theorem 5.20. *Let $(S, +, \circ)$ be a multiplicative hypersemiring, then the following conditions are equivalent:*

- **(i):** *S is regular;*
- **(ii):** *every hyperideal I of R is an idempotent;*
- **(iii):** *for every bi-hyperideal B of S, $B \circ S \circ B = B$;*
- **(iv):** *for every quasi-hyperideal Q of S, $Q \circ S \circ Q = Q$.*

Proof. $(i) \Leftrightarrow (ii)$ Let S be a regular multiplicative hypersemiring and I be any hyperideal of S. Obviously $I \circ I \subseteq I$. Let $x \in I \subseteq S$. Then $x \in x \circ S \circ x \subseteq (x \circ S) \circ x \subseteq I \circ I$. Thus $I = I \circ I$.

For the converse, let every hyperideal I of S is idempotent. Let R be a right hyperideal and L be a left hyperideal of S. Obviously $R \circ L \subseteq R \cap L$. Now we have $R \cap L = (R \cap L) \circ (R \cap L) \subseteq R \circ L$. Therefore $R \cap L = R \circ L$. By Theorem 4.3, S is regular.

$(i) \Rightarrow (iii)$ Suppose S is regular. Let B be a bi-hyperideal of S. Then $B \circ S \circ B \subseteq B \quad (1)$. Let $b \in B \subseteq S$. Then $b \in b \circ S \circ b \subseteq B \circ S \circ B \quad (2)$. From (1) and (2), $B = B \circ S \circ B$.

$(iii) \Rightarrow (iv)$ it follows from the Lemma 5.5.

$(iv) \Rightarrow (i)$ Suppose the conditions hold. Let R be a right hyperideal and L be a left hyperideal of S. Then by Theorem 3.10, $Q = R \cap L$ is a quasi-hyperideal of S. By hypothesis $Q \circ S \circ Q = Q$. Now $R \cap L = Q = Q \circ S \circ Q = (R \cap L) \circ S \circ (R \cap L) \subseteq (R \circ S) \circ L \subseteq R \circ L$, since R is a right hyperideal of S. Hence $R \cap L = R \circ L$ and S is regular. □

Theorem 5.21. *A subhypersemiring B of a regular multiplicative hypersemiring S is a bi-hyperideal of S if and only if B is a quasi-hyperideal of S.*

Proof. Let B be a bi-hyperideal of regular multiplicative hypersemiring S. Then by Theorem 4.3, $R \cap L = R \circ L$ for any right hyperideal R and any left hyperideal L of S.

Now $B \circ S \cap S \circ B = (B \circ S) \circ (S \circ B) \subseteq B \circ S \circ B \subseteq B$. Consequently B is a quasi-hyperideal of S.

Converse follows from Lemma 5.5. □

Theorem 5.22. *If Q_1 is a multiplicative subhypersemiring and Q_2 is a bi-hyperideal of a regular multiplicative hypersemiring S, then $Q_1 \circ Q_2$ and $Q_2 \circ Q_1$ are quasi-hyperideals of S.*

Proof. The theorem follows from Lemma 5.10 and Theorem 5.21. □

Corollary 5.23. *For any two quasi-hyperideal Q_1, Q_2 of a regular multiplicative hypersemiring S, $Q_1 \circ Q_2$ is a quasi-hyperideal of S.*

Proof. Corollary follows from Corollary 5.12 and Theorem 5.22. □

Acknowledgments

I am thankful to Prof T. K Dutta, Department of Mathematics, Calcutta University for his constant guidance throughout the preparation of this paper. Also I am thankful to referees for their valuable suggestions towards the improvement of this paper.

References

[1] A. M. Ibrahim and P. A. Ejegwa, *A survey on the concept of multigroup*, Journal of the Nigerian Association of Mathematical Physics. **38** (2016), 1-8.

[2] B. Davvaz and V. L. Fotea, *Hyperring Theory and Applications*, International Academic Press. Palm Harbor, Florida, USA. (2007).

[3] C. Dönges, *On Quasi-ideals of semirings*, Int. J. Math and Math. Sci. **17(1)** (1994), 47-58.

[4] E. S. Sevim, B. A. Ersoy and B. Davvaz, *Primary hyperideals of multiplicative hyperrings*, Eurasian Bulletin of Mathematics, **1(1)** (2018).

[5] F. Marty, *Sur une generalization de la notion de group*, 8^{th} congress des Math. Scandenaves stockholm, 45-49, (1934).

[6] F. M. Sioson, *Ideal theory in ternary semigroups*, Math. Japonica. **10** (1965), 63-84.

[7] H. Bordbar and I. Cristea, *Height of prime hyperideals in Krasner hyperrings*, Filomat, **31(19)**, 6153-6163.
DOI:10.2298/FIL1719153B.

[8] H. Bordbar, I. Cristea and M. Novak, *Height of Hyperideals in Noetherian Krasner Hyperrings*, University Politehnica of Bucharest Scientific Bulletin-Series A-Applied Mathematics and Physics, **79(2)** (2017), 31-42.

[9] K. Hila, B. Davvaz and K. Naka, *On quasi-hyperideals in semihypergroups*, Communication in Algebra. **39** (2011), 4183-4194.
DOI: 10.1080/00927872.2010.521932.

[10] M. Dresher and O. Ore, *Theory of multigroup*, Amer. J. of Mathematics, **60(3)** (1938), 705-733.

[11] M. Hamidi, R. Ameri and A. A. Tavakoli, *Boolean Rings Based on Multirings*, J. Sci .I. R. I., **32(2)** (2011), 159168.

[12] M. Hamidi, R. Ameri and A. A. Tavakoli, *Valued-potent (general) multirings*, J. alg. sys, **10(1)** (2022), 49-68.

[13] M. Krasner, *A class of hyperrings and hyperfields*, International J. Math and Math Sci. **6(2)** (1983), 307-312.

[14] M. Marshall, *Real reduced multirings and multifields*, J. Pure Appl. Algebra. **205** (2006), 452-468.

[15] Md Salim, T. Chanda and T. K. Dutta, *Regular equivalence and strongly regular equivalence on multiplicative ternary hyperring*, Journal of Hyperstructures. **4(1)** (2015), 20-36.

[16] N. Tamang and M. Mandal, *Hyperideals of a Ternary Hypersemiring*, Bull. Cal. Math. Soc., **110(5)** (2018), 385-398.

[17] O. Steinfeld, *Uber die Quasiideale Halbgruppen*, Publ. Math. Debrecen. **4** (1956), 262-275.

[18] P. Corsini, *Hypergraphs and hypergroups*, Algebra Universalis, **35**(1996), 548-555.

[19] P. Corsini, *Hypergroups réguliers et hypermodules*, Ann. Univ. Ferrara-Sez., VII-Sc. Mat, **20** (1975), 121-235.

[20] P. Corsini, *Sur les semihypergroupes*, Atti Soc. Pelor., Sc. Mat. Fis. Nat, **XXVI**, (1980).

[21] P. Corsini and V. L. Fotea, *Applications of Hyperstructure Theory*, Advances in Mathematics, Kluwer Academic, Dodrecht, The Netherlands, (2003).

[22] R. Ameri, A. Kordi and S. Sarka-Mayerova, *Multiplicative hyperring of fractions and coprime hyperideals*, An. Stiint. Univ. Ovidius Constanta Ser. Mat, **25(1)**(2017), 5-23.
DOI: 10.1515/auom-2017-0001.

[23] R. Procesi Ciampi and R. Rota, *The hyperring spectrum*, Rivis. Mat Pura ed Appl. **1** (1987) 71-80.
[24] R. Rota, *Strongly distribute multiplicative hyperrings*, Journal of Geometry. **39(1)** (1990).
[25] U. Dasgupta, *On certain classes of hypersemirings*, PhD. Thesis., University of Calcutta, (2011).
[26] V. N. Dixit and S. Dewan, *A note on quasi and bi-ideals in ternary semigroups*, Int. J. Math. Math. Sci., **18(3)** (1995), 501-508.

Md Salim Masud Molla
Department of Mathematics, Darjeeling Government College, P.O.Box 734101,
Darjeeling, India
Email: smpmath746@yahoo.in