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A COLLOCATION METHOD TO THE SOLUTION OF NONLINEAR FREDHOLM-HAMMERSTEIN INTEGRAL AND INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. This paper presents a computational technique for the solution of the nonlinear Fredholm-Hammerstein integral and integro-differential equations. A hybrid of block-pulse functions and the second kind Chebyshev polynomials (hereafter called as **HBC**) is used to approximate the nonlinear Fredholm-Hammerstein integral and integro-differential equations. The main properties of HBC are presented. Also, the operational matrix of integration together with the Newton-Cotes nodes are applied to reduce the computation of the nonlinear Fredholm-Hammerstein integral and integro-differential equations into some algebraic equations. The efficiency and accuracy of the proposed method have been shown by three numerical examples.

Key Words: Nonlinear Fredholm-Hammerstein integral equation; Block-pulse function; Second kind Chebyshev polynomial; Collocation method.

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1. Introduction

Integral equations play an important role in many branches of linear and nonlinear functional analysis and their applications in elasticity, engineering, mathematical physics and contact mixed problems [2, 13, 17, 23]. Several methods for solving such equations are available. Typical examples include spectral [18, 21] and transform methods

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[3, 25] as well as other methods [9, 14, 19, 26]. Many methods such as variational iteration [27], homotopy perturbation [8, 22], domain decomposition [5, 12], RBF [10, 11], wavelets [16, 29, 31] and others [1] have been developed and improved to obtain numerical solutions.

In the present paper, we consider application of HBC to obtain the numerical solution of the nonlinear Fredholm-Hammerstein integral equation:

$$y(t) = x(t) + \lambda \int_0^1 k(t,s)F(y(s))ds, \quad 0 \le t < 1 \quad , \tag{1.1}$$

and the nonlinear Fredholm-Hammerstein integro-differential equation:

$$\begin{cases} y'(t) = x(t) + \lambda \int_0^1 k(t,s)F(y(s))ds \\ \\ y(0) = y_0, \qquad 0 \le t < 1, \end{cases} \tag{1.2}$$

where λ and y_0 are constants and $x(t)$ and $k(t,s)$ are assumed to be in $L_2[0,1)$ and $L_2([0,1) \times [0,1))$, respectively. We assume that Eqs. (1.1) and (1.2) have a unique solution $y(t)$ as a linear combination of the HBC with some unknown coefficients [15]. The properties of HBC together with the Newton-Cotes nodes [24] are then applied to evaluate the unknown coefficients and find an approximate solution to Eqs. (1.1) and (1.2).

This paper is organized as follows: In Section 2, we describe the basic formulation of HBC required for our subsequent development. Sections 3 and 4 are devoted to the solution of Eqs. (1.1-1.2) by using HBC. In Section 5, we report our numerical results and demonstrate the accuracy of the proposed scheme by considering the numerical examples.

2. Properties of HBC

2.1. **HBC Definition.** A set of block-pulse functions $\phi_n(t), n = 1, 2, \ldots, N$, on interval $[0,1)$ is defined as (see [21]):

$$\phi_n(t) = \begin{cases} 1, & \frac{n-1}{N} \le t < \frac{n}{N} \\ 0, & otherwise \end{cases} \quad , \tag{2.1}$$

where N is an arbitrary positive integer.

The mth second kind Chebyshev polynomial in interval $[-1,1]$ is defined as follows (see [25]):

$$S_m(t) = \frac{\sin((m+1)\arccos(t))}{\sqrt{1-t^2}} \quad . \tag{2.2}$$

Also, we can, obtain the second kind Chebyshev polynomials by the following recursive formulas:

$$\begin{cases} S_0(t) = 1, \quad S_1(t) = 2t, \\ S_{m+1}(t) = 2tS_m(t) - S_{m-1}(t), \quad m = 1, 2, \dots \end{cases} . \tag{2.3}$$

Further, HBC $H_{nm}(t), n = 1, 2, \dots, N, m = 0, 1, \dots, M-1$, on the interval $[0, 1)$ are defined as:

$$H_{nm}(t) = \phi_n(t) S_m(2Nt - 2n + 1). \tag{2.4}$$

2.2. **Function approximation.** A function $f(t) \in L_2[0, 1)$ may be expanded as

$$f(t) \approx \sum_{n=1}^{N} \sum_{m=0}^{M-1} f_{nm} H_{nm}(t), \tag{2.5}$$

where

$$f_{nm} = \frac{2}{\pi} \int_{-1}^{1} f(\frac{t + 2n - 1}{2N}) S_m(t) \sqrt{1 - t^2} dt. \tag{2.6}$$

We can rewrite $f(t)$ as:

$$f(t) \approx \sum_{n=1}^{N} F_n H_k(t) = FH(t), \tag{2.7}$$

where

$$F_n = [f_{n0}, \dots, f_{n(M-1)}], \quad F = [F_1, \dots, F_N], \tag{2.8}$$

and

$$H_n(t) = [H_{n0}(t), \dots, H_{n(M-1)}(t)]^T, \quad H(t) = [H_1^T(t), \dots, H_N^T(t)]^T. \tag{2.9}$$

2.3. **The operational matrix of integration.** The integral part in Eq. (2.9) is given by:

$$\int_0^t H(\tau) d\tau \approx PH(t), \tag{2.10}$$

where

$$P = \frac{1}{2N} I \otimes \widehat{P} + \sum_{n=1}^{N-1} \sum_{i=1}^{N-n} \frac{1}{N} E^{(N)}_{i(i+n)} \otimes \sum_{n=1}^{[\frac{M+1}{2}]} \frac{1}{2n-1} E^{(M)}_{(2n-1)1}, \tag{2.11}$$

$$\widehat{P} = E_{11}^{(M)} - \frac{3}{4}E_{21}^{(M)} + \frac{1}{2}\left(\sum_{n=1}^{M-1}\frac{1}{n}E_{n(n+1)}^{(M)} - \sum_{n=2}^{M-1}\frac{1}{n+1}E_{(n+1)n}^{(M)}\right) + \sum_{n=3}^{M}\frac{(-1)^{n-1}}{n}E_{n1}^{(M)} \ . \tag{2.12}$$

Where $E_{ij}^{(m)}$ is the $m \times m$ matrix with entry 1 at (i, j) and zeros elsewhere and $\left[\frac{M+1}{2}\right]$ is the greatest integer part of $\frac{M+1}{2}$.

For more details, see [30].

In Eq. (2.11), notation $\otimes$ denotes Kronecker product which defined as:

$$A_{n\times p} \otimes B_{m\times q} = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1p}B \\ a_{21}B & a_{22}B & \dots & a_{2p}B \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1}B & a_{n2}B & \dots & a_{np}B \end{bmatrix}_{nm\times pq} .$$

2.4. **Expression of producing functions.** Let $g(t), f(t) \in L_2[0, 1)$, then the expressions of $g(t)$ and $f(t)$ are defined as:

$$g(t) \approx \sum_{n=1}^{N}\sum_{m=0}^{M-1} g_{nm}H_{nm}(t), \quad f(t) \approx \sum_{n=1}^{N}\sum_{m=0}^{M-1} f_{nm}H_{nm}(t). \tag{2.13}$$

Then

$$g(t)f(t) \approx \sum_{n=1}^{N}\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} g_{ni}f_{nj}H_{ni}(t)H_{nj}(t). \tag{2.14}$$

From

$$H_{ni}(t)H_{nj}(t) \approx \sum_{m=0}^{M-1} d_{nm}^{(i,j)}H_{nm}(t), \tag{2.15}$$

where

$$d_{nm}^{(i,j)} = \frac{2}{\pi}\int_{-1}^{1} S_i(t)S_j(t)S_m(t)\sqrt{1-t^2}dt, \tag{2.16}$$

we have:

$$g(t)f(t) \approx \sum_{n=1}^{N}\sum_{m=0}^{M-1}\left(\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} d_{nm}^{(i,j)}g_{ni}f_{nj}\right)H_{nm}(t) = \sum_{n=1}^{N}\sum_{m=0}^{M-1}\tilde{g}_{nm}H_{nm}(t), \tag{2.17}$$

whereas

(2.18)
$$\tilde{g}_{nm} = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} d_{nm}^{(i,j)} g_{ni} f_{nj} = \hat{g}_{nm} F_n^T, \quad F_n = [f_{n0}, \ldots, f_{n(M-1)}],$$

and

$$\hat{g}_{nm} = \left[\sum_{i=0}^{M-1} d_{nm}^{(i,0)} g_{ni}, \ldots, \sum_{i=0}^{M-1} d_{nm}^{(i,M-1)} g_{ni}\right]. \tag{2.19}$$

Therefore, using Eq. (2.5), it may be assumed that:

$$k(t,s)w(s) \approx \sum_{n=1}^{N}\sum_{m=0}^{M-1} \widetilde{k}_{nm}(t) H_{nm}(s), \tag{2.20}$$

where

$$w(s) \approx W H(s), \tag{2.21}$$

$W = [W_1, \ldots, W_N]$ and $H(s)$ are given in Eq. (2.8) and Eq. (2.9), respectively, and

$$\widetilde{k}_{nm}(t) = \widehat{k}_{nm}(t) W_n^T, \tag{2.22}$$

$$\widehat{k}_{nm}(t) = \left[\sum_{i=0}^{M-1} d_{nm}^{(i,0)} k_{ni}(t), \ldots, \sum_{i=0}^{M-1} d_{nm}^{(i,M-1)} k_{ni}(t)\right], \tag{2.23}$$

$$k_{ni}(t) = \frac{2}{\pi}\int_{-1}^{1} k(t, \frac{s+2n-1}{2N}) S_i(s)\sqrt{1-s^2}\,ds. \tag{2.24}$$

Let

$$v(t) = \int_0^1 k(t,s)w(s)ds \approx \sum_{n=1}^{N}\sum_{m=0}^{M-1} v_{nm} H_{nm}(t), \tag{2.25}$$

$$\widehat{k}_{nm}(t) \approx \sum_{j=1}^{N}\sum_{l=0}^{M-1} \widehat{k}_{nm}^{(j,l)} H_{jl}(t), \tag{2.26}$$

where

$$\widehat{k}_{nm}^{(j,l)} = \left[\sum_{i=0}^{M-1} d_{nm}^{(i,0)} k_{ni}^{(j,l)}, \ldots, \sum_{i=0}^{M-1} d_{nm}^{(i,M-1)} k_{ni}^{(j,l)}\right], \tag{2.27}$$

$$k_{ni}^{(j,l)} = \frac{2}{\pi}\int_{-1}^{1} k_{ni}(\frac{t+2j-1}{2N})S_l(t)\sqrt{1-t^2}dt. \tag{2.28}$$

By Eqs. (2.20) and (2.22) we have:

$$v(t) \approx \sum_{j=1}^{N}\sum_{l=0}^{M-1}\sum_{n=1}^{N}\sum_{m=0}^{M-1}\widehat{k}_{nm}^{(j,l)}\left(\int_{\frac{n-1}{N}}^{\frac{n}{N}} H_{nm}(t)dt\right)W_n^T H_{jl}(t). \tag{2.29}$$

Hence

$$V = \sum_{n=1}^{\left[\frac{M+1}{2}\right]}\sum_{i=0}^{N}\sum_{j=1}^{N} E_{ij}^{(N)} \otimes \frac{1}{N(2n-1)}\widehat{k}_{j(2n-2)}^{(i)}W^T, \tag{2.30}$$

where

$$\widehat{k}_{j(2n-2)}^{(i)} = \left[\widehat{k}_{j(2n-2)}^{T(i,0)}, \ldots, \widehat{k}_{j(2n-2)}^{T(i,M-1)}\right]^T. \tag{2.31}$$

Therefore

$$v(t) = \int_0^1 k(t,s)w(s)ds \approx W\widehat{V}^T H(t), \tag{2.32}$$

where

$$\widehat{V} = \sum_{n=1}^{\left[\frac{M+1}{2}\right]}\sum_{i=0}^{N}\sum_{j=1}^{N} E_{ij}^{(N)} \otimes \frac{1}{N(2n-1)}\widehat{k}_{j(2n-2)}^{(i)}. \tag{2.33}$$

3. Nonlinear Fredholm-Hammerstein integral equation

In this section we consider the Fredholm-Hammerstein integral Eq. (1.1). To solve this equation, we first consider the approximation:

$$w(t) = F(y(t)), \qquad 0 \le t < 1, \tag{3.1}$$

and then substitute it in Eq. (1.1).

From Eq. (1.1) we get

$$w(t) = F\left(x(t) + \lambda\int_0^1 k(t,s)w(s)ds\right). \tag{3.2}$$

By some manipulations as before, we have:

$$WH(t) = F(x(t) + \lambda W\widehat{V}^T H(t)). \tag{3.3}$$

Now, using Newton-Cotes nodes as

$$t_i = \frac{2i-1}{2NM}, \qquad i = 1, 2, \ldots, NM,$$

we have

$$WH(t_i) = F(x(t_i) + \lambda W\widehat{V}^T H(t_i)), \qquad i = 1, 2, \ldots, NM. \tag{3.4}$$

By solving this system of nonlinear equations, we find unknown vector W which has NM components. Substituting Eq. (3.1) into Eq. (1.1), yields:

$$y(t) = x(t) + \lambda \int_0^1 k(t, s)w(s)ds, \tag{3.5}$$

and applying Eq. (2.32) to Eq. (3.5),we get

$$y(t) = x(t) + \lambda W\widehat{V}^T H(t). \tag{3.6}$$

In other words, we could find the unknown function $y(t)$.

4. Nonlinear Fredholm-Hammerstein integro-differential equation

Here, the Fredholm-Hammerstein integro-differential Eq. (1.2) is considered.

In order to solve this equation, we suppose that:

$$w(t) = F(y(t)), \qquad 0 \leq t < 1, \tag{4.1}$$

when Eq. (4.1) implies that:

$$y(t) = y(0) + \int_0^t y'(\tau)d\tau. \tag{4.2}$$

We approximate $x(\tau)$ by Eq. (2.7) as follows:

$$x(\tau) = XH(\tau), \tag{4.3}$$

where X and $H(\tau)$ are given in Eq. (2.8) and Eq. (2.9), respectively. Consequently, we have:

$$\begin{aligned} y(t) &= y(0) + \int_0^t x(\tau)d\tau + \lambda \int_0^t \int_0^1 k(\tau, s)w(s)dsd\tau \\ &\approx y(0) + XPH(t) + \lambda W\widehat{V}^T PH(t), \end{aligned} \tag{4.4}$$

and

$$WH(t) = F(y(0) + XPH(t) + \lambda W\widehat{V}^T PH(t)). \tag{4.5}$$

In order to construct the appropriate approximations for $w(t)$, we collocate Eq. (4.5) in NM points. Now, using Newton-Cotes nodes, we have:

(4.6)
$$WH(t_i) = F(y(0) + XPH(t_i) + \lambda W\widehat{V}^T PH(t_i)), \quad i = 1, 2, \ldots, NM.$$

Finally, W is obtained by solving Eq. (4.6).

5. Numerical examples

To demonstrate the efficiency of our scheme, some numerical examples are presented in this section. The computations associated with the examples were performed by Matlab software on a personal computer.

Example 1. Consider the following nonlinear Fredholm integral equation [20]:

$$y(t) = \cos(t) - 0.4958t + \int_0^1 ts\tan(y(s))ds, \quad 0 \le t \le 1.$$

Taking $N = 1, M = 2$, we obtain the following approximate solution:

$$y_{N,M}(t) = y_{1,2}(t) \approx \cos(t) - 0.4958t + .278886658H_{10}(t) + 0.13944333H_{11}(t),$$

and for $N = 2, M = 4$, we obtain the approximate solution as:

$$y_{N,M}(t) = y_{2,4}(t) \approx \cos(t) - 0.4958t + 0.12394656H_{10}(t) + 0.06197328H_{11}(t)$$

$$+0.037183967H_{20}(t) + 0.06197328H_{21}(t).$$

Taking $N = 1, M = 2$ and $N = 2, M = 4$ the solutions are compared with the exact solution $y(t) = \cos(t)$ as presented in Table 1 and depicted graphically in Figure 1.

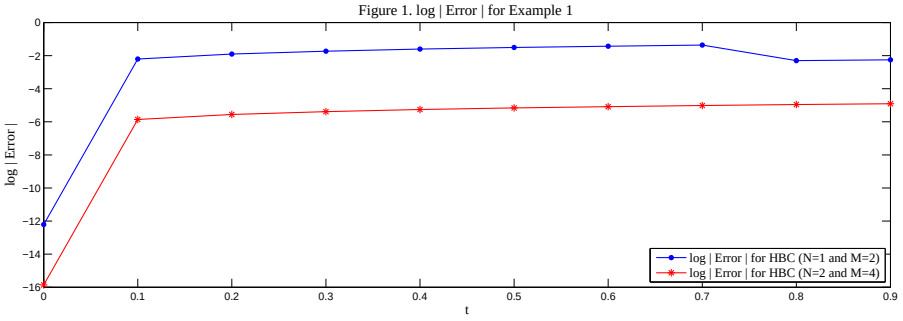
The $2-$norm errors for the proposed method are in agreement with those of the methods of Babolian *et al.* [6] and of Maleknejad *et al.* [20] as seen in Table 2.

Table 1: Numerical results of Example 1 with HBC

Nodes t	Approximate HBC for $N = 1$ and $M = 2$	Error HBC for $N = 1$ and $M = 2$	Approximate HBC for $N = 2$ and $M = 4$	Error HBC for $N = 2$ and $M = 4$
t = 0.0	1.00000000e-0	0.00000000e-0	1.00000000e-0	0.00000000e-0
t = 0.1	1.00120150e-0	6.19733165e-3	9.95002788e-1	1.37700726e-6
t = 0.2	9.92461241e-1	1.23946633e-2	9.80063824e-1	2.75401453e-6
t = 0.3	9.73928484e-1	1.85919950e-2	9.55332358e-1	4.13102179e-6
t = 0.4	9.45850321e-1	2.47893266e-2	9.21055486e-1	5.50802905e-6
t = 0.5	9.08569220e-1	3.09866583e-2	8.77575677e-1	6.88503632e-6
t = 0.6	8.62519605e-1	3.71839899e-2	8.25327353e-1	8.26204358e-6
t = 0.7	8.08223509e-1	4.33813216e-2	7.64832548e-1	9.63905084e-6
t = 0.8	7.46285363e-1	4.95786532e-3	6.96695693e-1	1.10160581e-5
t = 0.9	6.77385953e-1	5.57759849e-3	6.21597575e-1	1.23930654e-5

Table 2: Approximate norm-2 of absolute error for Example 1

Methods	$\|y(t) - y^*(t)\|$
Method of [6]	
m = 2	2.27187e-1
m = 3	1.34023e-1
m = 4	1.07511e-1
Method of [20]	
$j_u = 3$	3.54072e-5
$j_u = 4$	5.61248e-6
$j_u = 5$	7.83725e-7
Present method	
N = 1 , M = 2	7.11353e-2
N = 2 , M = 4	2.31441e-5
N = 3 , M = 4	6.84629e-8



Example 2. Consider the following nonlinear Fredholm integro-differential equation [4]:

$$y'(t) = 1 - \frac{1}{4}t + \int_0^1 tsy^2(s)ds, \quad y(0) = 0.$$

Taking $N = 1, M = 2$, we obtain the following approximate solution:

$$y_{N,M}(t) = y_{1,2}(t) \approx 0.494193699H_{10}(t) + 0.245354959H_{11}(t),$$

and for $N = 2, M = 4$, we obtain the approximate solution as:

$$y_{N,M}(t) = y_{2,4}(t) \approx 0.25H_{10}(t) + 0.125H_{11}(t) - 0.888595270e^{-31}H_{12}(t)$$
$$+0.75H_{20}(t) + 0.125H_{21}(t) - 0.888595270e^{-31}H_{22}(t).$$

The obtained solutions for $N = 1$, $M = 2$ and for $N = 2$, $M = 4$ are comparable with the exact solution $y(t) = t$ as have shown in Table 3 and Figure 2.

The 2−norm errors of the proposed method are in a good agreement with those of the methods of Babolian *et al.* [6] and of Saeedi *et al.* [28] as seen in Table 4.

Table 3: Numerical results of Example 2 with HBC

Nodes t	Approximate HBC for $N = 1$ and $M = 2$	Error HBC for $N = 1$ and $M = 2$	Error HBC for $N = 2$ and $M = 4$
t = 0.0	3.48378083e-3	3.48378083e-3	0.00000000e-00
t = 0.1	1.01625764e-1	1.62576438e-3	5.68700973e-32
t = 0.2	1.99767748e-1	2.32252056e-4	2.27480389e-31
t = 0.3	2.97909731e-1	2.09026850e-3	5.11830876e-31
t = 0.4	3.96051715e-1	3.94828495e-3	9.09921557e-31
t = 0.5	4.94193699e-1	5.80630139e-3	1.42175243e-30
t = 0.6	5.92335682e-1	7.66431784e-3	2.04732350e-30
t = 0.7	6.90477666e-1	9.52233428e-3	2.78663477e-30
t = 0.8	7.88619649e-1	1.13803507e-2	3.63968623e-30
t = 0.9	8.86761633e-1	1.32383672e-2	4.60647788e-30

Figure 2. log | Error | for Example 2

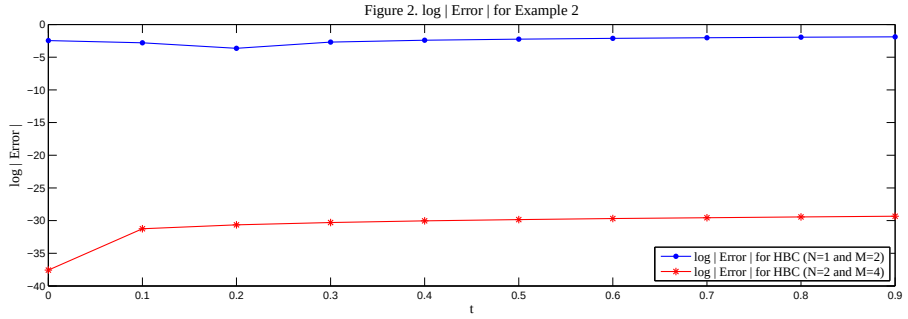


Table 4: Approximate norm-2 of absolute error for Example 2

Methods	$\|y(t) - y^*(t)\|$
Method of [6]	
m = 2	4.7434e-01
m = 3	3.0732e-01
m = 4	2.3717e-01
Method of [28]	
K = 2 , M = 1	2.7133e-03
K = 3 , M = 1	6.8179e-04
K = 4 , M = 1	1.6745e-05
Present method	
N = 1 , M = 2	2.2862e-02
N = 2 , M = 4	7.0420e-30
N = 3 , M = 4	3.4729e-38

Example 3. Consider the nonlinear Fredholm integro-differential equation [7]:

$$y'(t) = 2t + \frac{1}{8}(-\pi + \log(4)) + \int_0^1 s \arctan(y(s))ds, \quad y(0) = 0.$$

Taking $N = 1, M = 2$, we obtain the following approximate solution:

$$y_{N,M}(t) = y_{1,2}(t) \approx 0.299583352H_{10}(t) + 0.243516761H_{11}(t),$$

and for $N = 2, M = 4$, we obtain the following approximate solution:

$$y_{N,M}(t) = y_{2,4}(t) \approx 0.078131491H_{10}(t)+0.062503245H_{11}(t)+0.015625H_{12}(t)$$

$$+0.578144472H_{20}(t) + 0.187503245H_{21}(t) - 0.015625H_{22}(t).$$

The exact solution of this example is $y(t) = t^2$. As we can see from Table 5 errors for $N = 1$, $M = 2$ and $N = 2$, $M = 4$ are acceptable. Also, Figure 3 has a good agreement with the exact solution.
The $2-$norm errors of the proposed method are in a good agreement with those of the methods of Babolian *et al.* [6] and of Berenguer *et al.* [7] as seen in Table 6.

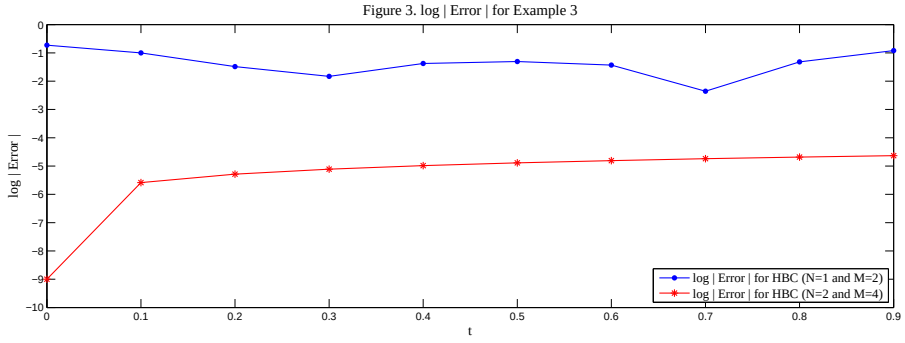
Table 5: Numerical results of Example 3 with HBC

Nodes t	Approximate HBC for $N = 1$ and $M = 2$	Error HBC for $N = 1$ and $M = 2$	Approximate HBC for $N = 2$ and $M = 4$	Error HBC for $N = 2$ and $M = 4$
t = 0.0	1.87500000e-1	1.87500000e-1	0.00000000e-0	0.00000000e-0
t = 0.1	9.00833296e-2	1.00083330e-1	1.00025963e-2	2.59632768e-6
t = 0.2	7.33334086e-2	3.26666591e-2	4.00051927e-2	5.19265536e-6
t = 0.3	1.04750011e-1	1.47500113e-2	9.00077890e-2	7.78898304e-6
t = 0.4	2.02121667e-1	4.21666817e-2	1.60010385e-1	1.03853107e-5
t = 0.5	2.99583352e-1	4.95833521e-2	2.50012982e-1	1.29816384e-5
t = 0.6	3.97000023e-1	3.70000226e-2	3.60015578e-1	1.55779661e-5
t = 0.7	4.94416693e-1	4.41669301e-3	4.90018174e-1	1.81742938e-5
t = 0.8	5.91833363e-1	4.81666366e-2	6.40020771e-1	2.07706214e-5
t = 0.9	6.89250034e-1	1.20749966e-1	8.10023367e-1	2.33669491e-5

Table 6: Approximate norm-2 of absolute error for Example 3

Methods	$\|y(t) - y^*(t)\|$
Method of [6]	
m = 2	3.40827-1
m = 3	2.43755e-1
Method of [7]	
j = 9 , h = 6	5.42945e-3
j = 17 , h = 6	1.53327e-3
Present method	
N = 1 , M = 2	2.62647e-1
N = 2 , M = 4	4.38311e-5

Figure 3. log | Error | for Example 3



6. Conclusion

In this paper, HBC method has successfully applied to compute the approximate solution of certain nonlinear Fredholm-Hammerstein integral and integro-differential equations. The accuracy and applicability of the method were investigated through some numerical examples. The numerical results showed that the accuracy of the obtained solutions was satisfactory. Furthermore, the current method can be used by increasing N and M until the results reach an appropriate accuracy.

This method can be easily extended and applied to a system of nonlinear Fredholm-Hammerstein integral and integro-differential equations.

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