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PRIME BI-INTERIOR IDEALS OF Γ-SEMIRINGS

MARAPUREDDY MURALI KRISHNA RAO

Abstract. In this paper, we introduce the notion of prime bi-interior ideal, bi-interior ideal,strongly prime bi-interior ideal,semiprime strongly irreducible bi-interior ideal and irreducible bi-interior ideal.we study these ideals properties and relation between them and also characterize regular Γ-semiring and Γ-semiring using prime bi-interior ideals.

Key Words: Γ-semiring, Bi-interior ideal, Prime ideals, Prime bi-interior ideal $\cdots$.
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1. Introduction

Many mathematicians proved important results and characterization of algebraic structures by using the concepts and the properties of generalization of ideals in algebraic structures. During 1950-1980, the concepts of bi-ideals, quasi ideals and interior ideals were studied by many mathematicians. The author introduced and studied weak interior ideals, tri-ideals, bi-interior ideals, bi-quasi ideals, quasi-interior ideals, bi-quasi interior ideals and tri- quasi ideals of $\Gamma-$semirings, $\Gamma-$semigroups, semigroups and semirings as a generalization of bi-ideal, quasi ideal and interior ideal of algebraic structures and charecterized regular algebraic structures as well as simple algebraic structures using these ideals. Semiring is the algebraic structure which is a common generalization of rings and distributive lattices, was first introduced by Vandiver[20] in 1934 but non-trivial examples of semirings had appeared in the studies on

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∗Address correspondence to M.M.K. Rao; E-mail: mmarapureddy@gmail.com.

the theory of commutative ideals of rings by Dedekind in 19th century. In 1995, M. Murali Krishna Rao[9, 10, 13] introduced the notion of $\Gamma-$semiring as a generalization of $\Gamma-$ ring, ternary semiring and semiring. As a generalization of ring, the notion of a $\Gamma-$ring was introduced by Nobusawa[18] in 1964. We know that the notion of a one sided ideal of any algebraic structure is a generalization of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals. In 1952, the concept of bi-ideals was introduced by Good and Hughes[2] for semigroups. The notion of bi-ideals in rings and semigroups were introduced by Lajos and Szasz[5, 6, **?**]. The concept of interior ideals was introduced by Lajos for semigroups. Steinfeld[19] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki[4] introduced the concept of quasi ideal for a semiring. In this paper,as a further generalization of ideals, M. Shabir[8] studied the prime bi-ideals of semigroups. In this paper, we introduced the notion of prime bi-interior ideals of $\Gamma-$semirings and extended the notion of prime bi-interior of semigroups to Γ-semirings.

2. PRELIMINARIES

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. A universal algebra $(S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws,
$i.e., a(b + c) = ab + ac,\ (a + b)c = ac + bc,$ for all $a, b, c \in S$.

Definition 2.2. A semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$.

Definition 2.3. A semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0x = x\ 0 = 0,$ for all $x \in M$.

Definition 2.4. An element $1 \in M$ is said to be unity if for each $x \in M$ such that $x\,1 = 1\,x = x$. In a semiring M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exists $b \in M$ such that $ba = 1(ab = 1)$.

Definition 2.5. An element $a \in M$ is said to be regular element of M if there exist $x \in M$ such that $a = axa$. If every element of semiring M is a regular, then M is said to be regular semiring.

Definition 2.6. An element $a \in M$ is said to be idempotent of M if $a = aa$. Every element of M is an idempotent of M then M is said to be idempotent semiring M.

Definition 2.7. Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. A Γ-semigroup M is said to be Γ-*semiring*, if it satisfies the following axioms: for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$.
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

A Γ-semiring M is said to be commutative Γ-semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$. Let M be a Γ-semiring. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$. An element $a \in M$ is said to be left invertible(right invertible) if there exists $b \in M, \alpha \in \Gamma$, such that $b\alpha a = 1(a\alpha b = 1)$. An element $a \in M$ is said to be invertible if there exist $b \in M, \alpha \in \Gamma$, such that $a\alpha b = b\alpha a = 1$.

Definition 2.8. A non-empty subset A of Γ-semiring M is called

(i) a Γ-subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $A\Gamma A \subseteq A$.
(ii) a quasi ideal of M if A is a Γ-subsemiring of M and $A\Gamma M \cap M\Gamma A \subseteq A$.
(iii) a bi-ideal of M if A is a Γ-subsemiring of M and $A\Gamma M\Gamma A \subseteq A$.
(iv) an interior ideal of M if A is a Γ-subsemiring of M and $M\Gamma A\Gamma M \subseteq A$.
(v) a left (right) ideal of M if A is a Γ-subsemiring of M and $M\Gamma A \subseteq A(A\Gamma M \subseteq A)$.
(vi) an ideal if A is a Γ-subsemiring of M, $A\Gamma M \subseteq A$ and $M\Gamma A \subseteq A$.
(vii) a $k-$ideal if A is a Γ-subsemiring of M, $A\Gamma M \subseteq A, M\Gamma A \subseteq A$ and $x \in M$, $x + y \in A, y \in A$ then $x \in A$.
(viii) a left(right) bi- quasi ideal of M if A is a Γ-subsemiring of M and $M\Gamma A \cap M\Gamma A\Gamma M(A\Gamma M \cap M\Gamma A\Gamma M) \subseteq A$.
(ix) a bi- quasi ideal of M if A is a left bi- quasi ideal and a right bi- quasi ideal of M

Definition 2.9. A semiring M is a left (right) simple Γ-semiring if M has no proper left (right) ideal of M A semiring M is a bi-quasi simple Γ-semiring if M has no proper bi-quasi ideal of M A semiring M is said to be simple Γ-semiring if M has no proper ideals.

Definition 2.10. A bi-interior ideal P of a Γ-semiring M is called a prime Bi-interior ideal if $x, y \in M$ and $x\Gamma M\Gamma y \subseteq P \Rightarrow x \in P$ or $y \in P$.

Definition 2.11. A non-empty subset B of M is said to be a bi-ideal of M if B is a sub Γ-semiring of M and $B\Gamma M\Gamma B \subseteq B$.

Example 2.12. Consider the semiring $S = M_{2*2}(N_0)$,where N denotes the set of all natural numbers and $N_0 = N \bigcup \{0\}$. if $\Gamma = S$, then S forms a Γ-semiring with $A\alpha B$=usual matrix product of A, α, B; for all $A, \alpha, B \in S$.

1 $C = \left\{ \begin{pmatrix} a & x \\ 0 & y \end{pmatrix} x, y \in N_0 \right\}$ is a bi-ideal of S.

2 $D = \left\{ \begin{pmatrix} a & 0 \\ 0 & x \end{pmatrix} x \in N_0 \right\}$ is a bi-ideal of S.

Definition 2.13. A bi-ideal B of M is called a prime bi-ideal if $B_1\Gamma B_2 \subseteq B$ implies $B_1 \subseteq B$ or $B_2 \subseteq B$, for any bi-ideals B_1 and B_2 of M.

Definition 2.14. A bi-ideal B of M is called a strongly prime bi-ideal if $(B_1\Gamma B_2) \bigcap (B_2\Gamma B_1) \subseteq B$ implies $B_1 \subseteq B$ or $B_2 \subseteq B$, for any bi-ideals B_1 *and* B_2 of M.

Definition 2.15. A bi-ideal B of M is called a semiprime bi-ideal if for any bi-ideal $(B_1 of M, B_1^2 = B_1\Gamma B_1) \subseteq B$ implies $B_1 \subseteq B$.

Obviously every strongly prime bi-ideal in M is a prime bi-ideal and every prime bi-ideal in M is a semiprime bi-ideal.

Definition 2.16. A bi-ideal B of M is called an irreducible bi-ideal if $(B_1 \bigcap B_2 = B)$ implies $B_1 = B$, or $B_2 = B$, for any bi-ideals B_1 and B_2 of M.

Definition 2.17. A bi-ideal B of M is called a strongly irreducible bi-ideal if for any bi-ideals B_1 *and* B_2 of M. $B_1 \bigcap B_2 \subseteq B$ implies $B_1 \subseteq B$ or $B_2 \subseteq B$.

Obviously every strongly irreducible bi-ideal is an irreducible bi-ideal.

3. PRIME BI-INTERIOR IDEALS OF Γ-SEMIRINGS

In this section, we introduction the notion of prime, strongly prime, semi prime, irreducible and strongyle irreducible bi-interior ideals of Γ-semirings. And we study the properties of prime ideals and relations between them.

Definition 3.1. Let B be a bi-interior ideal of a Γ-semiring M, and B_1 and B_2 be bi-interior ideals of a Γ-semiring M.

(i) If $B_1\Gamma B_2 \subseteq B \Rightarrow B_1 \subseteq B$ or $B_2 \subseteq B$, for any bi-interior ideals B_1, B_2 then B is prime bi-interior ideal .
(ii) If $(B_1\Gamma B_2) \cap (B_2\Gamma B_1) \subseteq B \Rightarrow B_1 \subseteq B$ or $B_2 \subseteq B$ then B is a strongly prime bi-interior ideal.

Definition 3.2. A bi-interior ideal B of a Γ-semiring M is called a semi prime bi-interior ideal if $B_1\Gamma B_1 \subseteq B \Longrightarrow B_1 \subseteq B$, for any bi-interior ideal B_1 of M.

NOTE:
(i). Every strongly prime bi-interior ideal of a Γ-semiring$M$ is a prime bi-interior ideal of M.

(ii). Every prime bi-interior ideal B of a Γ-semiring M is a semi prime bi-interior ideal of M.

Definition 3.3. A one sided ideal P of a Γ-semiring M is called a prime ideal if $x, y \in M, x\Gamma M\Gamma y \subseteq P \Rightarrow x \in P$ or $y \in P$.

Definition 3.4. A bi-interior ideal P of a Γ-semiring M is called a prime Bi-inferior ideals if $x, y \in M$ and $x\Gamma M\Gamma y \subseteq P \Rightarrow x \in P$ or $y \in P$.

Theorem 3.5. *A bi-interior ideal B of a Γ-semiring M is prime bi-interior ideal if and only if $R\Gamma L \subseteq B \Longrightarrow R \subseteq B$ or $L \subseteq B$ where R is a right ideal and L is a left ideal of M.*

Proof. Suppose that a prime bi-interior ideal B of the Γ-semiring M and $R\Gamma L \subseteq B$.

Since R and L are bi-interior ideals $R \subseteq B$ or $L \subseteq B$.

Conversely suppose that $R\Gamma L \subseteq B$ where R is a right ideal and L is a left ideal of M. $\Longrightarrow R \subseteq B$ or $L \subseteq B$.

Suppose $A\Gamma C \subseteq B$, A and C are bi-interior ideals and $a \in A$ and $c \in B$. Then

$$(a)_r\Gamma(c)_l \subseteq A\Gamma C \subseteq B$$
$$\Rightarrow (a)_r \subseteq B \text{ or } (c)_l \subseteq B$$

Then $a \in B$ or $c \in B$. Therefore $A \subseteq B$ or $C \subseteq B$.
Hence a bi-interior ideal B is a prime bi-interior ideal of the Γ-semiring M. □

Theorem 3.6. *A prime bi-interior ideal P of a Γ-semiring M is a prime one sided ideal of M.*

Proof. Suppose A prime bi-interior ideal P of the Γ-semiring M is not a prime one sided ideal of M. Then

$$P\Gamma M \nsubseteq P \text{ and } M\Gamma P \nsubseteq P$$
$$\Rightarrow P\Gamma M\Gamma M\Gamma P \nsubseteq P.$$
$$\text{We have } P\Gamma M\Gamma M\Gamma P \subseteq P$$
$$\Rightarrow P\Gamma M\Gamma M\Gamma P \subseteq P\Gamma M\Gamma M\Gamma M\Gamma P \nsubseteq P$$
$$P\Gamma M\Gamma P\Gamma M\Gamma P \nsubseteq P.$$

Which is a contradiction.

Hence $P\Gamma M \subseteq P$ or $M\Gamma P \subseteq P$, that is, P is a prime one sided ideal of M. □

Lemma 3.7. *A bi-interior ideal of P of M is a prime if and only if I is a right ideal of M and J is a left ideal of M, $I\Gamma J \subseteq P \Rightarrow I \subseteq P$ or $J \subseteq P$.*

Definition 3.8. A bi-interior ideal B of M is called an irreducible bi-interior ideal B if bi-interior ideals B_1, B_2
and $B_1 \cap B_2 \subseteq B \Longrightarrow B_1 \subseteq B or B_2 \subseteq B$.

Theorem 3.9. *If B_1 and B_2 be bi-interior ideal of a Γ-semiring M for any bi-interior ideal $B_1, B_2 of M$ such that $B_1 \cap B_2 \subseteq B$ then B is strongly irreducible ideal of M.*

Proof. Let B_1 and B_2 be bi-interior ideals of M such that $B_1 \cap B_2 \subseteq B$.
Then $(B_1\Gamma B_2) \cap (B_2\Gamma B_1) = B$.

$$(B_1\Gamma B_2) \cap (B_2\Gamma B_1) = B_1 \cap B_2 \subseteq B)$$
$$\Longrightarrow B_1 \subseteq B \text{ or } B_2 \subseteq B.$$

Therefore B is strongly irreducible ideal of M. □

Theorem 3.10. *Let M be a regular Γ-semiring and $B\Gamma B = B$), for all bi-interior idealB of M. Then any bi-interior ideal B of M is strongly irreducible bi-interior ideal if and only if B is strongly prime bi-interior ideal.*

Proof. Let M be a regular Γ-semiring and $B\Gamma B = B$, for any bi-interior ideal B of M.
Suppose B is a strongly irreducible bi-interior ideal of M and

$(B_1\Gamma B_2)\cap(B_2\Gamma B_1)\subseteq$, where B_1, B_2 are bi-interior ideal of M.

$$\begin{aligned}&(B_1\Gamma B_2)\cap(B_2\Gamma B_1)=B_1\cap B_2\\ \Rightarrow &B_1\cap B_2\subseteq B\\ \Rightarrow &B_1\subseteq B \text{ or } B_2\subseteq B\end{aligned}$$

Thus B is a strongly prime bi-interior ideals of M.

Conversely suppose B is a strongly prime bi-interior ideals of M.

$$\begin{aligned}(B_1\Gamma B_2)\cap(B_2\Gamma B_1)&=(B_1\Gamma B_2)\cap(B_2\Gamma B_1)\Gamma(B_1\Gamma B_2)\Gamma(B_2\Gamma B_1)\\ &\subseteq B_1\Gamma M\Gamma B_1\cap M\Gamma B_1\Gamma M\\ &\subseteq B_1\end{aligned}$$

Similarly we can prove that $(B_1\Gamma B_2)\cap(B_2\Gamma B_1)\subseteq B_2$.
Therefore $(B_1\Gamma B_2)\cap(B_2\Gamma B_1)\subseteq B_1\cap B_2$.
Hence $(B_1\Gamma B_2)\cap(B_2\Gamma B_1)=B_1\cap B_2$. □

Theorem 3.11. *If $(B_1\Gamma B_2)\cap(B_2\Gamma B_1)=B_1\cap B_2$ then every bi-interior ideal is semi prime.*

Proof. Let B be any bi-interior ideal of M and $B_1\Gamma B_1\subseteq B$, B_1 is a bi-interior of M. Then

$$\begin{aligned}B_1=&B_1\cap B_1\\ =&(B_1\Gamma B_1)\cap(B_1\Gamma B_1)\\ =&B\cap B\\ =&B.\end{aligned}$$

Hence every bi-interior ideal of M is semi prime. □

Theorem 3.12. *Every bi-interior ideal of a Γ-semiring M is a strongly prime bi-interior ideal if and only if M is a regular and the set of bi-interior ideals of a Γ-semiring M is a totally ordered under the inclusion of sets.*

Proof. Suppose every bi-interior ideal of the Γ-semiring M is a strongly prime bi-interior ideal. every bi-interior ideal semi prime.
M is regular.

Let $B_1 and B_2$ be two bi-interior ideals of Γ-semiring M is from the set of bi-interior ideals of M.

$B_1 \cap B_2$ is bi-interior ideal and strongly prime bi-interior ideal of M.

$$\begin{aligned}&(B_1\Gamma B_2) \cap (B_2\Gamma B_1) = B_1 \cap B_2\\ \Rightarrow &B_1 \subseteq B_1 \cap B_2 \text{ or } B_2) \subseteq B_1 \cap B_2\\ \Rightarrow &B_1 \subseteq B_2 \text{ or } B_2 \subseteq B_1.\end{aligned}$$

Therefore the set of bi-interior ideal M is a totally ordered under the inclusion of sets.

Conversely suppose that the set of bi-interior ideal of M is regular and totally ordered under the set inclusion.
Let $(B_1\Gamma B_2) \cap (B_2\Gamma B_1) \subseteq B$, where B_1, B_2 are bi-interior ideal of M.

$$\begin{aligned}&(B_1\Gamma B_2) \cap (B_2\Gamma B_1) = B_1 \cap B_2\\ \Rightarrow &B_1 \cap B_2 \subseteq B_1 \quad\text{or}\quad B_1 \cap B_2 \subseteq B_2\\ \Rightarrow &B_1 \subseteq B_2 \quad\text{or}\quad B_2 \subseteq B_1.\end{aligned}$$

By assume set of bi-interior ideal of M is totally ordered under the set inclusion.
Then $B_1 \cap B_2 \subseteq B \Rightarrow B_2 \subseteq B_1$ or $B_1 \subseteq B_2$.
Therefore $B_1 \subseteq B$ or $B_2 \subseteq B$. Hence B is a strongly prime bi-interior ideal of M. □

Theorem 3.13. *If $B\Gamma B = B$, for all bi-interior ideals of a Γ-semiring M then $B_1 \cap B_2 = (B_1\Gamma B_2)\Gamma(B_2\Gamma B_1)$, for any bi-interior ideals $B_1 and B_2$ of M and M is a regular.*

Proof. Suppose $B\Gamma B = B$, for all bi-interior ideal B of M. Let R be a right ideal and L be a left ideal of M. Then $R \cap L$ is a bi-interior ideal of M. Therefore

$$\begin{aligned}&(R \cap L)\Gamma(R \cap L)) = (R \cap L)\\ \Rightarrow &(R \cap L \subseteq (R\Gamma L)\end{aligned}$$

and we have $(R\Gamma L \subseteq (R \cap L)$. Therefore $(R \cap L = (R\Gamma L)$.

Hence by standard results M is a regular Γ-semiring.
Let B_1 and B_2 be a bi-interior ideals of M. Then $B_1 \cap B_2$ is a bi-interior ideal of M

$$\begin{aligned}B_1 \cap B_2 = (B_1 \cap B_2) &= (B_1 \cap B_2)\Gamma(B_1\Gamma B_2)\\ &\subseteq (B_1\Gamma B_2).\end{aligned}$$

Similarly we can prove $(B_1 \cap B_2) \subseteq B_2 \Gamma B_1$.
Hence $B_1 \cap B_2) \subseteq (B_1 \Gamma B_2) \cap ((B_2 \Gamma B_1)$. □

Theorem 3.14. *If B is a bi-interior ideal of M and $a \in M$ such that $a \notin B$ then there exists an irreducible bi-interior ideal I of M such that $B \subseteq I$ and $a \in I$.*

Theorem 3.15. *Any proper bi-interior ideal B of M is the intersection of all bi-interior ideals M containing B.*

Proof. Let B be a bi-interior ideal of M and $\{B_i / i \in \wedge\}$ be the collection of irreducible ideals containing B. Then $B \subseteq \bigcap\limits_{i \in \wedge} B_i$.

Suppose that $a \notin B$. there exists an irreducible bi-interior ideal I such that $B \subseteq I$ or $a \notin I and a \in I$. Then

$$a \notin \bigcap_{i \in \wedge} B_i$$
$$\Rightarrow \bigcap_{i \in \wedge} B_i \in I.$$

Hence $I = \bigcap\limits_{i \in \wedge} B_i$. □

Theorem 3.16. *Following statements are equivalents in a $\Gamma-$semiring M :*

(1). *The set of bi-interior ideals of M is totally ordered set under inclusion of sets.*
(2). *Each bi-interior ideal of M is strongly irreducible.*
(3). *Each bi-interior ideal of M is irreducible.*

Proof. Let M be a $\Gamma-$semiring.

$(1) \Rightarrow (2)$: Suppose that the set of bi-interior ideals of M is a totally ordered set under inclusion of sets.

Let B be any bi-interior ideal of M. To show that B is a strongly irreducible bi-interior ideal of M. Let B_1 and B_2 be any two bi-interior ideals of M such that $B_1 \cap B_2 \subseteq B$. But by the hypothesis we have either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$. Therefore $B_1 \cap B_2 = B_1$ or $B_1 \cap B_2 = B_2$.

Hence $B_1 \subseteq B$ or $B_2 \subseteq B$.

Thus B is a strongly irreducible bi-interior ideal of M.

$(2) \Rightarrow (3)$: Suppose that each bi-interior ideal of M is strongly irreducible. Let B be any bi-interior ideal of M such that $B = B_1 \cap B_2$, for any bi-interior ideals B_1 and B_2 of M.

Hence by (2), we have $B_1 \subseteq B$ or $B_2 \subseteq B$.
As $B \subseteq B_1$ and $B \subseteq B_2$, we have $B_1 = B$ or $B_2 = B$.
Hence B is an irreducible bi-interior ideal of M.

$(3) \Rightarrow (1)$: Suppose that each bi- interior ideal of M is an irreducible bi-ideal.
Let B_1 and B_2 be any two bi- interior ideals of M.
Then $B_1 \cap B_2$ is also a bi- interior ideals of M, (from remark).
Hence $B_1 \cap B_2 = B_1 \cap B_2$ implies $B_1 \cap B_2 = B_1$ or $B_1 \cap B_2 = B_2$, by our assumption.
Therefore either $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$.
This shows that the set of bi-interior ideals of M is a totally ordered set under inclusion of sets.

□

4. Conclusion

The algebraic structures play a prominent role in mathematics with wide range of applications. Generalization of ideals of algebraic structures and ordered algebraic structure plays a very remarkable role and also necessary for further advance studies and applications of various algebraic structures. The author, introduced and studied (weak interior, tri, bi-interior, bi-quasi, quasi-interior and bi-quasi interior) ideals of $\Gamma-$semirings, $\Gamma-$semigroups, semigroups and semirings as a generalization of bi-ideal, quasi ideal and interior ideal of algebraic structures and charecterized regular algebraic structures as well as simple algebraic structures using these ideals. In this paper, we introduced the notion of prime bi-interior ideal, semiprime biinterior ideal, irreducible bi-interior ideal and strongly prime bi-interior ideal of $\Gamma-$ semiring and studied these ideals properties, relations between them and also characterized regular $\Gamma-$semiring and $\Gamma-$semiring using prime bi-interior ideals.

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Marapureddy Murali Krishna Rao
Department of Mathematics, Sankethika Engineering College , Visakhapatnam, India
Email: mmarapureddy@gmail.com