Journal of Operation and Automation in Power Engineering

Vol. 10, No. 3, Dec. 2022, Pages: 214-218

http://joape.uma.ac.ir

Reactive Power Scheduling Using Quadratic Convex Relaxation

E. Limouzadeh, A. Rabiee[*]

Faculty of Engineering and Technology, Shahrekord University (SKU), Shahrekord, Iran.

Abstract- *In this paper, quadratic convex relaxation (QCR) is used to relax AC optimal power flow (AC-OPF) used for reactive power scheduling (RPS) of power system. The objective function is system active power losses minimization to optimally determine the tap position of tap-changers, reactive power output of generating units, synchronous condensers, shunt capacitor banks and reactors. The nonlinear and non-convex terms due to trigonometric functions causes the problem to be non-convex which results in trapping in local minimum or even not converging in large size power systems. Therefore, in this paper the nonlinear terms and trigonometric function are relaxed by linear and quadratic functions. Furthermore, the product of two variables and multi-variables are relaxed by McCormick bilinear and multi-linear expressions, converting the AC-OPF of RPS to quadratic constraint programming (QCP) optimization problem. The proposed RPS method is studied based on IEEE RTS 24-bus test system. The results show the accuracy of the proposed (QCR) method to relax AC-OPF optimization problem of RPS.*

Keyword: AC optima power flow, McCormic relaxation, quadratic convex relaxation, reactive power scheduling, tap-changer ratio

1. INTRODUCTION

Reactive power has an important role in security and stability of network voltage so that insufficient reactive power is reported as the main reason for blackouts [1]–[3].

In power system studies, some research works are devoted to RPS. In [4], the reactive output of synchronous generators and condenser is rescheduled to improve system voltage stability margin. A fuzzy RPS is proposed in [5] to improve system voltage security and reduce system losses. A robust OPF problem is proposed for scheduling of reactive in which reactive power capability limit is considered in [6] in which the coupling of active and reactive power is taken by using the concept of power factor. In [7], a zonal congestion management method is proposed wherein the zones are determined based on the sensitivity of real and reactive power flow of transmission lines respect to re-scheduling of active and reactive power called real and reactive transmission congestion distribution factors.

A voltage control and short and very short term RPS is presented in the form of tri-level scheme [8]. Regarding the fact that insufficient reactive would lead to limit power transfer, a successive fuzzy multi-objective RPS is proposed [9]. The CIGRE models of synchronous generators units is used for contingency scheduling of reactive power based on the capability curves and the reactive power margin of automatic voltage regulator (AVR) in [10].

Received: 21 Jul. 2021

Revised: 9 Oct. 2021

Accepted: 31 Oct. 2021

*Corresponding author: Abdorreza Rabiee

E-mail: rabiee@sku.ac.ir (Abdorreza Rabiee)

DOI: 10.22098/joape.2022.9252.1645

Research Paper

In [11] a hierarchical optimization strategy is proposed for energy and RPS problem in a photovoltaic-battery microgrid cluster (MGC) operating autonomously, based on decentralized control architecture in multi-microgrids (MMGs). A robust active and reactive power management method is proposed for distribution networks including electric vehicles (EVs) in which energy cost and the voltage deviation are simultaneously minimized [12]. A multi-objective optimal reactive power dispatch (ORPD) method is proposed for power systems including wind farm (WF) [13]. In the context of electricity market, a reactive power dispatch model is presented in which both technical and economic aspects related to reactive power dispatch in competitive electricity markets are considered [14].

The RPS in the electricity market is studied in [15] – [17] wherein technical and economic concerns related to reactive power is considered in the form of single objective and multi-objective reactive power market. Total payment to the generators for reactive power compensation, voltage deviation and overload indexes are minimized while system voltage stability margin (VSM) is maximized.

The work in [18] indicates the importance of dynamic reactive power support of wind power plants in a wind power dominated power system.

Owning to trigonometric terms in AC-OPF of RPS, it is a non-convex non-linear optimization problem that may tarp in local minima. It is so dependent to initial value and rarely reach to the global optimum, especially in large size system. For this reason research works are devoted to change the nonlinear non-convex optimization problem to a convex linear one, e.g. [13]. Another method is to use quadratic convex relaxations for mixed-integer nonlinear programing problems [19]. Inspired by

[19], the RPS problem is relaxed by quadratic convex method, make the nonlinear non-convex OPF of RPS problem to quadratic convex one, resulting in tractable and scalable optimization problem that is converged even for large size power systems.

The contribution of this paper are summarized as follows:

1. The QCR method proposed in [19] is used to relax AC-OPF of RPS problem.
2. McCormick and double McCormic relaxations are used to relax bilinear and tri-linear terms, respectively.

In the second section of this paper the QCR for AC-OPF RPS problem solution is taken in the third section. The IEEE RTS 24 bus test system is used as case study and the results are discussed. Conclusions are included in the last section.

2. AC-OPF FORMULATION FOR RPS

The RPS can be formulated in the form of an AC optimal power flow (AC-OPF) problem with the objective function of minimizing system active power losses as follows:

$$Obj\,Function:\ \min\left\{P_{Losses}=\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(p_{ij}+p_{ji}\right)\right. \quad (1)$$

Subjected to:

$$P_{gi}-P_{di}=\sum_{j=1,\,j\neq i}^{n}p_{ij} \quad (2)$$

$$Q_{gi}-Q_{di}+Q_i^{Comp}=\sum_{j=1,\,j\neq i}^{n}q_{ij} \quad (3)$$

$$p_{ij}=G_{ij}V_i^2-G_{ij}V_iV_j\cos(\theta_{ij})-B_{ij}V_iV_j\sin(\theta_{ij}) \quad (4)$$

$$q_{ij}=\frac{y_C}{2}V_i^2-B_{ij}V_i^2+B_{ij}V_iV_j\cos(\theta_{ij})-G_{ij}V_iV_j\sin(\theta_{ij}) \quad (5)$$

Branch flow equations of transformers with tap-changer α (as shown in Fig. 1):

$$p_{ij}=\alpha_{ji}^2G_{ij}V_i^2-\alpha_{ji}G_{ij}V_iV_j\cos(\theta_{ij})-\alpha_{ji}B_{ij}V_iV_j\sin(\theta_{ij}) \quad (6)$$

$$q_{ij}=-\alpha_{ji}^2B_{ij}V_i^2+\alpha_{ji}B_{ij}V_iV_j\cos(\theta_{ij})-\alpha_{ji}G_{ij}V_iV_j\sin(\theta_{ij}) \quad (7)$$

$$p_{ji}=G_{ji}V_j^2-\alpha_{ji}G_{ji}V_iV_j\cos(\theta_{ji})-\alpha_{ji}B_{ji}V_iV_j\sin(\theta_{ji}) \quad (8)$$

$$q_{ji}=-B_{ji}V_j^2+\alpha_{ji}B_{ji}V_iV_j\cos(\theta_{ji})-\alpha_{ji}G_{ji}V_iV_j\sin(\theta_{ji}) \quad (9)$$

$$Tap_{ji}^{\min}\leq\alpha_{ji}\leq Tap_{ji}^{\max} \quad (10)$$

$$V_i^{\min}\leq V_i\leq V_i^{\max} \quad (11)$$

$$\theta_{ij}^l\leq\theta_{ij}\leq\theta_{ij}^u \quad (12)$$

$$p_{ij}^2+q_{ij}^2\leq S_{\mathrm{ij,max}}^2 \quad (13)$$

$$Q_{gi}^{\min}\leq Q_{gi}\leq Q_{gi}^{\max} \quad (14)$$

$$P_{gi}^{\min}\leq P_{gi}\leq P_{gi}^{\max} \quad (15)$$

In Eq. (1), the system losses is formulated as objective function to be minimized. Eqs. (2)-(3) are nodal active and reactive power balance equations. Eqs. (4)-(5) are active and reactive power flow of transmission lines, while Eqs. (6)-(9) are related to active/reactive power flow of transformer(s) connected to bus *i* and *j* of network. Equation (10) determines the tap-changer limit to its minimum and maximum values as shown in Fig. 1. Eqs. (11)-(13) are security limit of network and Eqs. (14)-(15) are synchronous generator technical limits. Equation Eqs. (1)-(15) is a non-convex AC-OPF formulation for RPS problem.

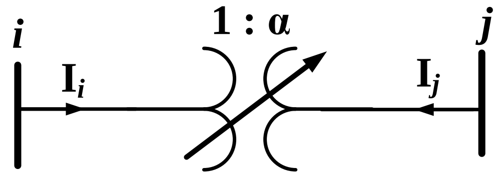

Fig. 1. Diagram of transformer with tap-changer.

3. QCR METHOD FOR RELAXATION OF AC-OPF

All nonlinear terms in equations (1) to (16) are relaxed as follows [19]:

Quadratic terms:

$$\hat{V}_i^2\geq V_i^2 \quad (16)$$

$$\hat{V}_i^2\leq\left(V_i^u+V_i^l\right)V_i-V_i^uV_i^l \quad (17)$$

Trigonometric terms:

$$\hat{\cos}(\theta_{ij})\geq\cos(\theta_{ij}^u) \quad (18)$$

$$\hat{\cos}(\theta_{ij})\leq 1-\frac{1}{1-\cos^2\theta_{ij}^u}\theta_{ij}^2 \quad (19)$$

$$\hat{\sin}(\theta_{ij})\geq\cos\left(\frac{\theta_{ij}^u}{2}\right)\left(\theta_{ij}+\frac{\theta_{ij}^u}{2}\right)-\sin\left(\frac{\theta_{ij}^u}{2}\right) \quad (20)$$

$$\hat{\sin}(\theta_{ij})\leq\cos\left(\frac{\theta_{ij}^u}{2}\right)\left(\theta_{ij}-\frac{\theta_{ij}^u}{2}\right)+\sin\left(\frac{\theta_{ij}^u}{2}\right) \quad (21)$$

McCormic relaxation for the multiplication of two variables xy [20]:

$$\hat{xy} = < x, y >^M \quad (22)$$

$$x \in [x^l, x^u] \Rightarrow (x - x^l) \geq 0 \ , \ (x - x^u) \leq 0$$
$$y \in [y^l, y^u] \Rightarrow (y - y^l) \geq 0 \ , \ (y - y^u) \leq 0$$
$$\Rightarrow \begin{cases} (x - x^l)(y - y^l) \geq 0 \\ (x - x^l)(y - y^u) \leq 0 \\ (x - x^u)(y - y^l) \leq 0 \\ (x - x^u)(y - y^u) \geq 0 \end{cases} \quad (23)$$
$$\Rightarrow \begin{cases} \text{xy} \geq x^l \text{ y} + y^l x - x^l y^l \\ \text{xy} \leq x^l \text{ y} + y^u x - x^l y^u \\ \text{xy} \leq x^u \text{ y} + y^l x - x^u y^l \\ \text{xy} \geq x^u \text{ y} + y^u x - x^u y^u \end{cases}$$

$$\begin{aligned} \hat{\text{xy}} &\geq x^l \text{ y} + y^l x - x^l y^l \\ \hat{\text{xy}} &\leq x^l \text{ y} + y^u x - x^l y^u \\ \hat{\text{xy}} &\leq x^u \text{ y} + y^l x - x^u y^l \\ \hat{\text{xy}} &\geq x^u \text{ y} + y^u x - x^u y^u \end{aligned} \quad (24)$$

$$\hat{V_i V_j} = < V_i , V_j >^M \quad (25)$$

$$WCos_{ij} = < \hat{V_i V_j} , \hat{\cos(\theta_{ij})} >^M \quad (26)$$

$$WSin_{ij} = < \hat{V_i V_j} , \hat{\sin(\theta_{ij})} >^M \quad (27)$$

Therefore, the proposed convex quadratic relaxation of AC-OPF problem for RPS can be written as:

Minimize (1)

Subjected to:

(11) - (12), (13), (14), (15), (16), (17)

Branch flow equations:

$$p_{ij} = G_{ij} \hat{V}_i^2 - G_{ij} \, WCos_{ij} - B_{ij} \, WSin_{ij} \quad (28)$$

$$q_{ij} = \frac{y_C}{2} \hat{V}_i^2 - B_{ij} \hat{V}_i^2 + B_{ij} \, WCos_{ij} - G_{ij} \, WSin_{ij} \quad (29)$$

Flow equations of transformers with tap-changer:

$$p_{ij} = G_{ij} < \hat{\alpha}_{ji}^2 , \hat{V}_i^2 >^M - G_{ij} < \alpha_{ji} , WCos_{ij} >^M - B_{ij} < \alpha_{ji} , WSin_{ij} >^M \quad (30)$$

$$q_{ij} = - B_{ij} < \hat{\alpha}_{ji}^2 , \hat{V}_i^2 >^M + B_{ij} < \alpha_{ji} , WCos_{ij} >^M - G_{ij} < \alpha_{ji} , WSin_{ij} >^M \quad (31)$$

$$p_{ji} = G_{ji} \hat{V}_j^2 - G_{ji} < \alpha_{ji} , WCos_{ji} >^M - B_{ji} < \alpha_{ji} , WSin_{ji} > \quad (32)$$

$$q_{ji} = - B_{ji} \hat{V}_j^2 + B_{ji} < \alpha_{ji} , WCos_{ij} > - G_{ji} < \alpha_{ji} , WSin_{ji} > \quad (33)$$

Current magnitude constraint [19]:

$$p_{ij}^2 + q_{ij}^2 \leq \hat{V}_i^2 \, l_{ij} \quad (34)$$

$$r_{ij} \, l_{ij} = p_{ij} + p_{ji} \quad (35)$$

The proposed method in [22] is modified by including the following equations (36)-(41):

$$\theta_{ji} = - \theta_{ij} \quad (36)$$

$$\hat{\sin}(\theta_{ji}) = - \hat{\sin}(\theta_{ij}) \quad (37)$$

$$\hat{\cos}(\theta_{ji}) = \hat{\cos}(\theta_{ij}) \quad (38)$$

$$V_i \hat{V}_j = V_j \hat{V}_i \quad (39)$$

$$WCos_{ij} = WCos_{ji} \quad (40)$$

$$WSin_{ij} = - WSin_{ji} \quad (41)$$

Equations (36)-(41) are added to the model proposed in [22], to reach to a solution that cope with the KVL and KCL laws. For example, if equation (36) is not considered, θ_{ij} and θ_{ji} are determined based on equations (19) and (20) and θ_{ji} is not necessarily equals to $-\theta_{ij}$ that results in reaching to a solution without physical interpretation. This matter is observed even in small size system with only three buses. According to (20) and (21), any value between these two curves can be considered as approximate relaxation of $\sin(\theta_{ij})$. Therefore, it is possible that $\sin(\theta_{ij})$ is approximated by a value that is not exactly equal to $-\sin(\theta_{ji})$ and accordingly θ_{ij} is not equal to $-\theta_{ji}$. The same justification can be considered for equation (38) related to $\cos(\theta_{ij})$ and also for equations (39)-(41).

4. CASE STUDY

The proposed method is studied based on IEEE RTS 24-bus [21]. This system is shown in Fig. 2 which includes five transformers with tap-changer. The minimum and maximum value of tap position is 0.9 and 1.1 ($a \in [0.9, 1.1]$). The AC-OPF problem is nonlinear programming (NLP) problem which is solved by CONOPT solver in GAMS. The proposed method is quadratic-constrained programming (QCP) problem solved by CPLEX solver in GAMS [22]. The results of the proposed QCR method for RPS is compared with those of AC-OPF method.

As shown in Table 1, the results in QCR method are close to those of AC-OPF method. The objective function (active power losses) in QCR method is only 0.17 MW is greater than that of AC-OPF, showing the accuracy of the proposed relaxation method. This negligible error is only 0.6 percent of active power losses in AC-OPF method (27.79 MW).

Also, the reactive power losses of QCR method is only 3.7 MVar greater than that of AC-OPF one, which is 0.7 percent of reactive power losses in AC-OPF method (i.e. 513.30 MVar), validating the effectiveness of QCR relaxation method. It is noted in QCR method, since the main non-convex AC-OPF problem is relaxed to quadratic convex one, the obtained result of QCR method is optimal solution. However, in AC-OPF, there is no grantee that the obtained result is optimal solution.

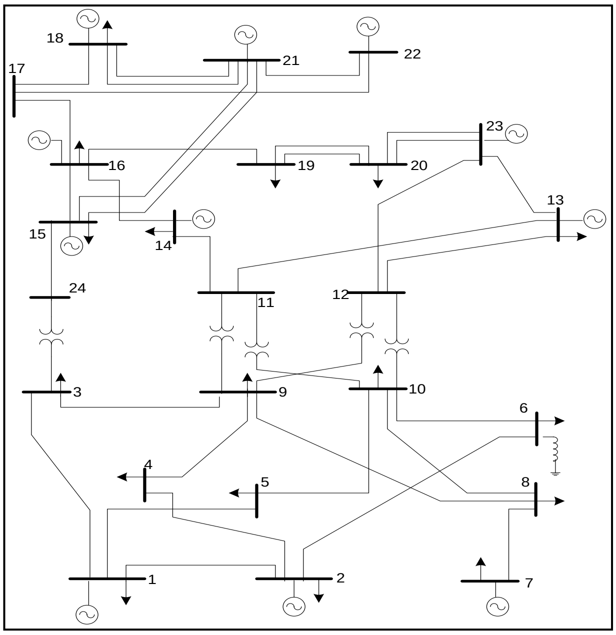

Fig. 2. IEEE RTS 24-bus test system.

Table 1. The results of RPS solution for AC-OPF and the proposed QCR method.

	AC OPF method	The proposed QCR method
Total generation (MW)	2877.79	2877.96
Total active power Load (MW)	2850	2850
Total reactive power Load (MVar)	580	580
Active power losses (Objective Function) (MW)	27.79	27.96
Reactive power losses (MVar)	513.30	516.70

In Table 2, the tap position of five transformers are reported. From Table 2, it can be seen that the obtained tap position in the QCR method is approximately similar to those of AC-OPF method, again justifying the applicability of the proposed QCR method to relax AC-OPF of RPS non-convex optimization problem.

Table 2. Obtained Tap changer value of ULTP transformers.

Transformer Number	from bus	to bus	Tap changer value (a)	
			AC OPF	The proposed QCR
1	3	24	0.96	0.96
2	9	11	0.95	0.94
3	9	12	0.97	0.97
4	10	11	0.94	0.93
5	10	12	0.96	0.97

5. CONCLUSIONS

In this paper QCR method is used to relax the non-convex AC-OPF optimization problem for RPS. All the equations in the objective function and equality and inequality constraints are converted to linear and quadratic equations to make the problem convex. The results show the effectiveness and accuracy of QCR method for relaxation of AC-OPF problem. Although the solution obtained by QCR is not exact but it is so close to exact result. Furthermore, the QCR solution is optimal. The main advantage to the proposed QCR is that it is converged in large size power systems including thousands of buses. Also the QCR method can be more improved by tightening the boundaries of variables, especially those of phase angles. These modification are considered as future work.

REFERENCES

[1] J. Wang, R.G. Wen and R. S. Yang, "On the procurement and pricing of reactive power service in the electricity market environment," *IEEE PES Gen. Meet.*, vol. 1, pp. 1120-1124, 2004.

[2] J. Zhong and K. Bhattacharya, "Reactive power management in deregulated power systems-A Review," *IEEE PES Wint. Meet.*, vol. 2, pp. 1287-1292, 2002.

[3] K. L. Lo and Y. A. Alturki, "Toward reactive power markets. Part 1: reactive power allocation", *IET Gener., Transm. Distrib.*, vol. 153, no. 1, pp. 59-70, 2006.

[4] T.V. Menezes, L.C.P. da Silva and V.F. da Costa, "Dynamic VAr sources scheduling for improving voltage stability margin," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 969 – 971, 2003.

[5] Ch.-T. Su and Ch.-T. Lin, "Fuzzy-based voltage/reactive power scheduling for voltage security improvement and loss reduction," *IEEE Trans. Power Deliv.*, vol. 16, no. 2, pp. 319-323, 2001.

[6] V. Sarkar and S.A. Khaparde, "Reactive Power Constrained OPF Scheduling With 2-D Locational Marginal Pricing," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 503-512, 2013.

[7] A. Kumar, S.C. Srivastava and S.N. Singh, "A Zonal Congestion Management Approach Using Real and Reactive Power Rescheduling," *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 554-562, 2004.

[8] S. Corsi, P. Marannino, N. Losignore, G. Moreschini and G. Piccini, "Coordination between the reactive power scheduling function and the hierarchical voltage control of the EHV ENEL system," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 686-694, 1995.

[9] B. Venkatesh, G. Sadasivam and M.A. Khan, "A new optimal reactive power scheduling method for loss minimization and voltage stability margin maximization using successive multi-objective fuzzy LP technique," *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 844-851, 2000.

[10] R.D. Youssef, "Implicit generator and SVC modelling for contingency scheduling of reactive

power dispatch," *IET Gener. Transm. Distribut*., vol. 142, no. 5, pp. 527-534, 1995.

[11] M. V. Castro, C. Moreira and L. M. Carvalho, "Hierarchical optimisation strategy for energy scheduling and volt/var control in autonomous clusters of microgrids," *ET Renew. Power Gener*., vol. 14, no. 1, pp. 27-38, 2020.

[12] S. Pirouzi, J. Aghaei, M. A. Latify, G. R. Yousefi and G. Mokryani, "A Robust Optimization Approach for Active and Reactive Power Management in Smart Distribution Networks Using Electric Vehicles," *IEEE Syst J*., vol. 12, no. 3, pp. 2699-2710, 2018.

[13] M. Mahzouni-Sani, A. Hamidi, D. Nazarpour and S. Golshannavaz, "Multi-objective linearised optimal reactive power dispatch of wind-integrated transmission networks," *IET Gener. Transm. Distrib*., vol. 13, no. 13, pp. 2686-2696, 2019.

[14] C.A. Cañizares, K. Bhattacharya, I. El-Samahy, H. Haghighat, J. Pan and C. Tang, "Re-defining the reactive power dispatch problem in the context of competitive electricity markets," *IET Gener. Transm. Distrib.*, vol. 4, no. 2, pp. 162-177, 2010.

[15] A. Rabiee, H. A. Shayanfar and N. Amjady, "Reactive power pricing – problems and a proposal for a competitive market," *IEEE Power Energy Magaz*., pp. 19-32, 2009.

[16] N. Amjady, A. Rabiee and H. A. Shayanfar, "Pay-as-bid based reactive power market," *Energy Convers. Manage.*, vol. 51, no. 2, pp. 367-81, 2010.

[17] A. Rabiee, H. A. Shayanfar and N. Amjady, "Coupled energy and reactive power market clearing considering power system security," *Energy Convers. Manage.*, vol. 50, no. 4, pp. 907-15, 2009.

[18] Z. H. Rather, Z. Chen, P. Thogersen and P. Lund, "Dynamic Reactive Power Compensation of Large-Scale Wind Integrated Power System," *IEEE Trans. Power Syst*., vol. 30, no. 5, pp. 2516-2526, 2015.

[19] H. Hijazi, C. Coffrin and P. V. Hentenryck, "Convex quadratic relaxations for mixed-integer nonlinear programs in power systems," *Math. Program. Comput.*, vol. 9, pp. 321-367, 2017.

[20] G. McCormick, "Computability of global solutions to factorable nonconvex programs: part i—convex underestimating problems," *Math. Program*., vol. 10, pp. 146–175, 1976.

[21] Reliability Test System Task Force, "The IEEE Reliability Test System–1996," *IEEE Trans. Power Syst*., vol. 14, no. 3, pp. 1010-1020, 1999.

[22] Generalized Algebraic Modeling Systems (GAMS), [Online] Available: http://www.gams.com.